Chapter 3 DYNAMICS

Content

- Newton's laws of motion
- Linear momentum and its conservation

Learning Outcomes

Candidates should be able to:

(a) state and apply each of Newton's laws of motion.

(b) show an understanding that mass is the property of a body which resists change in motion (inertia).

(c) describe and use the concept of weight as the force experienced by a mass in a gravitational field.

(d) define and use linear momentum as the product of mass and velocity.

(e) define and use impulse as the product of force and time of impact.

(f) relate resultant force to the rate of change of momentum.

(g) recall and solve problems using the relationship $F = ma$, appreciating that resultant force and acceleration are always in the same direction.

(h) state the principle of conservation of momentum.

(i) apply the principle of conservation of momentum to solve simple problems including inelastic and (perfectly) elastic interactions between two bodies in one dimension. (Knowledge of the concept of coefficient of restitution is not required.)

(j) show an understanding that, for a (perfectly) elastic collision between two bodies, the relative speed of approach is equal to the relative speed of separation.

(k) show an understanding that, whilst the momentum of a closed system is always conserved in interactions between bodies, some change in kinetic energy usually takes place.

3 Introduction

Dynamics is the study of the motion of a body. In this chapter, we will study the motion of objects by analysing the forces acting on them.

As early as 1600s, Galileo Galilei performed experiments that convinced him that a moving body needs no push to keep it moving. However, the body would need a force to change its motion (i.e. a change in speed and/or direction).

Hence, the essence of dynamics can be stated simply as

Force causes a *change* in the state of motion

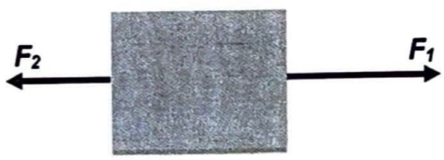
3.1 Newton's Laws of Motion

3.1.1 Force

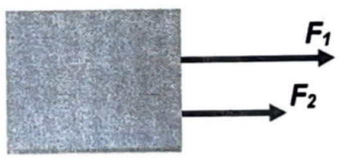
Net force

A force is any action that causes a change in the physical shape or the state of motion of a body.

If we push or pull a body, we are exerting a force on it. Forces are vectors. If two or more forces are acting on a body, the **resultant** (or net) force is the **vector sum** of all the forces acting on the body.



For 2 forces acting in **opposite** directions, the resultant force would be in the direction of the larger force and its magnitude would be equal to the *difference* of the magnitudes of the two forces.



For 2 forces acting in the **same** direction, the resultant force would be in the same direction as the two forces, and its magnitude would be the *sum* of the magnitudes of the two forces.

The S.I. unit for force is the **newton (N)**.

3.1.2 Newton's First Law of Motion

Newton's First Law of Motion

Newton's First Law of motion states that

> **A body continues in its state of rest or uniform motion in a straight line, unless a resultant external force acts on it**

The first law implies that
1. the **state of rest** requires no resultant force to maintain.
2. the **state of uniform velocity** also requires no resultant force to maintain.

The first law gives rise to the concept of *force*, whereby *force* is a quantity that can cause a body to undergo a change in velocity (or momentum).

Inertia and Mass

Newton's 1st Law of motion also expresses the concept of **inertia**.

The inertia of a body can be described as its **reluctance to start moving**, or **to change its motion** once it has started.

The **mass** m of a body is the intrinsic property of a body which resists change in motion, i.e. it is a measure of the inertia of a body. The same force, when applied to a body of larger mass will cause a smaller change in motion.

Weight

The **weight** W of a body is the force experienced by a mass in a gravitational field. It is an extrinsic property as it depends not just on the mass of the body, but also the gravitational field strength at the point where the body is.

$$W = mg$$

where m is the mass of the object measured in kilograms (kg) and
g = 9.81 m s–2 is the acceleration of free fall near the surface of the Earth.

It is important not to confuse weight with mass.

Equilibrium

When the state of a body remains unchanged even though two or more forces are acting upon it, the body is said to be in **equilibrium**. In the chapter on Forces, the conditions necessary for equilibrium of a body are stated as:

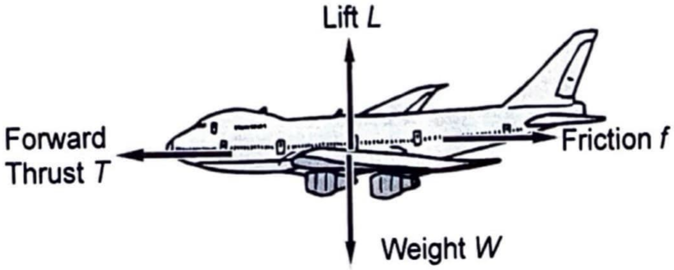
1. The **resultant force** on the body must be zero.
   (The first condition stems from Newton's First Law of motion, where the net force acting on a body must be zero in order that the translational motion of the body remains unchanged.)
2. The **resultant torque** on the body about any axis must be zero.
   (The second condition comes about if we are dealing with an extended body that can rotate. The net torque acting on the body must be zero such that rotational motion remains unchanged.)

Scenario 1: A woman standing on the floor (static equilibrium).

Since the woman is in *equilibrium*, the force N exerted by the floor on her is equal in magnitude to her weight W and opposite in direction.

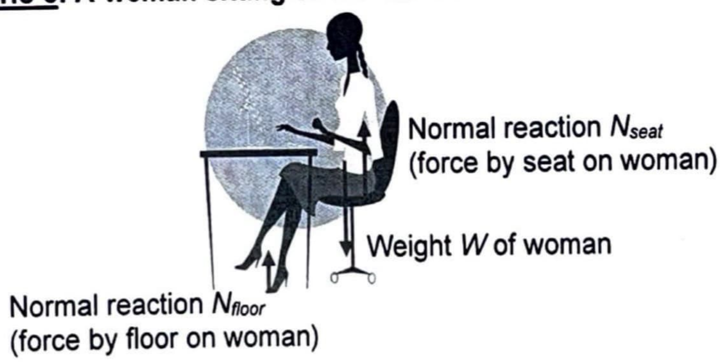
$$W = N$$

Scenario 2: An aeroplane moving horizontally with constant velocity (translational equilibrium).

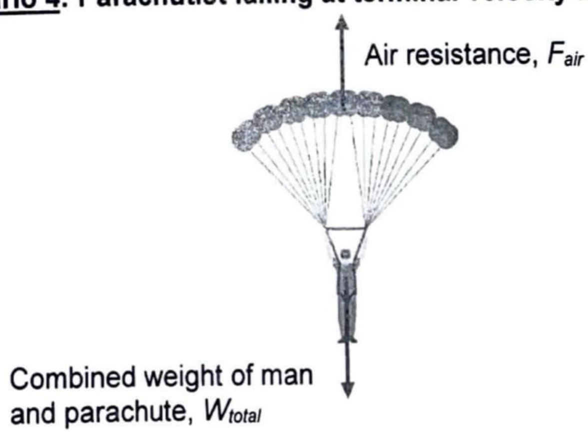


The vector sum of the forces is zero.

Scenario 3: A woman sitting on her chair.



Scenario 4: Parachutist falling at terminal velocity in mid-air.



3.1.3 Newton's Third Law of Motion

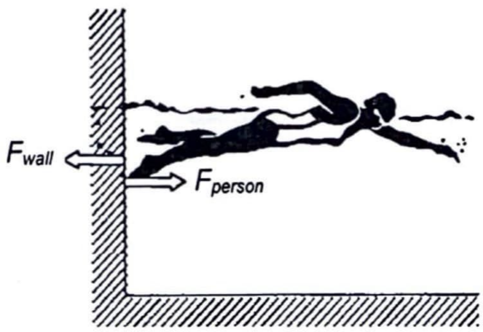
Newton's Third Law of Motion

Newton's Third Law of motion states that:

> **If a body A exerts a force on body B, then body B exerts an equal and opposite force on body A.**
>
> $$|F_{\text{by A on B}}| = |F_{\text{by B on A}}|$$
> $$|F_{AB}| = |F_{BA}|$$
> *(the 2 forces are equal in magnitude)*
>
> $$F_{AB} = -F_{BA}$$
> *(the 2 forces are opposite in direction)*

F_{AB} and F_{BA} form an *action-reaction* pair of forces.

Newton's Third Law Illustrated



The force exerted by the wall on the person (F_{person}) is equal in magnitude and opposite in direction to the force exerted by the person on the wall (F_{wall}).

The force exerted by the ground on the person (F_{person}) is equal in magnitude and opposite in direction to the force exerted by the person on the ground (F_{ground}).

Newton's Third Law clarified

Note the following concerning an action-reaction pair:

1. The two forces must be of the **same type**
2. The two forces act on **different** bodies and they therefore **do not cancel** each other out.

There are many situations where two forces are equal in magnitude but opposite in direction, but they are not action-reaction pairs. This often happens when a body is in equilibrium (e.g. Scenarios 1,2 and 4 above).

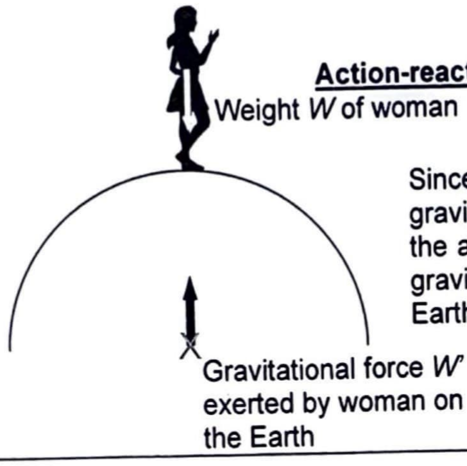
Scenario 5: A woman standing on the floor.

The forces W and N acting on the woman are equal in magnitude and opposite in direction.

However, they are not the same type of force and they both act on the same body.

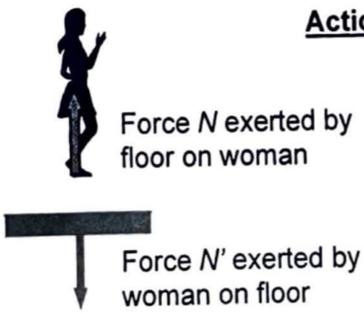
Hence, they are **not** an action-reaction pair.

<u>Action-reaction pair</u>



Since her weight W is the downward gravitational force exerted on her by the Earth, the action-reaction force to this is the upward gravitational force W' exerted by her on the Earth.

<u>Action-reaction pair</u>



Similarly, the action-reaction force to the normal force N exerted on her by the floor is the downward force N' exerted by the woman on the floor.

