Additional Practice Questions

1. **[HCI/2007/JC2/CT]**

Loy is a member of his school's soccer team, and the team is participating in an invitation tournament where each team is to play a total of 3 matches. The probabilities that Loy gets selected to play in the first match and the second match are $\frac{7}{9}$ and $\frac{4}{7}$ respectively. The probability that Loy gets selected to play in the third match, given that he was selected to play in the second match is $\frac{1}{3}$; and the probability that Loy gets selected to play in the third match, given that he was not selected to play in the second match is $\frac{4}{5}$.

Find the probability that

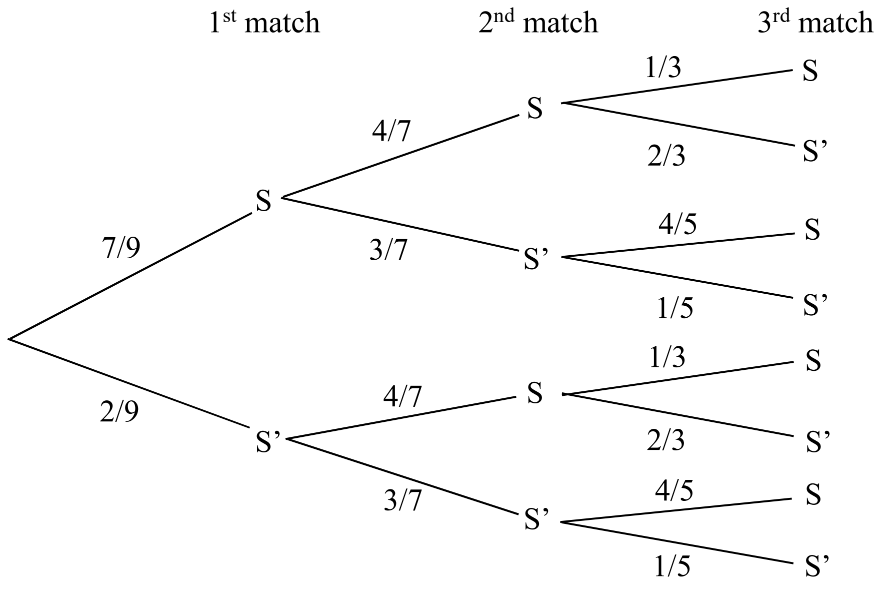
(i) Loy is not selected to play in the third match,

(ii) Loy gets selected to play in all the 3 matches,

(iii) Loy gets selected to play in at most 1 of the 3 matches.

[(i) $\frac{7}{15}$ (ii) $\frac{4}{27}$ (iii) $\frac{233}{945}$]

Let event S = Loy gets selected to play



(i) P(2nd S, 3rd S') + P(2nd S', 3rd S') = (4/7)(2/3) + (3/7)(1/5)
= 7/15

(ii) (7/9)(4/7)(1/3) = 4/27

(iii) P(1st S', 2nd S', 3rd S') + P(S, S', S') + P(S', S, S') + P(S', S', S)
= (2/9)(3/7)(1/5) + (7/9)(3/7)(1/5) + (2/9)(4/7)(2/3) + (2/9)(3/7)(4/5)
= 233/945

2. **[ACJC/2010/Prelim/8]**

A girl is given two toys, a doll and a teddy bear. The probability that she plays with the doll is 0.5 and the probability that she plays with the teddy bear is 0.6. The probability that she plays with the doll or the teddy bear or both is 0.8. Find the probability that

(i) She plays with the doll but not the teddy bear.

(ii) She plays with the doll given that she does not play with the teddy bear.

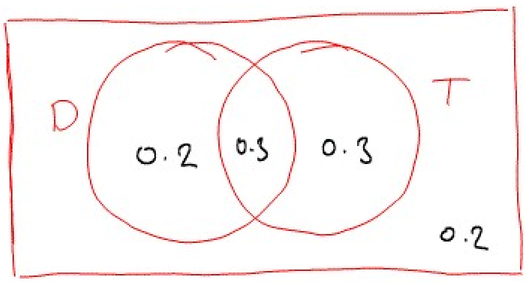
[(i) 0.2; (ii) 0.5]

Let D be the event she plays with the doll

" T " " teddy

$P(D) = 0.5 \quad P(T) = 0.6 \quad P(D \cup T) = 0.8$

$\therefore \quad P(D \cap T) = 0.5 + 0.6 - 0.8 = 0.3$



(i) $P(D \cap T') = 0.2$

(ii) $P(D \mid T') = \dfrac{P(D \cap T')}{P(T')}$

$= \dfrac{0.2}{0.4} = \dfrac{1}{2}$

3. **[RI(JC)/2011/Promo/9]**

A group of students take a physical fitness test. A student who fails the test on the first attempt is allowed one further attempt. For a randomly chosen student, the probability of passing the test on the first attempt is 0.6 and the probability of passing on the second attempt is 0.8.