Quiz

Consider the following pairs of forces and state whether they constitute an action-reaction pair:

1.	The forces of repulsion between a North pole of a magnet and a North pole of a second magnet.	Y / N
2.	The forces of repulsion between an atom in the surface of a table and an atom in the surface of a book resting on a table.	Y / N
3.	The weight of an object floating in water and the buoyant force (upthrust) which the water exerts on the object.	Y / N
4.	The forces of attraction between an electron and a proton in a hydrogen atom.	Y / N
5.	The weight of a parachutist and the force which the parachute exerts on him as he descends with constant velocity.	Y / N

3.1.4 Force Diagrams

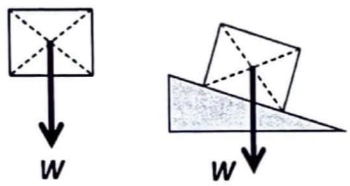
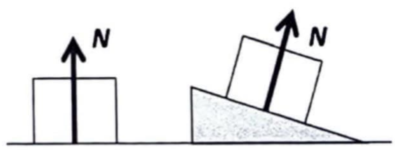
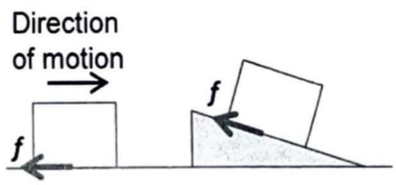
Free Body Diagrams

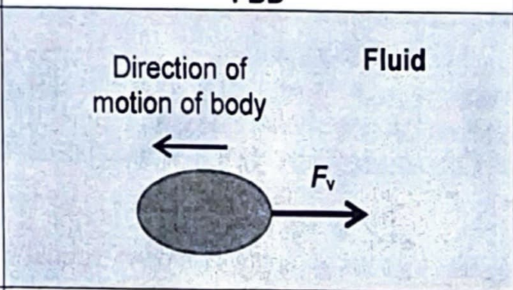
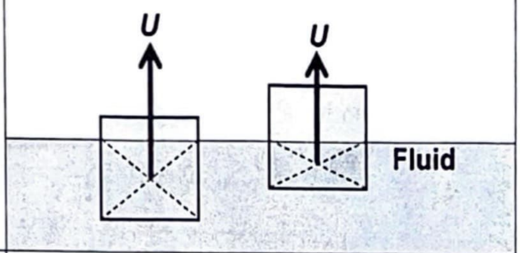
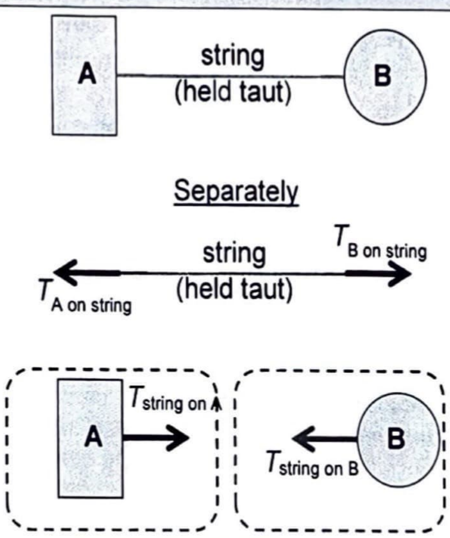
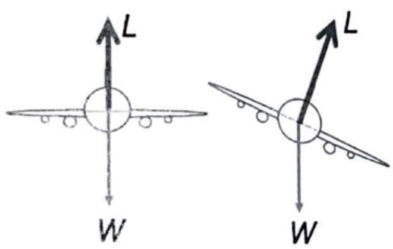
A free body diagram shows the forces acting on a body. It is the first step to solve Physics problems involving forces. Hence it is very important for you to be able to sketch these diagrams accurately.

Guidelines to Drawing Free Body Diagrams

1. Isolate the body of interest. **Identify all forces acting only on that body.**
    - DO NOT show forces that the body exerts on other bodies.
    - DO NOT show the resultant force on the body.
2. Draw and label clearly the vector representing each force
    - Ensure that the **magnitude** and **direction** of the force are clearly represented by the length and the direction of the arrowhead of the vector respectively.
    - Be mindful of where the force **originates**.

Commonly Encountered Forces

Force	FBD	To Note:
Weight, *W* • Gravitational force of attraction by Earth.		**Origin**: center of gravity **Direction**: vertically downwards
Normal contact force, *N* • Usually exists when a body is contact with another body.		**Origin**: contact surface **Direction**: perpendicular from contact surface through the body of interest *if objects are described to be <u>*just*</u> touching, it can be assumed that $N = 0$
Friction, *f* • Force resisting the relative motion or impending motion between surfaces in contact.	 *box is about to slide down the slope	**Origin**: contact surfaces **Direction**: along contact surfaces, opposite to the direction of motion or impending motion of the body of interest

Force	FBD	To Note:
Viscous force, F_v **(or Drag force, F_D)** • Exists when a body is moving relative to a liquid or gas.		**Direction**: opposite to the direction of motion * when both the body and fluid are stationary relative to each other, there is no viscous force.
Upthrust, *U* • Buoyant force provided by a liquid or gas when a body is partially or fully immersed in it.		**Origin**: centre of buoyancy (centre of immersed section of body) **Direction**: vertically upwards
Tension, *T* • Force that is transmitted through a rope, string or wire when pulled by forces acting from opposite sides. **Forces in Equilibrium**: $T_{\text{A on string}} = T_{\text{B on string}}$ **Action-reaction pairs**: $T_{\text{string on A}}$ & $T_{\text{A on string}}$ $T_{\text{string on B}}$ & $T_{\text{B on string}}$		**Origin**: where the string etc touches the body of interest **Direction**: over the length of the string etc & pulls equally on the bodies at the ends (away from the body along the line of the string etc) **Assumption:** string etc is massless and the tension *T* along the string is constant
Lift, *L* • A fluid flowing around the surface of an object exerts a force on it. Lift is the component of this force that is perpendicular to the oncoming flow direction.		**Direction**: perpendicular to the surface of the aircraft wings (or blades of a helicopter)

3.1.5 Momentum

Momentum

In ancient times, the term 'motion' of a body was quantified based on the velocity of the body. However, during the late 17th and early 18th centuries, as the science of dynamics was being established, the kinematic concept of motion was deemed unsatisfactory.

Newton proposed that the quantity of motion should arise simultaneously from the velocity and 'quantity of matter' (i.e. mass) of the body. This is what is known as the **momentum** of a body.

> The **momentum** of a body is defined as the **product of its mass, *m*, and its velocity, *v*.**
>
> $$p = mv$$

Since velocity is a vector, momentum is also a vector. Its direction follows the direction of the velocity. The S.I. unit of momentum is **kg m s^{-1} or N s.**

If we have a system of n bodies with masses m_1, m_2, e.t.c., the total momentum p of the system is the vector sum of the momenta of the individual bodies.

$$\begin{aligned} p &= p_1 + p_2 + \ldots + p_n \\ &= mv_1 + mv_2 + \ldots + mv_n \end{aligned}$$

Did you know?
The concept of momentum was actually first introduced by the French philosopher Jean Buridan (1295 – 1358), about four centuries before Issac Newton was born. Buridan described the quantity of motion as 'impetus', and that is a function of the quantity of matter and velocity of a body.

More info:

Example 1

A body of A of mass 2.0 kg is moving to the left with a speed of 5.0 m s^{-1}. A body B of mass 4.0 kg is moving to the right with a speed of 2.5 m s^{-1}.

Calculate
(a) the momentum of A,
(b) the momentum of B,
(c) the total momentum of the system of A and B.

Solution:

Taking motion to the right as positive,

(a) $p_A = m_A v_A = (2.0)(-5.0) = -10 \text{ kg m s}^{-1}$

(b) $p_B = m_B v_B = (4.0)(2.5) = 10 \text{ kg m s}^{-1}$

(c) $p_{Total} = p_A + p_B = -10 + 10 = 0.0 \text{ kg m s}^{-1}$

3.1.6 Newton's Second Law of Motion

Newton's Second Law of Motion

Newton's Second Law of motion states that:

> **The rate of change of momentum of a body is proportional to the resultant force acting on it and occurs in the direction of the force.**

$$F_{net} \propto \frac{d(p)}{dt}$$

$$\therefore F_{net} = k\frac{d(p)}{dt}$$ where k is the proportionality constant

In S.I. units, $k = 1$. Thus,

$$F_{net} = \frac{d(p)}{dt} = \frac{d(mv)}{dt} \Rightarrow F_{net} = m\frac{dv}{dt} + v\frac{dm}{dt}$$

> **One newton (N)** is defined as the force which will give a mass of 1 kg an acceleration of 1 m s^{-2} in the direction of the force.

Note:

(i) When **mass** is constant, $F_{net} = m\frac{dv}{dt} = ma$

(F_{net} and a are in the same direction)

(ii) When **velocity** is constant, $F_{net} = v\frac{dm}{dt}$

Newton's 2nd Law implies that a body with a larger momentum (compared to one with a smaller momentum) will either require

- a larger force to stop it in the same time, or
- a longer time to stop it using the same force.

3.1.7 Solving Dynamical Problems

Strategy when applying Newton's 2nd Law

1. Select a body or a system of bodies for analysis.
2. Draw a free-body diagram of the body or system showing all the forces acting on the body or system.
3. Select a direction to be positive. (It will be more convenient to take the direction of the acceleration as positive.)
4. Determine $F_{net} = \sum(\text{all forces})$
5. Apply Newton's 2nd Law $F_{net} = ma$ to the chosen body or system.

Example 2

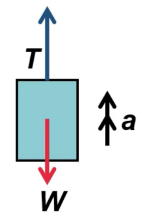
An elevator has a mass of 1000 kg. Calculate the force T exerted by the cable on the elevator when the elevator is accelerating

(a) upward at 3.0 m s^{-2},
(b) downward 3.0 m s^{-2}.

(g = 9.81 m s^{-2})

Solution:

(a) Taking upwards as positive,



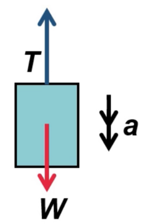
$$F_{net} = T - W \quad \boxed{F_{net} = \sum_i F_i}$$

$$F_{net} = ma \quad \boxed{\text{Newton's 2nd Law}}$$

$$T - W = ma$$

$$T = m(a+g) = 1000(3.0+9.81) = 13000 \text{ N}$$

(b) Taking downwards as positive,



$$F_{net} = ma$$

$$W - T = ma$$

$$T = m(g-a) = 1000(9.81-3.0) = 6800 \text{ N}$$

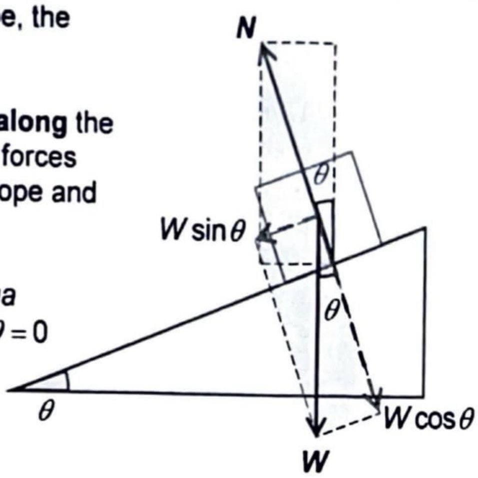
Masses on slopes

When a mass is placed on a smooth slope, the mass will accelerate down the slope

Since we are concerned with the motion **along** the slope, it is more convenient to resolve all forces with respect to the slope (i.e. along the slope and perpendicular to the slope).

Along the slope: $W\sin\theta = ma$

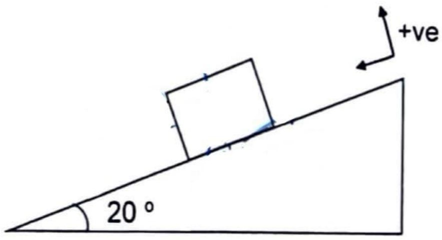
Perpendicular to slope: $N - W\cos\theta = 0$



Example 3

A wooden block of mass 50 g is sliding down a slope inclined at an angle of 20 ° with a frictional force of 0.10 N acting on it. Determine the acceleration of the wooden block along the slope. (g = 9.81 m s^{-2})

Solution:



Since motion is along the slope, resolve forces parallel to the slope and perpendicular to the slope.

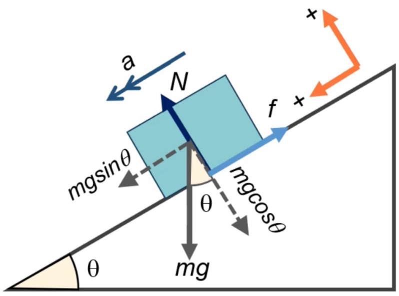
Resolving // to the incline:

$$mg\sin\theta - f = ma$$

$$a = \frac{mg\sin\theta - f}{m}$$

$$= \frac{(50\times10^{-3})(9.81)\sin 20^\circ - 0.10}{50\times10^{-3}}$$

$$= 1.4\ \text{ms}^{-2}$$



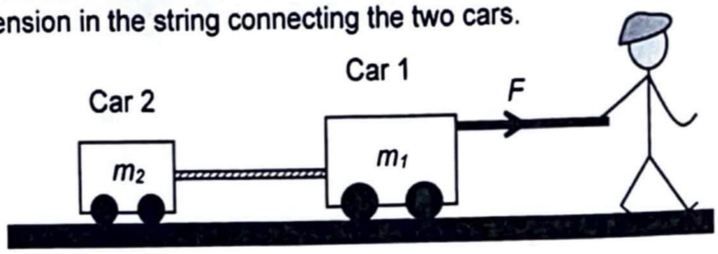
Example 4

A child pulls a train of 2 cars with a horizontal force F = 10 N. Car 1 has a mass m_1 = 3.0 kg, and Car 2 has a mass of m_2 = 1.0 kg. Assume that the frictional forces acting on the cars are negligible. Determine

(a) the normal forces exerted on each car by the floor,

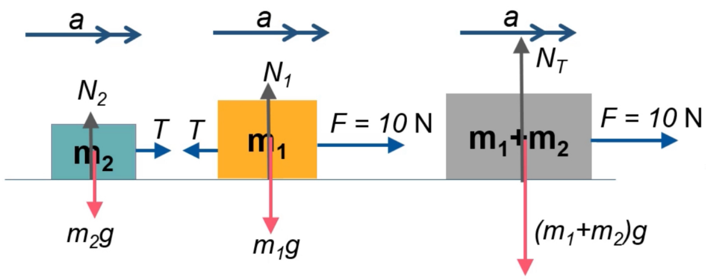
(b) the acceleration of the train,

(c) the tension in the string connecting the two cars.



Solution:

Solution



(a) Resolving vertically for m_1

$$N_1 = m_1 g = (3.0)(9.81) = 29.4 \text{ N}$$

Resolving vertically for m_2

$$N_2 = m_2 g = (1.0)(9.81) = 9.8 \text{ N}$$

(b) Considering m_1+m_2 system

Taking right as positive

$$F = (m_1 + m_2)a$$

$$a = \frac{F}{(m_1 + m_2)} = \frac{10}{(3.0 + 1.0)} = 2.5 \text{ m s}^{-2}$$

(c) Taking right as positive

Considering m_2

$$T = m_2 a = (1.0)(2.5) = 2.5 \text{ N}$$

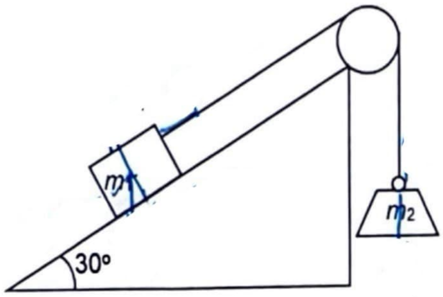
OR

Considering m_1

$$F - T = m_1 a$$

$$T = F - m_1 a = 10 - (3.0)(2.5) = 2.5 \text{ N}$$

Example 5 Two objects of masses m_1 = 5.0 kg and m_2 = 10.0 kg are connected by a light inextensible string that passes over a light frictionless pulley. The 5.0 kg object lies on a smooth incline of angle 30°. Determine the acceleration of the two objects and the tension in the string.



Solution:

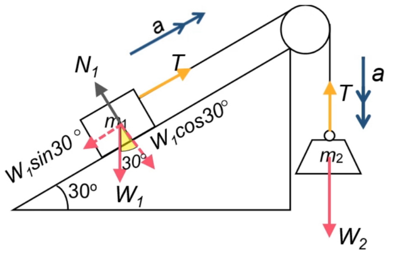
Solution

Considering m_2 Taking downwards positive

$$W_2 - T = m_2 a \qquad \cdots(1)$$

Considering m_1 Taking up the incline as positive

$$T - W_1 \sin 30^\circ = m_1 a \qquad \cdots(2)$$



$$(1)+(2) \quad \boxed{W_2 - W_1 \sin 30^\circ = (m_1 + m_2)a}$$

$$a = \frac{m_2 g - m_1 g \sin 30^\circ}{(m_1 + m_2)} = \frac{(10.0)(9.81) - (5.0)(9.81)\sin 30^\circ}{(10.0 + 5.0)}$$

$$a = 4.9 \text{ m s}^{-2}$$

From (1) $T = m_2 g - m_2 a$

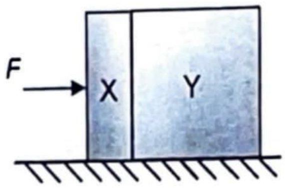
$= (10.0)(9.81 - 4.905)$

$= 49$ N

Important note

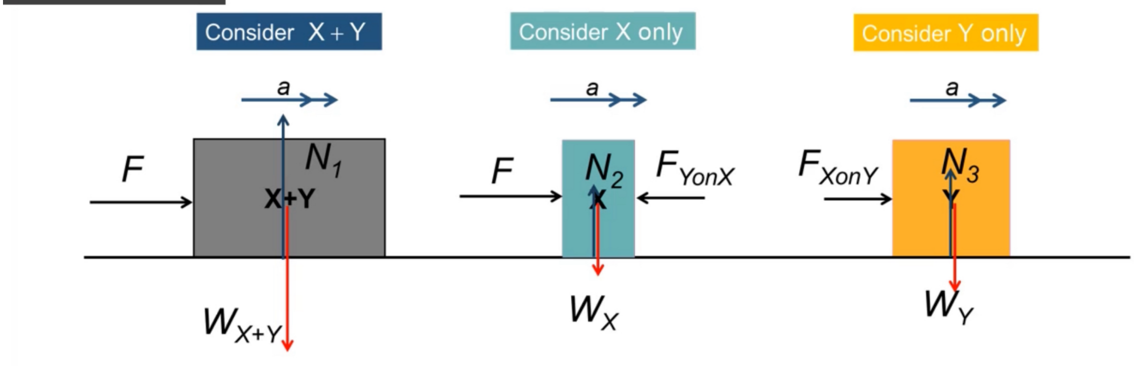
We should not skip equations (1) and (2) and simply write down $W_2 - W_1\sin30° = (m_1 + m_2)a$ as W_2, $W_1\sin30°$ and a (though a is the same magnitude for both masses) are not along the same direction.

Example 6 Two blocks X and Y, of masses M and $3M$ respectively, are accelerated along a smooth horizontal surface by a force F applied to block X as shown. In terms of F, determine the magnitude of the force exerted by block Y on block X during this acceleration.



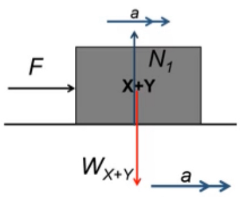
Solution:

Solution



Consider $X + Y$ Taking right as positive

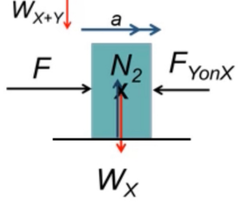
$$F = (m_x + m_y)a \quad \Rightarrow \quad a = \frac{F}{4M}$$



Consider X only

$$F - F_{YonX} = m_X a$$

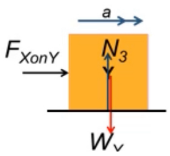
$$F_{YonX} = F - M\left(\frac{F}{4M}\right) = 0.75F$$



OR

Consider Y only

$$\left|F_{YonX}\right| = \left|F_{XonY}\right| = m_y a = 3M\left(\frac{F}{4M}\right) = 0.75F$$



Apparent Weight

When an elevator starts to move upwards, it accelerates briefly and then moves at a constant velocity until it approaches the desired floor where it decelerates to a stop.

During the upward acceleration, we feel heavier than usual. Similarly, when the acceleration is downward, we have the feeling that our weight is reduced.

Our weight is the gravitational force exerted on us by the earth, and that, of course, is not changed by being in the elevator. However, our perception of weight is determined by the forces exerted on us by the floor or chair or whatever is supporting us. These forces are not equal to our weight when we are accelerating.

We define the **apparent (or effective) weight** of a body as the total force that the body exerts on a spring scale.

When we stand on a weighing scale, what the scale measures is not our true weight, but rather this apparent weight.

Example 7

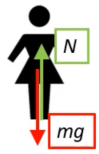
A woman of mass m stands on a weighing scale in an elevator. Determine the apparent weight of the woman if the elevator is

(a) stationary,
(b) moving upwards with constant velocity,
(c) moving upwards with an acceleration of $0.2g$,
(d) moving upwards with a deceleration of $0.3g$,
(e) moving downwards with acceleration g.

Solution:

Solution

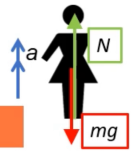
Let force by the scale on the woman be N.



(a) and (b) When the elevator is stationary or moving upwards with constant velocity

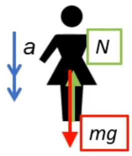
Taking upwards as positive

$$N - mg = 0$$
$$N = mg$$ Apparent weight = True weight



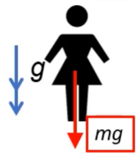
(c) Taking upwards as positive

$$N - mg = ma$$
$$N = m(g + 0.2g) = 1.2mg$$ Apparent weight > True weight



(d) Taking downwards as positive

$$mg - N = ma$$
$$N = m(g - 0.3g) = 0.7mg$$ Apparent weight < True weight



(e) Taking downwards as positive

$$mg - N = ma$$
$$N = m(g - g) = 0$$ Weightlessness

Note: the true weight of a body remains the same always. But the apparent weight changes with acceleration. When the body is undergoing free fall, it will experience weightlessness.

3.1.8 Impulse and Momentum Change

Impulse

Impulse is defined as **the product of a force *F* acting on an object and the time Δt** for which the force acts.

$$\text{Impulse} = F\,\Delta t$$

For a constant force

Newton's Second Law of Motion states the relationship between the force acting on body and its change in momentum. For a *constant* force,

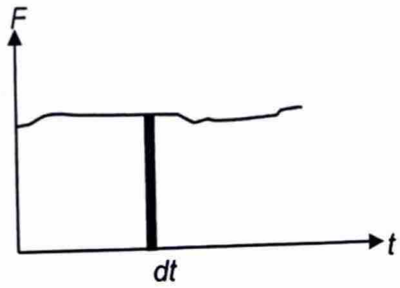
$$F = \frac{\Delta p}{\Delta t}$$

Thus, $\Delta p = F\Delta t$ ⇒ This is **IMPULSE**

The impulse of the force acting on a body is **equal to its change in momentum after time Δt**. Impulse has the same unit as momentum i.e. **kg m s^{-1} or N s**. It is a vector quantity.

For a time-varying force

Consider a force *F* acting on a body for a time *t* as shown in the graph.



We know that $F = \frac{dp}{dt}$. Therefore, during the small time interval d*t*, the momentum of the body changes by $dp = Fdt$, where *F* remains constant in the small time interval d*t*.

To find the change in momentum during the **entire** time interval *t*,

$$\int_{p_i}^{p_f} dp = \int_0^t Fdt \Rightarrow p_f - p_i = \int_0^t Fdt$$

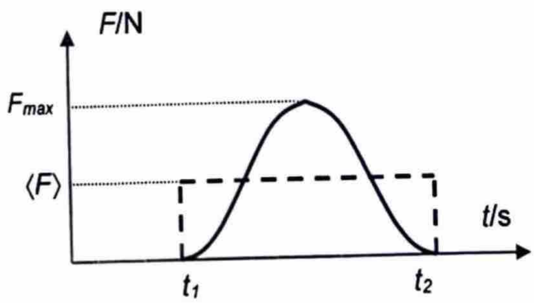
$$\Delta p = \text{area under } F - t \text{ graph}$$

*Mathematically, the quantity $\int$ **Fdt** is the **area under the force-time graph**.*

By Newton's second law, it is equal to the change in momentum Δp.

Thus, $\boxed{\text{Impulse} = \Delta p = \text{Area under } F\text{ - }t \text{ graph}}$

When a collision between two objects occur, the force normally rises abruptly to a maximum value and then abruptly decreases to zero again as they leave each other.



Since the force is not constant, in such instances, it is necessary to define an average force, $\langle F \rangle$, during the collision.

$$\langle F \rangle = \frac{\text{change in momentum, } \Delta p}{\text{time interval, } \Delta t}$$

The average force $\langle F \rangle$ over the same time interval, $\Delta t = t_2 - t_1$, produces the same change in momentum.

Example 8 (constant force)

A block of mass 2.0 kg is moving with an initial velocity of 5.0 m s^{-1} towards the right on a smooth horizontal surface. A constant force of 10 N towards the right is then applied on it for 2.0 s.