(i) Draw a tree diagram to represent the possible outcomes.

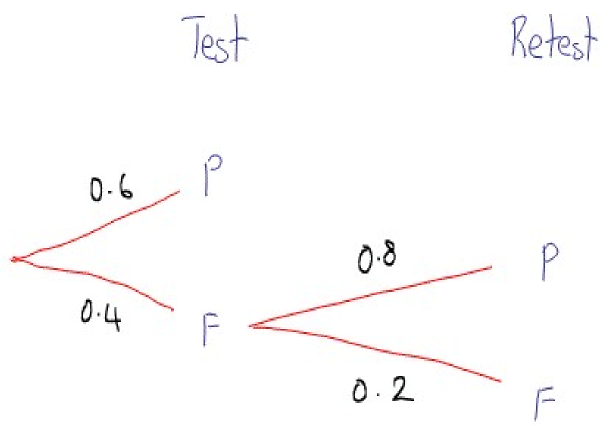
(ii) Find the probability that a randomly chosen student fails the test on both attempts.

(iii) Given that a student passes the test, find the probability that it is on the second attempt.

(iv) Three students taking the test are chosen at random. Find the probability that two of them pass on the first attempt and the other passes on the second attempt.

[(ii) 0.08; (iii) 0.348; (iv) 0.3456]

(i)



(ii) $P(FF) = 0.4 \times 0.2 = 0.08$

(iii) $P(FP \mid \text{Passed}) = \dfrac{P(FP)}{P(\text{Pass})}$

$= \dfrac{0.4 \times 0.8}{0.4 \times 0.8 + 0.6} \approx 0.348$ (3 sf)

(iv) $P(P, P, FP) + P(P, FP, P) + P(FP, P, P)$

$= 3 \times 0.6 \times 0.6 \times (0.4 \times 0.8)$

$= 0.3456$

4. **[AJC/2011/Promo/10]**

(a) E and F are two events such that

$$\mathrm{P}(E)=\frac{1}{4},\ \mathrm{P}(E\cup F)=\frac{1}{3} \text{ and } \mathrm{P}(F)=p.$$

Find p if

(i) E and F are mutually exclusive;

(ii) E and F are independent;

(iii) E is a subset of F.

(b) The following information is known about two events X and Y.

$$\mathrm{P}(X)=\frac{2}{5},\ \mathrm{P}(Y|X)=\frac{7}{10} \text{ and } \mathrm{P}(X'\cap Y)=\frac{1}{4}.$$

Find the following probabilities:

(i) $\mathrm{P}(X\cap Y)$

(ii) $\mathrm{P}(Y)$

(iii) $\mathrm{P}(X'\cap Y')$

[(a) (i) $\frac{1}{12}$; (ii) $\frac{1}{9}$; (iii) $\frac{1}{3}$; (b) (i) $\frac{7}{25}$; (ii) $\frac{53}{100}$; (iii) $\frac{7}{20}$]

(a) (i) If E, F mutually exclusive

$P(E) + P(F) = P(E \cup F)$

$p = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$

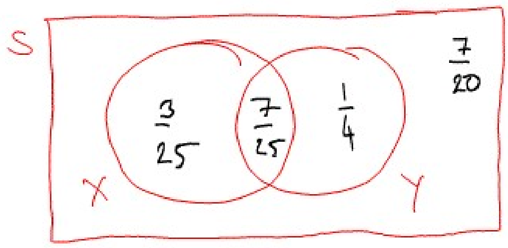
(ii) If E, F independent $P(E \cap F) = P(E) \times P(F)$

$P(E \cup F) = P(E) + P(F) - P(E) \times P(F)$

$\frac{1}{3} = \frac{1}{4} + p - \frac{1}{4}p \Rightarrow p = \frac{1}{9}$

(iii) If $E \subseteq F$ then $\frac{1}{3} = P(E \cup F) = P(F) = p$.