(a) Calculate the change in momentum experienced by the block.
(b) Determine the final velocity of the block.
(c) Determine the final velocity of the block if its initial velocity is 5.0 m s^{-1} towards the *left*.

Solution:

Solution

(a) $\Delta p = F\Delta t = 10(2.0) = 20$ N s

(b) Taking right as positive

$\Delta p = p_f - p_i = mv_f - mv_i$

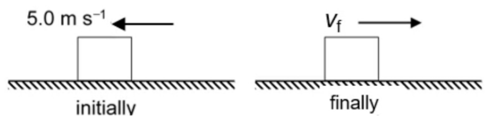
$$v_f = \frac{\Delta p + mv_i}{m} = \frac{20 + (2.0)(5.0)}{2.0} = 15 \text{ m s}^{-1}$$

(c) Taking right as positive

From (b) $v_f = \dfrac{\Delta p + mv_i}{m}$

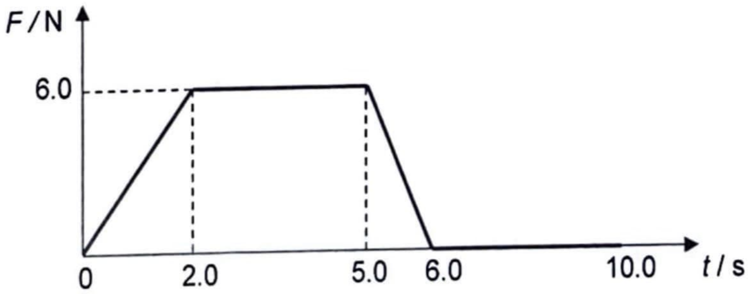
$$= \frac{20 + (2.0)(-5.0)}{2.0}$$

$= 5$ m s^{-1}



Example 9 (time varying force)

A body of mass 3.0 kg is moving with an initial speed of 1.0 m s^{-1} when it is acted upon by a force *F* which varies with time *t* as shown below:



If the force acts in the same direction as the initial velocity of the body, what is the body's velocity at *t* = 10.0 s?

Solution:

Solution

$\Delta p =$ Area under F-t graph

$\Delta p = (0.5 \times 2.0 \times 6.0) + (6.0)(3.0) + (0.5 \times 1.0 \times 6.0)$

$= 27$ N s

$\Delta p = p_f - p_i = mv_f - mv_i$

$$v_f = \frac{\Delta p + mv_i}{m}$$

$$= \frac{27 + (3.0)(1.0)}{3.0}$$

$= 10$ m s^{-1}

Example 10 (average force of a time varying force)

A man throws a ball of mass 3.0 kg with a speed of 5.0 m s^{-1}.

(a) His hand is in contact with the ball for a time interval of 0.20 s while throwing the ball. Calculate the average force he exerts on the ball.

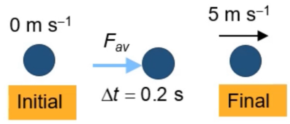
(b) The ball thrown by the man hits a vertical wall at right angles with a speed of 5.0 m s^{-1} and bounces back horizontally with a speed of 3.0 m s^{-1}. The ball is in contact with the wall for 0.10 s. Determine the magnitude of the average force exerted by the wall on the ball.

Solution

(a) Taking right as positive

$\Delta p = F_{av}\Delta t$

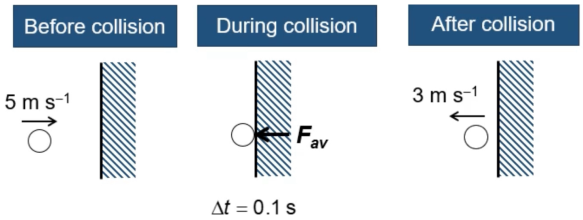
$$F_{av} = \frac{m(v_f - v_i)}{\Delta t} = \frac{3.0(5.0-0)}{0.20} = 75\text{ N}$$



(b) Taking right as positive

$\Delta p = F_{av}\Delta t$

$$F_{av} = \frac{m(v_f - v_i)}{\Delta t} = \frac{3.0[-3-5]}{0.10} = -240\text{ N}$$



Note that the force exerted by the wall on the ball is most likely not constant throughout the 0.10 s of contact. However, we are finding the average force which produces the same change in momentum of the ball.

Example 11

A parachutist of weight W strikes the ground with her knees bent and comes to rest with a deceleration of $3g$.

(a) Determine, in terms of W, the force exerted on her by the ground during landing.

(b) Suggest and explain what may happen if the parachutist did not bend her knees upon landing?

Solution

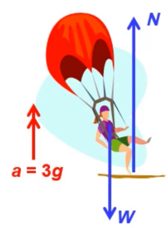
(a) Considering forces upon landing,

Taking upwards as positive

$N - W = ma$

$N = m(3g) + mg = 4mg$

$\therefore N = 4W$

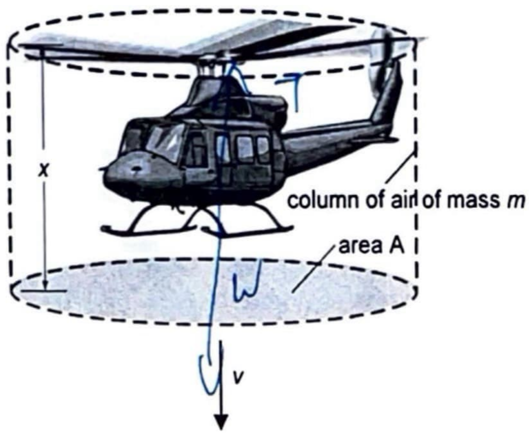


(b) If she did not bend her knees, she would be brought to rest in a shorter time Δt. This means that for the same change in momentum $|mv - 0|$, where v is the velocity of the parachutist just before impact, the force acting on her by the ground

$N = \dfrac{|mv-0|}{\Delta t} + W$ will be larger, potentially causing injury.

(The resultant force F_R acting on the parachutist is $F_R = N - W = \dfrac{|mv-0|}{\Delta t}$.

Example 12 A helicopter of mass 1000 kg hovers by imparting a downward velocity v to the air displaced by its rotating blades. The area swept out by the blades is 80 m^2. Calculate the value of v.
(Density of air = 1.3 kg m^{-3}, g = 9.81 m s^{-2}.)



Solution:

As the blades push the air downwards, the air exerts an upward thrust T on the blades, according to Newton's Third Law. Since the helicopter hovers, the net force exerted on it is zero, hence $\boldsymbol{T = W}$. The magnitude of T is equal to the rate of change of momentum of air.

Solution

Since the helicopter is hovering

$$T = W = (1000)(9.81) = 9810 \text{ N}$$

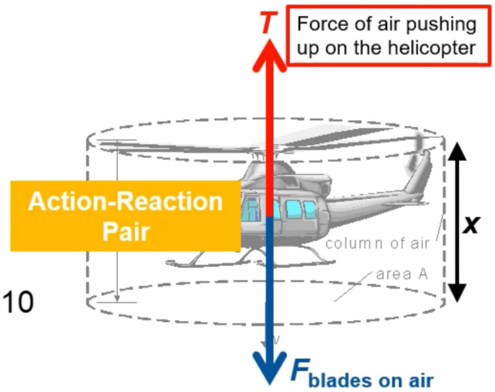
By Newton's 3rd Law $F_{\text{blades on air}} = T = 9810$ N

By Newton's 2nd Law $F_{\text{blades on air}} = \dfrac{dp_{air}}{dt} = \dfrac{d(mv)}{dt}$

$$m_{air} = \rho_{air} V_{air} = \rho_{air}(Ax)$$

$$\frac{d\rho_{air} Axv}{dt} = \rho_{air} Av\frac{dx}{dt} = \rho_{air} Av^2 = 9810$$

$$v = \sqrt{\frac{9810}{1.3(80)}} = 9.7 \text{ m s}^{-1}$$



3.2 Collisions and Conservation of Momentum

3.2.1 Conservation of Momentum

Collisions

In a collision, two bodies come close to each other and interact by means of forces. The time interval Δt is relatively short. Since $F = \frac{\Delta p}{\Delta t}$, for a very small Δt, the force of interaction is relatively large.

The motion of at least one of the colliding bodies changes so abruptly that we can make a clean distinction between "before the event" and "after the event". The laws of conservation of momentum and energy permit us to learn much about such processes by studying the "before" and "after" situations.

Conservation of Momentum During Collisions

Suppose that a moving body A, of mass m_A and initial velocity u_A, collides head-on with a body B, of mass m_B and initial velocity u_B, moving in the same direction as shown in Fig. 1(a).

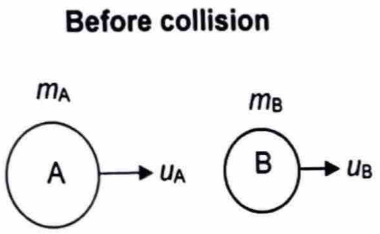


Fig. 1(a)

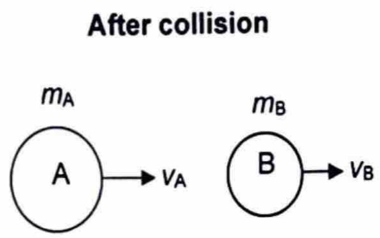


Fig. 1(b)

After the collision, they move off with final velocities v_A and v_B as shown in Fig. 1(b).

During the brief collision, these bodies exert large forces on one another. By Newton's Third Law of motion, the force F_{BA} exerted by B on A is equal and opposite to the force F_{AB} exerted by A on B (see Fig. 2).

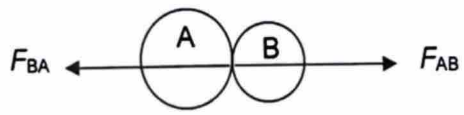
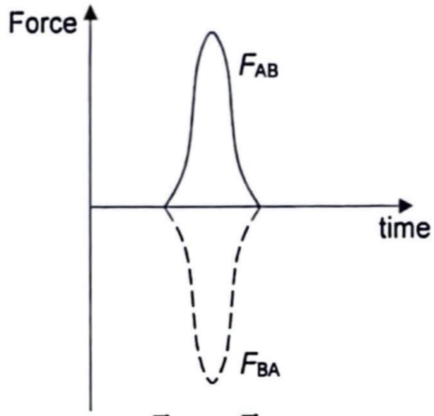


Fig. 2



$$\vec{F}_{BA} = -\vec{F}_{AB}$$

Since the time interval during which the force acted on B is equal to the time during which the force of reaction acted on A, the impulse on A is equal and opposite to that on B.

$$\int_{t_i}^{t_f} F_{BA} dt = -\int_{t_i}^{t_f} F_{AB} dt$$

Since impulse = change in momentum

$$\Delta p_A = -\Delta p_B$$

$$m_A v_A - m_A u_A = -(m_B v_B - m_B u_B)$$

Re-arranging the equation,

$$m_A u_A + m_B u_B = m_A v_A + m_B v_B$$

Net initial momentum of bodies = Net final momentum of bodies

Principle of Conservation of Momentum

In an **isolated** system of bodies (one where no resultant external force is acting), when the bodies interact with one another, their individual momentum may change but the total momentum of the bodies remains constant. This is the principle of conservation of momentum.

The **Principle of Conservation of Momentum** states that

> **when bodies in a system interact, the total momentum of the system remains constant, provided no net external force acts on it.**

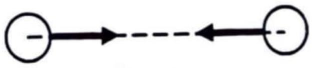
The principle of conservation of momentum is a consequence of Newton's 2nd Law.

3.2.2 Types of Collisions

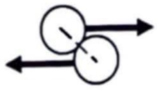
Types of collisions

Collisions can occur in any planes and in any directions. In particular, we consider collisions made *head-on* and at a *glancing angle*.

A *head-on* collision is one where the velocities of the bodies before and after collision are along the line joining their centre of masses.



A head-on collision



A collision made at a glancing angle

In all collisions, the total energy and momentum are conserved.

Collisions can be any one of the following 3 types:

1) **Elastic Collision** in which both total momentum and total kinetic energy are conserved.