(b)



(i) $P(X \cap Y) = P(X)\,P(Y|X) = \frac{2}{5} \times \frac{7}{10} = \frac{7}{25}$

(ii) $P(Y) = P(X \cap Y) + P(X' \cap Y) = \frac{7}{25} + \frac{1}{4} = \frac{53}{100}$

(iii) $P(X' \cap Y') = 1 - P(X \cup Y) = 1 - (P(X) + P(X' \cap Y)) = \frac{7}{20}$

5. **[CJC/2019/Prelim/02/Q11]**
A palindrome is a string of letters or digits that is the same when you read it forwards or backwards. For example: HHCCHH, RACECAR, STATS are palindromes.

A computer is instructed to use any of the letters **R**, **O**, **F**, **L** to randomly generate a string of 5 letters. Repetition of any letter is allowed, but the string cannot contain only one letter. For example, RRRRR is not allowed.

Events A and B are defined as follows:
A: the string generated contains 2 distinct letters
B: the string generated is a palindrome

(i) Find $\mathrm{P}(A)$ and $\mathrm{P}(B)$. [5]

(ii) Find $\mathrm{P}(A \cap B)$ and hence determine if A and B are independent. [3]

(iii) Find the probability that the string generated *either* contains 2 distinct letters, *or* that it is a palindrome, or *both*. [2]

(iv) Find the probability that the string generated contains 2 distinct letters, given that it is a palindrome. [2]

[(i) $\mathrm{P}(A)=\frac{3}{17}, \mathrm{P}(B)=\frac{1}{17}$ (ii) $\frac{3}{85}$ (iii) $\frac{1}{5}$ (iv) $\frac{3}{5}$]

Solution

(i) No of ways to form the string unrestricted $= 4^5 - 4 = 1020$

No of ways to form the string using 2 letters $= {}^4C_2 \times (2^5 - 2) = 180$

OR $\quad {}^4C_2 \times \frac{5!}{4!} \times 2 + {}^4C_2 \times \frac{5!}{2!3!} \times 2 = 180$

Hence $\mathrm{P}(A) = \frac{180}{1020} = \frac{3}{17} = 0.176$

No of ways to have a palindrome $= 4^3 - 4 = 60$ OR ${}^4C_2 \times 2 \times \frac{3!}{2!} + {}^4C_3 \times 3! = 60$

Hence $\mathrm{P}(B) = \frac{60}{1020} = \frac{1}{17} = 0.0588$

(ii) No of ways to form palindrome of 2 letters $= {}^4C_2 \times (2^3 - 2)$ or ${}^4C_2 \times \frac{3!}{2!} \times 2 = 36$

Hence $\mathrm{P}(A \cap B) = \frac{36}{1020} = \frac{3}{85} = 0.0353$

$\mathrm{P}(A) \times \mathrm{P}(B) = 0.0104$

Since $\mathrm{P}(A \cap B) \neq \mathrm{P}(A) \times \mathrm{P}(B)$, A and B are not independent.

(iii) $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B) = \frac{3}{17} + \frac{1}{17} - \frac{3}{85} = \frac{1}{5} = 0.2$

(iv) $\mathrm{P}(A \mid B) = \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)} = \frac{3/85}{1/17} = \frac{3}{5} = 0.6$

6. **[HCI/2020/Prelim/02/Q7]**

A team of 15 students was selected for an outdoor education trip. One student volunteered to be the trip leader while another volunteered as the assistant trip leader. They decided to have some ice-breaker games, where all 15 students sat in a circle.

(a) Find the probability that both leaders were not seated together. [2]

During the ice-breaker games, it was realised that 3 of the other 13 students belonged to the same class.

(b) Find the probability that none of these three students sat next to each other, given that both leaders were not seated together. [3]

(c) After the ice-breaker games, both leaders decided to randomly break the team up into two groups to discuss about administrative matters and training program for the trip. The group which discussed about the training program consisted of 7 students. Find the probability that the trip leader was heading the training program group while the assistant leader was in charge of the administrative matters group. [2]

[(a) $\frac{6}{7}$ (b) $\frac{95}{156}$ (c) $\frac{4}{15}$]

7(a)	Required probability $=\dfrac{\frac{13!}{13}\times {}^{13}C_2\times 2!}{\frac{15!}{15}}$ $=\dfrac{6}{7}$ or 0.857 (3 s.f.) **Alternative** Total numbers of ways = $(15-1)!=14!$ Number of ways leaders seated together $=(14-1)!\times 2=13!\times 2$ Probability = $1-\dfrac{13!\times 2}{14!}=1-\dfrac{1}{7}=\dfrac{6}{7}$
7(b)	Let X be the event that the 3 students from the same class are seated separately. Let Y be the event that both leaders are seated separately. Required probability $=\dfrac{P(X\cap Y)}{P(Y)}$

	$=\dfrac{\mathrm{P}(X)-\mathrm{P}(X\cap Y')}{P(Y)}$ $=\dfrac{\frac{12!}{12}\times {}^{12}C_3\times 3!\ -\frac{11!}{11}\times 2!\ \times {}^{11}C_3\times 3!}{\frac{\frac{15!}{15}}{6/7}}$ $=\dfrac{95}{156}$ or 0.609 (3 s.f.)
7(c)	Required probability $=\dfrac{{}^{13}C_6}{{}^{15}C_7}$ $=\dfrac{4}{15}$ or 0.267 (3 s.f)

7. **[VJC/2016/Prelim/7]**

Box A contains 10 red, 8 blue and 7 green balls. Box B contains 2 white and 3 black balls. All the balls are indistinguishable except for their colours. Three balls are taken from Box A and two balls are taken from Box B, at random and without replacement.
Mr Wong guesses that there are at least 1 red ball and exactly 2 black balls taken, while Mr Tan guesses that all the balls taken are of different colours.

(i) Show that the probability that Mr Wong is correct is 0.241, correct to 3 significant figures. [3]
(ii) Find the probability that Mr Tan is correct. [2]
(iii) Find the probability that Mr Wong is correct, given that Mr Tan is wrong. [3]

7i	P(Mr Wong is correct) = $\left(1-\dfrac{{}^{15}C_3}{{}^{25}C_3}\right)\times\dfrac{{}^{3}C_2}{{}^{5}C_2}=0.24065=0.241$
ii	P(Mr Tan is correct) = $\dfrac{{}^{10}C_1\times{}^{8}C_1\times{}^{7}C_1}{{}^{25}C_3}\times\dfrac{{}^{3}C_1\times{}^{2}C_1}{{}^{5}C_2}=\dfrac{84}{575}=0.146$ Alternative method: P(Mr Tan is correct) $=\dfrac{10\times8\times7}{25\times24\times23}\times3!\ \times\dfrac{3\times2}{5\times4}\times2!\ =\dfrac{84}{575}=0.146$
iii	P(Mr Wong's guess is right, given that Mr Tan's guess is wrong) $=\dfrac{\text{P(Mr Wong is correct and Mr Tan is wrong)}}{\text{P(Mr Tan is wrong)}}$ $=\dfrac{0.24065}{1-0.14609}$ $=0.282$

8. **[MJC/2014/Prelim/5]**

A box contains 45 balls, which are distinctly labelled with integers 1 to 45. Seven balls are drawn randomly from the box without replacement, and the observed integers are recorded as 'winning' numbers.

Andy chooses eight distinct integers randomly from the set of integers $\{1,2,3,...,44,45\}$.

(i) Show that the probability that none of Andy's chosen integers matches the 'winning' numbers is 0.227. [1]

(ii) Hence find the probability that Andy's chosen integers match exactly four of the 'winning' numbers, given that he has at least one winning number. [3]

[(i) 0.227 (ii) 0.0155]

8(i)	$\mathrm{P}(\text{no number matched}) = \dfrac{\binom{38}{8}}{\binom{45}{8}} = 0.226874 = 0.227 \text{ (to 3 s.f.)}$
(ii)	$\mathrm{P}(4 \text{ numbers matched} \mid \text{at least 1 number matched}) = \dfrac{\mathrm{P}(4 \text{ numbers matched})}{1-\mathrm{P}(\text{no number matched})}$ $= \dfrac{\dfrac{\binom{7}{4}\binom{38}{4}}{\binom{45}{8}}}{1-0.227} = 0.0155 \text{ (to 3 s.f.)}$

9. **[2013 HCI/2/10]**

A bag contains 15 tokens that are indistinguishable apart from their colours. 2 of the tokens are blue and the rest are either red or green. Participants are required to draw the tokens randomly, one at a time, from the bag without replacement.

(i) Given that the probability that a participant draws 2 red tokens on the first 2 draws is $\frac{1}{35}$, show that there are 3 red tokens in the bag.

(ii) Find the probability that a participant draws a red or green token on the second draw.

Events A and B are defined as follows.

A: A participant draws his/her second red token on the third draw.

B: A participant draws a blue token on the second draw.

(iii) Find $\mathrm{P}(A \cup B)$.

(iv) Determine if A and B are independent events.