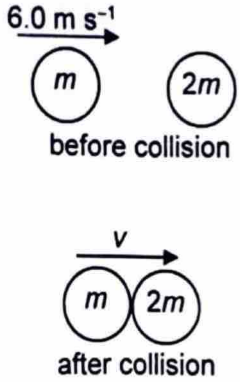
2) **Inelastic Collision** in which total momentum is conserved but total kinetic energy is not conserved.

3) **Completely Inelastic Collision** in which **total kinetic energy is not conserved** and **the particles stick together after collision** so that their final velocities are the same. Total momentum is conserved.

<u>**Note:**</u>

- **Total energy** is always conserved in all types of collisions.
- The **total momentum** of the colliding bodies is always conserved in all types of collisions, as long as there is no resultant external force acting on the system (i.e. the system is isolated). However, the **momentum of each individual body may change.**
- When finding the total kinetic energy of the colliding bodies, the direction of travel of the bodies is not taken into account since kinetic energy is a scalar quantity, unlike the case when we are finding the total momentum (vector).
- In an **elastic collision**, the kinetic energy lost by one body during collision is transferred to the other body so that the total kinetic energy of the colliding bodies is conserved after the collision.
- In inelastic collisions, some of the kinetic energy is converted into internal energy or sound when the objects are deformed. Therefore, in an inelastic collision, the total kinetic energy is not conserved.
- In a super-elastic collision, the total kinetic energy of the system increases due to the conversion of potential energies into kinetic energy (eg in an explosion).

Example 13 A body of mass m moving with a velocity of 6.0 m s^{-1} makes a head-on collision with a stationary body of mass $2m$. The particles coalesce and move off with a common velocity. Determine the velocity of the bodies after the collision.



Solution:

In a completely inelastic collision, the bodies coalesce and move off with a **common velocity.**

By the principle of conservation of momentum.

$$p_{\text{initial}} = p_{\text{final}}$$

$$(p_A + p_B)_{\text{initial}} = (p_A + p_B)_{\text{final}}$$

$$m_A u_A + 0 = (m_A + m_B)V$$

$$V = \frac{m_A u_A}{(m_A + m_B)} = \frac{(m)(6.0)}{(m + 2m)} = 2.0 \text{ m s}^{-1}$$

Elastic collisions In one dimension

Consider an **elastic** collision between two bodies of masses m_1 and m_2 initially moving with velocities u_1 and u_2 as shown in Fig. 3(a). After the collision they move off with velocities as shown in Fig. 3(b).

Before collision

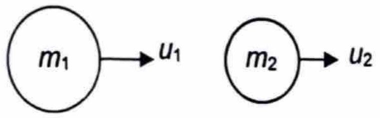


Fig. 3(a)

After collision

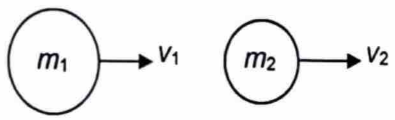


Fig. 3(b)

Take motion to the right as positive.
Applying the principle of conservation of momentum, we have

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \quad ...(1)$$

Since the collision is an elastic collision, the total kinetic energy of the bodies remains constant.

$$\tfrac{1}{2}m_1u_1^2 + \tfrac{1}{2}m_2u_2^2 = \tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2 \quad ...(2)$$

Eliminating m_1 and m_2 from equations (1) and (2) [*see Appendix*] results in the equation

$u_1 - u_2 = v_2 - v_1$
Relative speed of approach = Relative speed of separation

Note:

- Be careful with the subscripts. A common mistake made by students is writing $u_1 - u_2 = v_1 - v_2$, which is **wrong**.

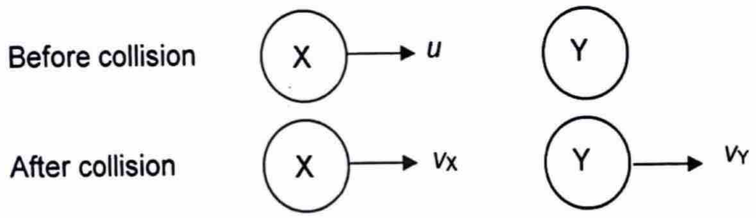
- The expression is derived based on the scenario where the **two particles are moving in the same direction before and after collision**. Mistakes often occur when the velocities are in opposite directions, and students forget to include a negative sign for these velocities.

Solving Collision Problems

When solving collision problems, first identify the type of collision. Then apply the relevant laws:

Type of collision	Laws to apply	Remarks
Elastic	1. Conservation of momentum $\left[\sum p_{initial} = \sum p_{final}\right]$ 2. Relative speed of approach = Relative speed of separation $[u_1 - u_2 = v_2 - v_1]$ **OR** Conservation of kinetic energy $\left[\sum KE_{initial} = \sum KE_{final}\right]$	
Inelastic	1. Conservation of momentum $\left[\sum p_{initial} = \sum p_{final}\right]$	
Completely Inelastic	1. Conservation of momentum $\left[\sum p_{initial} = \sum p_{final}\right]$	Final velocities of both bodies are the same

Example 14 An ice-puck X, traveling with velocity *u*, strikes head-on with an identical stationary ice-puck Y, as shown below. The collision is elastic. Determine the velocities of X and Y immediately after the collision.



Solution

Taking right as positive,

By the Principle of Conservation of Momentum

$$mu = mv_X + mv_Y$$

$$u = v_X + v_Y \text{ -------[1]}$$

Since the collision is elastic

relative speed of approach = relative speed of separation

$$u - 0 = v_Y - v_X$$

$$u = v_Y - v_X \text{ -------[2]}$$

Solving simultaneous equations

Add [1] and [2]

$$2u = 2v_Y$$

Subst $v_Y = u$ in [1]

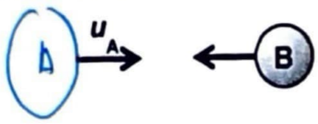
$$\therefore v_Y = u$$

and $v_X = 0$

Example 15 A 2.0 kg body moving with velocity 6.0 m s^{-1} collides head-on with a 1.0 kg body moving with velocity 10 m s^{-1} in the opposite direction. The collision is elastic. Calculate the velocity of each body after the collision.

Solution:

Before Collision:



After Collision:

Solution

Taking right as positive,

By the Principle of Conservation of Momentum

$$m_A u_A + m_B u_B = m_A v_A + m_B v_B$$

$$(2.0)(6.0) + (1.0)(-10) = (2.0)v_A + (1.0)v_B$$

$$2.0 = 2v_A + v_B \text{ -----[1]}$$

Since the collision is elastic

relative speed of approach = relative speed of separation

$$u_A - u_B = v_B - v_A$$

$$6.0 - (-10) = v_B - v_A$$

$$16 = v_B - v_A \text{ -----[2]}$$

Solving

$$v_A = -4.67 \text{ m s}^{-1}$$

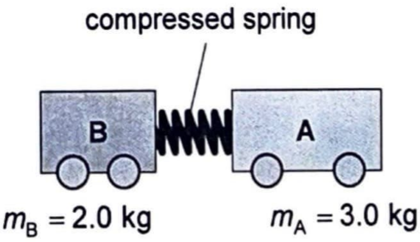
$$v_B = 11.3 \text{ m s}^{-1}$$

Note: The negative sign for v_A simply means v_A is in the opposite direction to its assumed direction (right), i.e. v_A is leftwards.

Example 16 (e.g. of an super-elastic interaction) The figure below shows two trolleys A and B are held at rest, with a light compressed spring between them. The trolleys are now released and the 3.0 kg trolley moves with a velocity of 1.0 m s^{-1} to the right after separation. Calculate

(a) the velocity of the 2.0 kg trolley after separation,
(b) the total kinetic energy of the trolleys after separation.

Neglect the mass of the spring and any friction forces.



Solution:

Solution

(a) Taking right as positive,

By the Principle of Conservation of Momentum

$$0 = m_A v_A + m_B v_B$$

$$v_B = -\frac{m_A v_A}{m_B} = -\frac{(3.0)(1.0)}{(2.0)} = -1.5 \text{ m s}^{-1}$$

The velocity of the 2.0 kg trolley is 1.5 m s^{-1} to the left.

(b) Total kinetic energy of the trolleys after separation

$$= \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2$$

$$= \frac{1}{2}(3.0)(1.0)^2 + \frac{1}{2}(2.0)(1.5)^2$$

$$= 3.8 \text{ J}$$

Explosions (e.g. of a super-elastic interaction)

When a body explodes, large forces act on the fragments causing them to be ejected at high speed in different directions. Each fragment possesses momentum but since the forces acting on them are **internal forces**, the **total momentum of the system is conserved**, i.e. the total momentum of the fragments is equal to the momentum of the body before explosion.

The energy released by the explosion is carried away mainly as kinetic energies of the fragments. The loud sound produced in the explosion only constitutes a fraction of the total energy released. There is also thermal energy produced. In our calculations, for simplicity, we neglect both sound and thermal energies and equate total energy to the total kinetic energies of the fragments.

Example 17 A 10 kg rock initially at rest is broken into 2 pieces after a small explosion. If a 6.8 kg piece of rock flies to the right at 12 m s^{-1}, calculate the velocity of the other piece of rock.

Solution Taking right as positive,

By the Principle of Conservation of Momentum

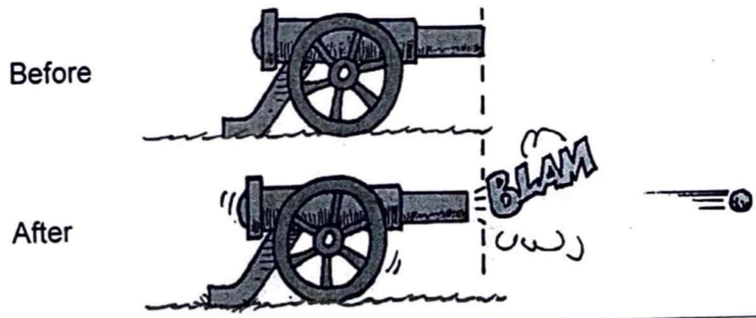
$$0 = m_A v_A + m_B v_B$$

$$v_A = -\frac{m_B v_B}{m_A} = -\frac{(6.8)(12)}{(3.2)} = -26 \text{ m s}^{-1}$$

The other piece of rock flies to the left with a speed of 26 m s^{-1}.

Recoil (e.g. of an inelastic interaction)

Recoil is the backward motion of a gun when it is discharged.



Example 18 A shell of mass 1.6 kg is fired with a velocity of 250 m s^{-1} from an artillery gun of mass 1000 kg. If the gun is free to move, calculate its recoil velocity.

Solution Taking right as positive,

By the Principle of Conservation of Momentum

$$0 = m_{gun} v_{gun} + m_{shell} v_{shell}$$

$$v_{gun} = -\frac{m_{shell} v_{shell}}{m_{gun}} = -\frac{(1.6)(250)}{(1000)} = -0.40 \text{ m s}^{-1}$$

Appendix — Elastic Collisions in One Dimension

Consider an elastic collision of two particles of masses m_1 and m_2 initially moving with velocities u_1 and u_2 as shown in Fig. 4(a). After the collision they move off with velocities as shown in Fig. 4(b).

Before collision

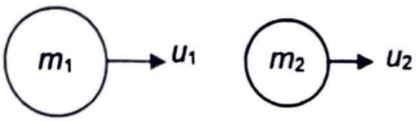


Fig. 4(a)

After collision

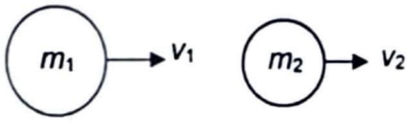


Fig. 4(b)

Taking motion to the right as positive.
Applying the principle of conservation of momentum, we have

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \quad ...(1)$$

Since the collision is an elastic collision, the total kinetic energy of the bodies remains constant.

$$\tfrac{1}{2}m_1u_1^2 + \tfrac{1}{2}m_2u_2^2 = \tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2 \quad ...(2)$$

$$\Rightarrow \quad m_1u_1^2 + m_2u_2^2 = m_1v_1^2 + m_2v_2^2 \quad ...(3)$$

Re-arranging equations (1) and (3):

$$m_1(u_1 - v_1) = m_2(v_2 - u_2) \quad ...(4)$$

$$m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2) \quad ...(5)$$

Dividing equation (5) by equation (4):

$$\frac{u_1^2 - v_1^2}{u_1 - v_1} = \frac{v_2^2 - u_2^2}{v_2 - u_2}$$

$$\frac{(u_1 + v_1)(u_1 - v_1)}{u_1 - v_1} = \frac{(v_2 + u_2)(v_2 - u_2)}{v_2 - u_2}$$

$$u_1 + v_1 = v_2 + u_2$$

$$u_1 - u_2 = v_2 - v_1 \quad ...(6)$$

relative speed of approach = relative speed of separation

To obtain expressions for v_1 and v_2 in terms of m_1, m_2, u_1 and u_2:

From eqn (6), $v_2 = u_1 - u_2 + v_1$. Substituting expression into eqn (1) yields

$$m_1v_1 + m_2(u_1 - u_2 + v_1) = m_1u_1 + m_2u_2$$

$$(m_1 + m_2)v_1 = (m_1 - m_2)u_1 + 2m_2u_2$$

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2 \qquad \cdots(7)$$

Similarly substituting $v_1 = v_2 - u_1 + u_2$ into eqn (1) and then expressing v_2 in terms of the masses and initial velocities give

$$v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 \qquad \cdots(8)$$

Cases of Interest: [deduced from equations (7) and (8)]

Cases	Final velocities	Remark
Case 1: $m_1 = m_2$	$v_1 = u_2$ $v_2 = u_1$	In a one-dimensional elastic collision of two particles of equal mass, the particles simply exchange velocities during collision.
If $u_2 = 0$	$v_1 = 0$ $v_2 = u_1$	When a particle collides with a second identical particle initially at rest, the first particle is stopped cold and the second particle took off with the original velocity of the first particle.
Case 2: $m_1 << m_2$ and $u_2 = 0$ (i.e. m_1 can be approximated to zero)	$v_1 \approx -u_1$ $v_2 \approx 0$	When a light particle collides with a massive particle at rest, the light particle rebounds with little change in speed whilst the massive particle remains approximately at rest.
Case 3: $m_1 >> m_2$ and $u_2 = 0$ (i.e. m_2 can be approximated to zero)	$v_1 \approx u_1$ $v_2 \approx 2u_1$	When a massive particle collides with a light particle at rest, the velocity of the massive particle is virtually unchanged whilst the light particle moves off with approximately twice the velocity of the incident particle.

RAFFLES INSTITUTION
YEAR 5-6 PHYSICS DEPARTMENT

Tutorial 3 DYNAMICS

Self-Check Questions

S1 State Newton's Laws of Motion.

S2 Distinguish between the mass and weight of an object.

S3 State the characteristics of the two forces of an action-reaction pair.

S4 What is meant by the linear momentum of a body? List down two expressions of the unit of momentum.

S5 How is force related to momentum?

S6 How does Newton's Second Law lead to the commonly used relationship

$$\text{Force} = \text{mass} \times \text{acceleration ?}$$

S7 What is meant by the impulse of a force? How is it obtained from the force-time graph?

S8 State the Principle of Conservation of Momentum.

S9 Use Newton's Laws to derive the Principle of Conservation of Momentum.

S10 Distinguish between elastic, inelastic and completely inelastic collisions.

Self-Practice Questions

SP1 A trolley runs down a slope with a constant acceleration *a*. The mass of the trolley is now doubled and the trolley is allowed to run down the same slope. In both cases the effects of friction and air resistance are negligible.

Which statement is correct for the second experiment?

A The accelerating force is the same.

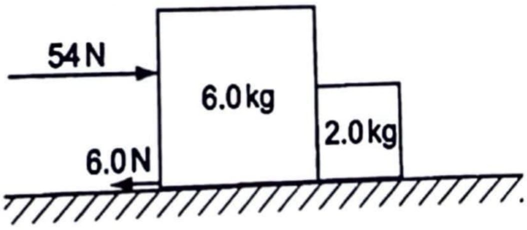
B The acceleration is ½ *a*.

C The acceleration is *a*.

D The acceleration is 2*a*.

N02/I/6

SP2 A force of 54 N pushes two touching blocks of masses 6.0 kg and 2.0 kg along a flat surface. The frictional force between the blocks and the surface is 6.0 N.



What is the magnitude of the resultant force on the 6.0 kg mass?

A 12 N **B** 36 N **C** 45 N **D** 48 N

N10/I/7

SP3 A rocket stands vertically on its launching pad. The mass of the rocket inclusive of the mass of a man in it and its fuel is 3.0×10^4 kg. On ignition, gas is ejected from the rocket at a speed of 6.0×10^3 m s^{-1} relative to the rocket, and the fuel is consumed at a constant rate of 70 kg s^{-1}.

Calculate

(a) the thrust on the rocket,

(b) the initial acceleration of the rocket,

(c) the initial net force on the man if he has a mass of 50 kg.

SP4 A ball of mass 80 g collides with a vertical wall. The ball has a velocity of 23 m s^{-1} in a horizontal direction. After hitting the wall the ball moves with a velocity of 18 m s^{-1} in the opposite direction.

What is the impulse provided by the wall?

A 0.40 N s in a direction away from the wall

B 3.3 N s in a direction away from the wall

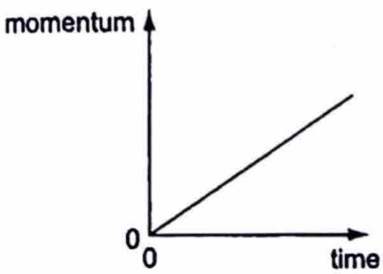
C 33 N s in a direction towards the wall

D 3300 N s in a direction towards the wall

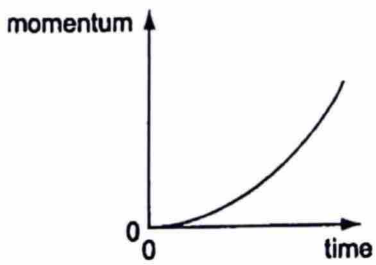
N11/I/7

SP5 Which graph best shows the variation with time of the momentum of a body accelerated by a constant force?

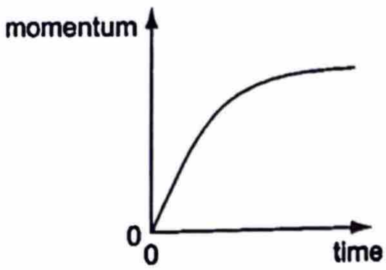
A



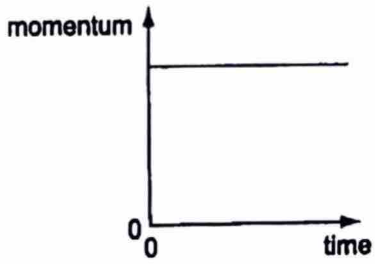
B



C

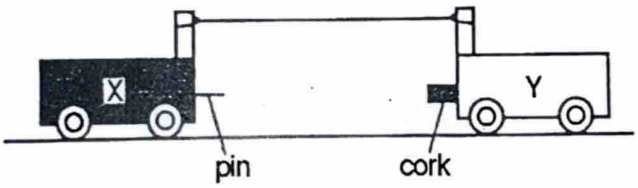


D



N04/I/5

SP6 The diagram shows two trolleys X and Y held stationary and connected by an extended elastic cord. The mass of X is twice that of Y.



The trolleys are released at the same instant. They move towards each other and stick together on impact. Just before the collision, the speed of X is 20 cm s^{-1}.

What is the speed of Y after the collision?

A zero **B** 5 cm s^{-1} **C** 7 cm s^{-1} **D** 10 cm s^{-1}

N03/I/4

SP7 A particle of mass m travelling with velocity u collides elastically and head-on with a stationary particle of mass M.

Which expression gives the velocity of the particle of mass M after the collision?

A u

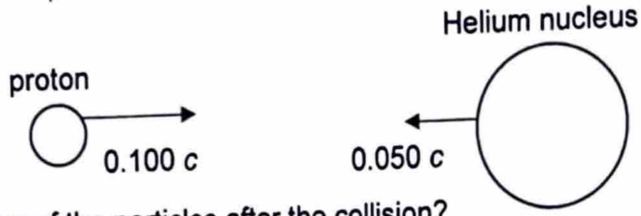
B $\dfrac{mu}{M}$

C $\dfrac{2mu}{M+m}$

D $\dfrac{(M-m)u}{M+m}$

N07/I/7

SP8 A proton (mass 1u) travelling with velocity +0.100 *c* collides elastically head-on with a helium nucleus (mass 4u) travelling with velocity −0.050 *c*.

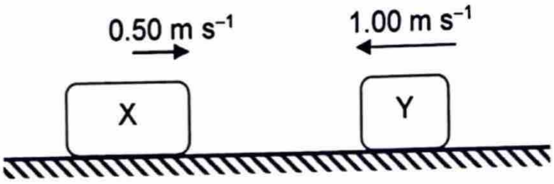


What are the velocities of the particles after the collision?

	Proton	Helium nucleus
A	-0.140 c	+0.010 c
B	+0.140 c	+0.010 c
C	+0.233 c	−0.083 c
D	−0.233 c	+0.083 c

N09/I/7

SP9 The diagram shows two blocks, X of mass 0.30 kg and Y of mass 0.20 kg moving toward each other along a frictionless horizontal surface with speeds of 0.50 m s^{-1} and 1.00 m s^{-1}, respectively.



(a) If the blocks collide and stick together, calculate

(i) the final velocity of the blocks,

(ii) the loss of kinetic energy during the collision.

(b) If the blocks collide elastically, calculate the final velocity of each block.

SP10 [J94/II/1]
The principle of conservation of momentum can be applied in different types of interaction. These are illustrated by the following examples.

(a) A piece of plasticine of mass 0.20 kg falls to the ground and hits the ground with a velocity of 8.0 m s^{-1} vertically downward. It does not bounce but sticks to the ground.

(i) State the type of collision between the plasticine and Earth. [1]

(ii) Calculate the momentum of the plasticine just before it hits the ground. [1]

(iii) State the transfers of momentum and kinetic energy of the plasticine which occur as a result of the collision. [2]

(b) Two strong magnets are held stationary with the north pole of one pushed against the north pole of the other. On letting go, the magnets spring apart. It is apparent that the kinetic energy of the magnets has increased.
Explain how the law of conservation of momentum applies in this case. [2]

Discussion Questions

Newton's Laws of Motion

D1 Suppose you were seated in a car in which a plumbline (string with a bob) was hanging from the roof. Sketch how you expect the plumbline to incline (if it does) in the following scenarios:

(a) the car was accelerating at a constant rate in the positive x-direction, [1]

(b) the car was decelerating at a constant rate in the positive x-direction, [1]

(c) the car was traveling at constant velocity in the negative x-direction, [1]

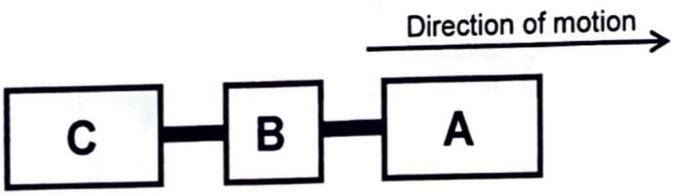
(d) the car was driven at constant speed along a roundabout, of which the centre of curvature is to the right of the driver. [1]

D2 A man of mass 80 kg is falling vertically. He opens his parachute and experiences an upward acceleration of 2.4 m s^{-2}. The mass of the parachute is 5.0 kg.

(a) What is the value of the upward force exerted on the parachute by the air? [2]

(b) What is the value of the upward force exerted by the parachute on the man? [2]

D3 A train, consisting of three coaches A, B and C linked together via tow-bars, is moving towards the right. A and C are coaches with engines and have a mass of 1.0×10^4 kg each, while B is an un-powered coach having a mass of 8.0×10^3 kg. The driving force of the engines in A and C are 2.0×10^4 N and 1.8×10^4 N respectively. The drag force on A is 9.0×10^3 N while B and C experience the same drag force of 6.0×10^3 N.