(i)	Let n be the number of red balls. $\text{P}\left(\text{drawing two red balls in first two draws}\right)$ $=\frac{n}{15}\times\frac{n-1}{14}$ $=\frac{n^2-n}{210}$ $\frac{n^2-n}{210}=\frac{1}{35}$ $n^2-n-6=0$ $\left(n-3\right)\left(n+2\right)=0$ $n=3$ or -2 (rej.)
(ii)	**Method 1:** P(red or green token on second draw) = P(blue on first draw and red or green on second draw) + P(non-blue on first draw and red or green on second draw) $=\frac{2}{15}\times\frac{13}{14}+\frac{13}{15}\times\frac{12}{14}=\frac{13}{15}$ or 0.867 **Method 2:** P(red or green token on second draw) $=1-\frac{2}{15}\times\frac{1}{14}-\frac{13}{15}\times\frac{2}{14}$ = 0.867
(iii)	$\text{P}\left(A\right)=\frac{3}{15}\times\frac{12}{14}\times\frac{2}{13}\times 2=\frac{24}{455}$ $\text{P}\left(B\right)=\frac{13}{15}\times\frac{2}{14}+\frac{2}{15}\times\frac{1}{14}=\frac{2}{15}$ $\text{P}\left(A\cap B\right)=\frac{3}{15}\times\frac{2}{14}\times\frac{2}{13}=\frac{2}{455}$ $\text{P}\left(A\cup B\right)=\text{P}\left(A\right)+\text{P}\left(B\right)-\text{P}\left(A\cap B\right)$ $=\frac{248}{1365}$ or 0.182
(iv)	Since $\text{P}\left(A\right)\times\text{P}\left(B\right)=\frac{16}{2275}\neq\frac{2}{455}=\text{P}\left(A\cap B\right)$ A and B are not independent events.

10. **[2013MI/II/7]**

A group of students take an examination in Science. A student who fails the examination at the first attempt is allowed one further attempt. For a randomly chosen student, the probability of passing the examination at the first attempt is p. If the student fails the examination at the first attempt, the probability of passing at the second attempt is 0.3 more than the probability of passing the examination at the first attempt.

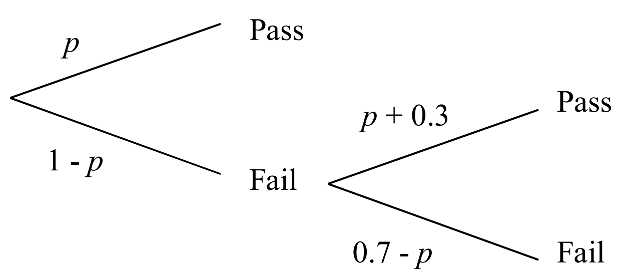
Show that the probability that a randomly chosen student passes the examination is $0.3+1.7p-p^2$.

(a) Find the value of p such that the probability that a randomly chosen student passes the examination on the first attempt given that the student passes is 0.6.

(b) Two students are randomly chosen.

(i) Find the probability that one passes the examination on the first attempt and the other passes the examination on the second attempt, leaving your answer in terms of p.

(ii) Find the value of p such that the value of the probability in part (i) is maximum.



$P(\text{student passes exam}) = p+(1-p)(p+0.3) = 0.3+1.7p-p^2$ (shown)

(a)

P(Student passes 1st attempt | Student passes exam) = 0.6

$$\frac{P(\text{Student passes 1st attempt})}{\text{P(Student passes exam)}} = 0.6$$

$$\frac{p}{0.3+1.7p-p^2} = 0.6$$

$$0.6p^2 - 0.02p - 0.18 = 0$$

$$p = 0.565 \text{ or } -0.531 \text{ (rejected)}$$

(b)(i)	Let the required probability be C $C = [p(1-p)(p+0.3)] \times 2$ $= 0.6p + 1.4p^2 - 2p^3$ $\frac{\mathrm{d}C}{\mathrm{d}p} = 0.6 + 2.8p - 6p^2$
(b)(ii)	$\frac{\mathrm{d}^2C}{\mathrm{d}p^2} = 2.8 - 12p$ For maximum / minimum p, $\frac{\mathrm{d}C}{\mathrm{d}p} = 0.6 + 2.8p - 6p^2 = 0$ $p = 0.626$ or -0.159 (rejected) At $p = 0.626$, $\frac{\mathrm{d}^2C}{\mathrm{d}p^2} = 2.8 - 12(0.626) = -4.712 < 0$ $\therefore C$ is maximum when $p = 0.626$