Neglecting the masses of the tow-bars, determine

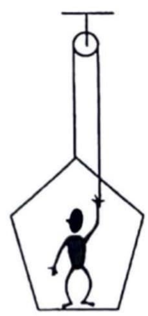
(a) the acceleration of the train, [2]

(b) the magnitude of the force that A exerts on B, [2]

(c) the magnitude of the force that C exerts on B. [2]

D4 The figure shows a painter in a crate which hangs alongside a building. When the painter pulls on the rope, the force he exerts on the floor of the crate is 300 N. The masses of the painter and the crate are 75 kg and 25 kg respectively.

Calculate the acceleration of the crate. [3]



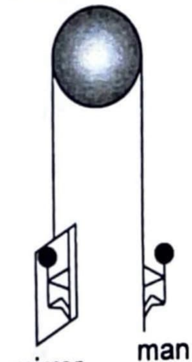
D5 **[2011 RI Common Test]**

(a) Bob stands on a weighing scale and it reads W_1. Alice then passes him a dumbbell. Bob, while holding the dumbbell, reads W_2 on the weighing scale. Bob then throws the dumbbell up into the air. In the process of throwing the dumbbell upwards, state and explain what happens to the reading on the weighing scale

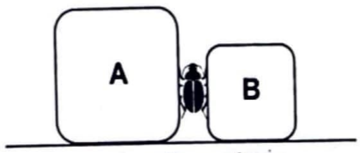
(i) before the dumbbell leaves his hand; and [2]
(ii) after the dumbbell leaves his hand. [2]

(b) A man is holding on to a light rope, and at the other end of the rope is a mirror which has the same mass as the man. The rope passes over a smooth pulley as shown in the figure on the right. The mirror and the man are at the same level, and the man sees his full image in the mirror.

The man then starts to climb up the rope. State and explain whether the man can still see his full image as he climbs up. [2]

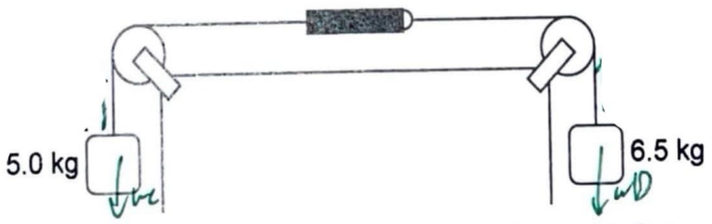


(c) A bug is trapped between two blocks **A** and **B** as shown. The mass of **A** is larger than **B**.



In which scenario below, **(1)** or **(2)**, is the bug more likely to survive when a force *F* is applied to one of the blocks? Explain your answer. [2]

(d) A 5.0 kg mass and a 6.5 kg mass are connected via light inextensible strings with a spring balance attached in the middle. The set-up rests on two smooth pulleys as shown. Both masses are initially held at rest. The mass of the spring balance can be assumed to be negligible.

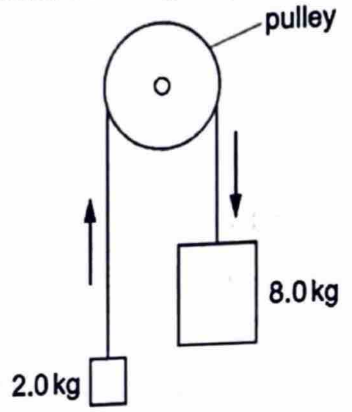


(i) Both masses are now released simultaneously from rest. Determine the tension in the string after release. [3]

(ii) State the reading on the spring balance in Newtons. [1]

D6 **[H1 N2020/1/7]**

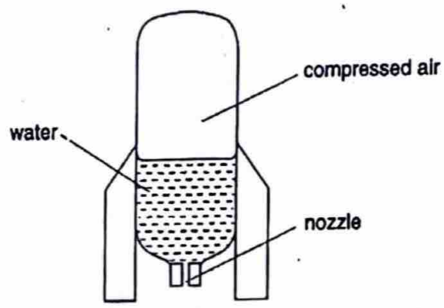
Masses of 8.0 kg and 2.0 kg hand from a light inextensible cord passing over a pulley.



The pulley exerts friction on the cord that is equivalent to a force of 3.0 N at the circumference. When the masses are released, they move in the direction of the arrows.
Determine the acceleration of the masses. [1]

D7 **[J96/3/1]**

A toy rocket consists of a plastic bottle which is partially filled with water. The space above the water contains compressed air, as shown in the figure below.



(a) At one instant during the vertical flight of the rocket, water of density ρ is forced through the nozzle of radius r at speed v relative to the nozzle. Determine, in terms of ρ, r and v,

(i) the mass of water ejected per unit time from the nozzle. [1]

(ii) the rate of change of momentum of the water. [1]

(b) Hence show that the accelerating force F acting on the rocket is given by the expression

$$F = \pi r^2 \rho v^2 - mg$$

where m is the mass of the rocket and its contents at the instant considered. [1]

(c) The toy manufacturer recommends that the rocket should contain about 550 cm^3 of water before take-off. If the initial air pressure is 1.6×10^5 Pa, all of this water will be expelled and the pressure is just reduced to atmospheric pressure as the last of the water is expelled.

However, on one flight, the initial volume of water was 750 cm^3 but the initial air pressure in the rocket was still 1.6×10^5 Pa. State, without calculation but with a reason, the effect of this increased volume of water on

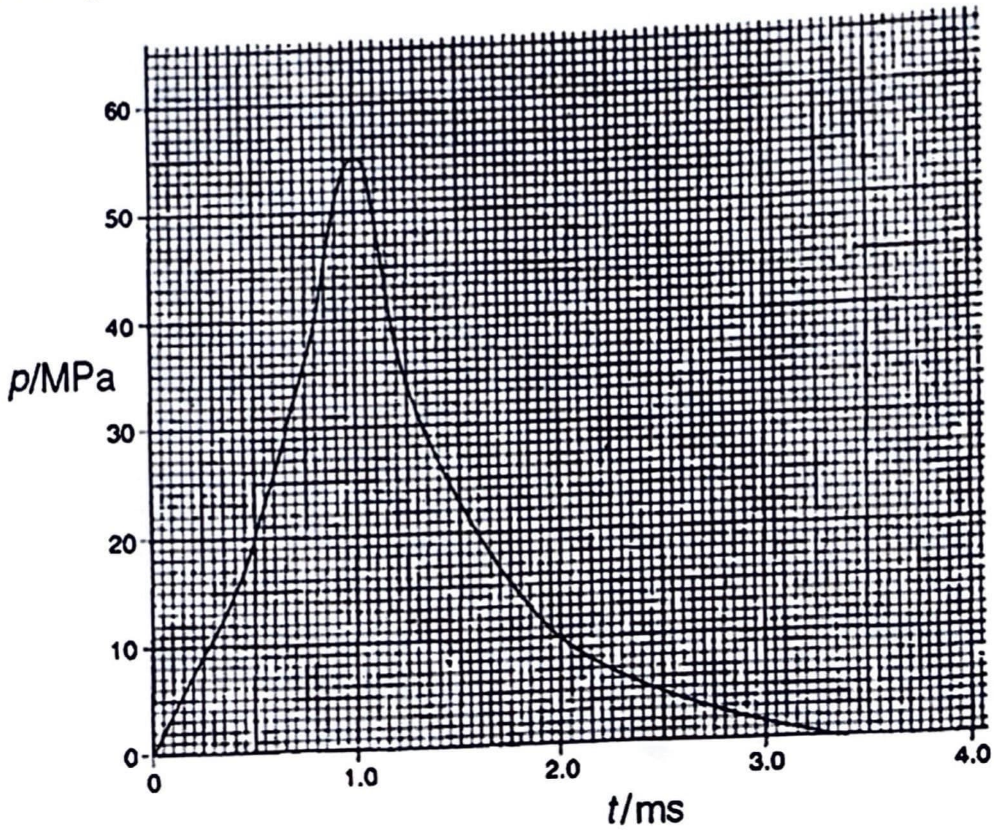
(i) the initial thrust, [2]

(ii) the initial resultant acceleration, [2]

(iii) the final mass of the rocket and its contents. [2]

D8 [N94/2/1]

A projectile of mass 3.2×10^{-2} kg is fired from a horizontal cylindrical barrel of cross-sectional area 2.8×10^{-4} m^2 by means of compressed air. The variation with time of the excess pressure p of the gas in the barrel above atmospheric pressure is shown in the figure below.



(a) Calculate

(i) the maximum force exerted on the projectile, [2]

(ii) the acceleration of the projectile which would result from the force calculated in **(i)**. [2]

(b) From the graph, estimate the total change in momentum experienced by the projectile, due to the compressed gas. [3]

(c) The speed of the projectile changes from zero to 270 m s^{-1} as it leaves the barrel. What is the change in momentum of the projectile? [1]

(d) Compare your answers in **(b)** and **(c)** and suggest a possible reason for the difference. [2]

Collisions and Conservation of Momentum

D9 [J87/II/9]

A body A of mass m moving with velocity u makes an elastic head-on collision with an identical body B which is initially at rest.

(a) Describe in words the motion of the bodies after the collision. [2]

(b) The elastic collision mentioned above is one in which the bodies become temporarily compressed and remain in contact for a short time. On the same axes of velocity against time, sketch labelled graphs of velocity of A and the velocity of B. The time axis should extend from a time before the bodies come into contact to a time after they separate; mark on this axis the time t_c at which they first touch, the time t_0 at which they suffer maximum compression and the time t_s at which they separate. [4]

(c) Explain why bodies have the same velocity at t_0 (the time of maximum compression). [1]

(d) What is the velocity of the two bodies at t_0? [1]

(e) Hence, find in terms of m and u, the total kinetic energy of the bodies at t_0 and again at a time after they have completely separated. [2]

(f) Account for the difference between the energies in **(e)**. [1]

D10 [N82/1/1]

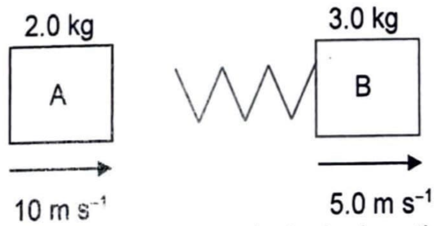
A particle of mass m moving with speed u makes a head-on collision with an identical particle which is initially at rest. The particles coalesce and move off with a common velocity.

(a) Find the common speed of the particles after the collision. [1]

(b) Find the ratio of the kinetic energy of the system after the collision to that before it. [2]

(c) What happens to the kinetic energy that is 'lost'? [1]

D11 Block A of mass 2.0 kg moves with a velocity of 10 m s^{-1} on a smooth horizontal table. Block B of mass 3.0 kg moves with a velocity of 5.0 m s^{-1} in front of A in the same direction. A light spring of force constant, k = 1000 N m^{-1} is attached to B as shown in the following figure.



When A collides with B, there will be an instant when the spring experiences maximum compression.

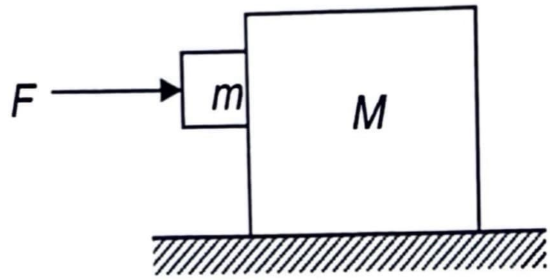
(a) Calculate x, the maximum compression of the spring. (For a spring that is compressed or extended by an amount x, the elastic potential energy stored in it is ½ kx^2.) [4]

(b) Calculate the velocity of A and B after they are separated. [4]

Challenging Questions

C1 The two blocks, m = 1.6 kg and M = 8.8 kg are free to move. The coefficient of static friction μ_s between the blocks is 0.38, but the surface beneath M is frictionless.

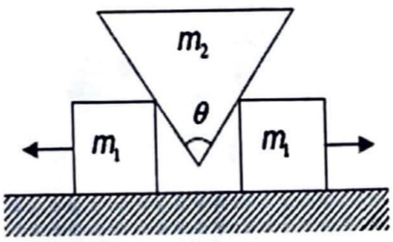
[Note: The maximum static friction f_{max} acting on an object is given by $f_{max} = \mu_s N$ where N is the normal reaction on the object.]



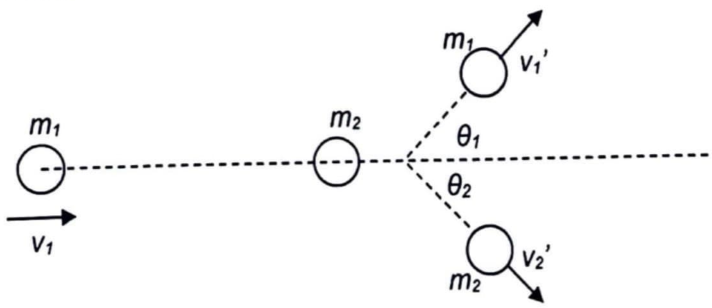
(a) Draw free-body diagrams of forces on m and M.

(b) What is the minimum horizontal force F is needed to hold m against M and what is the minimum acceleration produced by this force?

C2 Express the accelerations, a_1 and a_2 of the rectangular blocks of mass m_1 and the triangular block of mass m_2 in terms of m_1, m_2 and θ. Neglect friction.



C3 A mass m_1 having an initial velocity v_1 collides with a stationary mass m_2. After the collision, m_1 and m_2 are deflected as shown below.



The velocity of m_1 after collision is v_1'. Show that

$$\tan\theta_2 = \frac{v_1' \sin\theta_1}{v_1 - v_1' \cos\theta_1}$$

Given that θ_1 is 52.0° and θ_2 is 60.0°, calculate the percentage loss in the kinetic energy of m_1.

Numerical Answers

D2 **(a)** 1040 N
(b) 977 N

D3 **(a)** 0.61 m s^{-2}
(b) 4.9 × 10^{3} N
(c) 5.9 × 10^{3} N

D4 2.2 m s^{-2} upwards

D5 **(d)(i)** 55.4 N
(d)(ii) 55.4 N

D6 5.6 m s^{-1}

D8 **(a)(i)** 1.5 × 10^{4} N
(a)(ii) 4.8 × 10^{5} m s^{-2}
(b) 15 N s
(c) 8.6 N s

D11 **(a)** 0.173 m
(b) 4.0 m s^{-1}, 9.0 m s^{-1}

C1 **(b)** 49 N, 4.7 m s^{-2}

C2

$$a_1 = \frac{m_2 g \tan\frac{\theta}{2}}{m_2 + 2m_1 \tan^2\frac{\theta}{2}}, \quad a_2 = \frac{m_2 g}{m_2 + 2m_1 \tan^2\frac{\theta}{2}}$$

C3 12.8%

Tutorial 3 Dynamics Suggested Solutions

S1 Newton's First Law of Motion states that a body continues in its state of rest or uniform motion in a straight line unless a resultant external force acts on it,

Newton's Second Law states that the rate of change of momentum of a body is proportional to the resultant force acting on it and occurs in the direction of the force.

Newton's Third Law states that if body A exerts a force on body B, body B exerts an equal and opposite force on body A.

S2 The mass of an object is the property of the object which resists change in motion. The weight of an object refers to the force experienced by a mass in a gravitational field.

The mass of an object is the same regardless of its location but the weight of the object depends on the strength of the gravitational field it is placed in.

S3 The two forces must be equal in magnitude and opposite in direction, be of the same type and they must act on two different bodies.

Both forces must act at the same time.

S4 Linear momentum of a body is the product of its mass and velocity.

Its unit can be expressed as either N s or kg m s^{-1}.

S5 The rate of change of momentum of a body is proportional to the resultant force acting on it. When there is no net external force acting on a system, the total momentum remains constant. (However, it should be noted that within the system, each body can exert an internal force on each other; the momentum of the individual bodies can thus change.)

S6 From Newton's Second Law,

$$F_{net} = \frac{d(mv)}{dt}$$

$$= m\frac{dv}{dt} + v\frac{dm}{dt}$$

$$= m\frac{dv}{dt} \quad \text{(for constant mass, } \frac{dm}{dt}\text{=0)}$$

$$= ma$$

S7 Impulse of a force is the product of the force and the time interval for which the force acts on the body. It is mathematically equal to the area under the force-time graph.

S8 The principle of conservation of momentum states that the total momentum of a system remains constant provided no net external force acts on it.

S9 Suppose that a moving object A, of mass m_A and velocity u_A, collides with an object B, of mass m_B and velocity u_B, moving in the same direction as shown in Fig. 1(a).

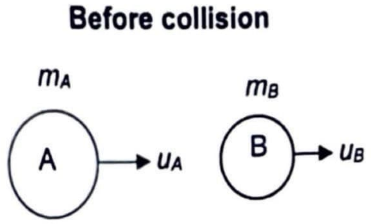


Fig. 1(a)

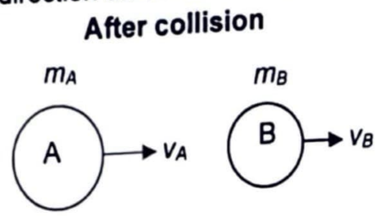


Fig. 1(b)

After the collision, they move off with velocities v_A and v_B as shown in Fig. 1(b).

During the brief collision, these objects exert large forces on one another. By Newton's Third Law, the force F_{BA} exerted by B on A is equal and opposite to the force F_{AB} exerted by A on B (see Fig. 2).

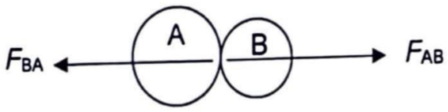


Fig. 2

$$F_{BA} = -F_{AB}$$

$$\frac{\Delta p_A}{\Delta t} = -\frac{\Delta p_B}{\Delta t}$$

$$\frac{m_A v_A - m_A u_A}{\Delta t} = -\frac{(m_B v_B - m_B u_B)}{\Delta t}$$

$$m_A v_A - m_A u_A = -(m_B v_B - m_B u_B)$$

Re-arranging the equation, $m_A u_A + m_B u_B = m_A v_A + m_B v_B$

Total initial momentum of bodies = Total final momentum of bodies

(provided no external force act on bodies A and B)

S10 Kinetic energy is conserved in elastic collisions but not in inelastic collisions. Objects stick together in completely inelastic collisions. For all collisions, total momentum and total energy are conserved.

SP1 Resolving forces parallel to slope,

$F = ma$

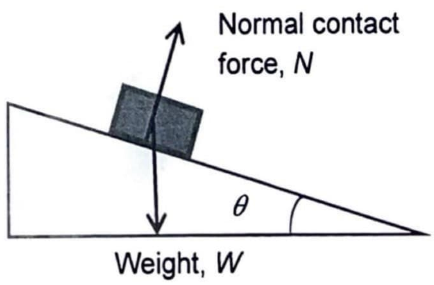
$W \sin \theta = ma$

$mg \sin \theta = ma$

$a = g \sin \theta$

The acceleration of the trolley is independent of the mass of the trolley.

Answer: C



SP2 Taking the two blocks as a system,

$F = ma$

$54 - 6.0 = (6.0 + 2.0)\,a$

$a = 6.0 \text{ m s}^{-2}$

resultant force on 6.0 kg mass = $ma = 6.0 \times 6.0 = 36$ N

Answer: B

SP3 **(a)** thrust on the rocket = rate of change of momentum of gas

$$= \frac{\Delta p}{\Delta t} = \frac{70 \times 6.0 \times 10^3}{1} = 4.2 \times 10^5 \text{ N}$$

(b) initial acceleration of the rocket

$$= \frac{F - Mg}{M} = \frac{4.2 \times 10^5 - (3.0 \times 10^4)(9.81)}{3.0 \times 10^4} = 4.19 \text{ m s}^{-2}$$

(c) initial net force on the man = $ma = 50 \times 4.19 = 210$ N

SP4 change in velocity $\Delta v = v - u = 18 - (-23) = 41 \text{ m s}^{-1}$

impulse provided by the wall

= change in momentum of ball

$= p_f - p_i$

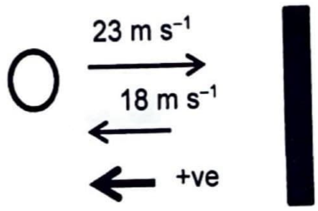
$= m(v_f - v_i)$

$= m \times \Delta v$

$= 0.080 \times 41$

= 3.3 N s away from wall

Answer: B



SP5 For a constant force, acceleration is constant and the rate of change of momentum (mv/t) is constant. So a graph of momentum against time will give a straight line graph with a constant gradient that is numerically equal to the constant force applied.

Answer: A

SP6 Initial total momentum of the two trolleys is zero as they are held stationary.
After the collision, by principle of conservation of momentum, the final total momentum of the two trolleys must also be zero.
Since this is a completely inelastic collision, the speed of Y is zero.

Answer: A

SP7 By conservation of momentum,

$mu + 0 = mv_1 + Mv_2$ …(1)

Using relative speed of approach = relative speed of separation,

$u - 0 = v_2 - v_1$

$v_1 = v_2 - u$ … (2)

Substitute (2) into (1): $mu + 0 = m(v_2 - u) + Mv_2$

$2mu = mv_2 + Mv_2$

$$v_2 = \frac{2mu}{M+m}$$

Answer: C

SP8 By conservation of momentum , $1\text{u} \times 0.100c - 4\text{u} \times 0.050c = 1\text{u} \times v_p + 4\text{u} \times v_{He}$

$-0.100c = v_p + 4v_{He}$ ...(1)

Using relative speed of approach = relative speed of separation,

$0.100c + 0.050c = v_{He} - v_p$ ...(2)

(1) + (2): $0.050\,c = 5v_{He}$

$v_{He} = 0.010c$

subst. into (2): $v_p = -\,0.140c$

Answer: A

SP9 **(a)** **(i)** $(0.30)(0.50) + (0.20)(-1.00) = (0.30 + 0.20)v$

$v = -0.10\ \text{m s}^{-1}$ (i.e. final velocity is in the initial direction of Block Y)

(ii) Loss in Kinetic Energy

$= [\ ½\,(0.30)(0.50)^2 + ½\,(0.20)(1.00)^2] - ½\,(0.30 + 0.20)\,(0.10)^2$

$= 0.135$ J

(b) By conservation of momentum

$(0.30)(0.50) + (0.20)(-1.00) = (0.30)v_1 + (0.20)v_2$

$-0.05 = 0.3v_1 + 0.2v_2$ ----- (1)

relative speed of approach = relative speed of separation

$0.50 - (-1.00) = v_2 - v_1 \rightarrow v_2 = 1.50 + v_1$ ----- (2)

subt. (2) into (1): $-0.05 = 0.3v_1 + 0.2\,(1.50 + v_1)$

$v_1 = -0.70\ \text{m s}^{-1}$

subt. into (2): $v_2 = 1.50 + (-0.70) = 0.80\ \text{m s}^{-1}$

SP10 **(a)** **(i)** Completely inelastic collision

(ii) $p = mv = (0.20)(8.0) = 1.6\ \text{kg m s}^{-1}$

(iii) Consider the system to be consisting of the plasticine and the Earth.

On collision with the Earth, the initial momentum of the plasticine is transferred to the Earth.

The total kinetic energy is not conserved as some of the initial kinetic energy of the plasticine is transferred to other forms of energy, such as work done in deforming the plasticine.

(The Earth is attracting the plasticine with a gravitational force and the plasticine is attracting the Earth with a gravitational force that is equal in magnitude and opposite in direction. The Earth and plasticine experience the same impulse. Since Earth and plasticine was initially at 'rest', the total momentum of the system is always zero. So the momentum of the Earth is equal in magnitude and opposite in direction to the momentum of the plasticine before impact, keeping the total momentum zero.)

(b) Consider the two magnets as a system and that there is no resultant force acting on the system (assume all surfaces are smooth). When both magnets are held stationary, total momentum of the system is zero.

By Newton's Third Law, each magnet will experience a repulsive force due to the other, and the forces are of the same magnitude but opposite in direction. Hence, they experience the same impulse due to the repulsive force. This means that when they spring apart from rest, they will have equal and opposite momentum.

Total momentum of the system remains zero and momentum is thus conserved.