MERIDIAN JUNIOR COLLEGE
JC2 Preliminary Examination
Higher 2

H2 Mathematics 9740/02

Paper 2 **18 September 2015**

3 Hours

Additional Materials: Writing paper
List of Formulae (MF 15)

READ THESE INSTRUCTIONS FIRST

Write your name and civics group on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

Section A: Pure Mathematics [40 marks]

1 The sequence $u_1,\ u_2,\ u_3,\ldots$ is such that $u_1 = 3$ and

$$u_{n+1} = 3 - \frac{2}{u_n}, \quad \text{for all } n \geq 1.$$

(i) By writing the first 3 terms of this sequence, show that a possible conjecture for u_n is $\dfrac{a^{n+1}-1}{a^n-1}$, where a is a positive integer to be determined. [2]

(ii) Using the value of a found in part **(i)**, prove by induction that $u_n = \dfrac{a^{n+1}-1}{a^n-1}$ for all $n \geq 1$. [4]

(iii) Hence determine if the limit of $u_1u_2u_3\ldots u_n$ exists as $n \to \infty$. [2]

2

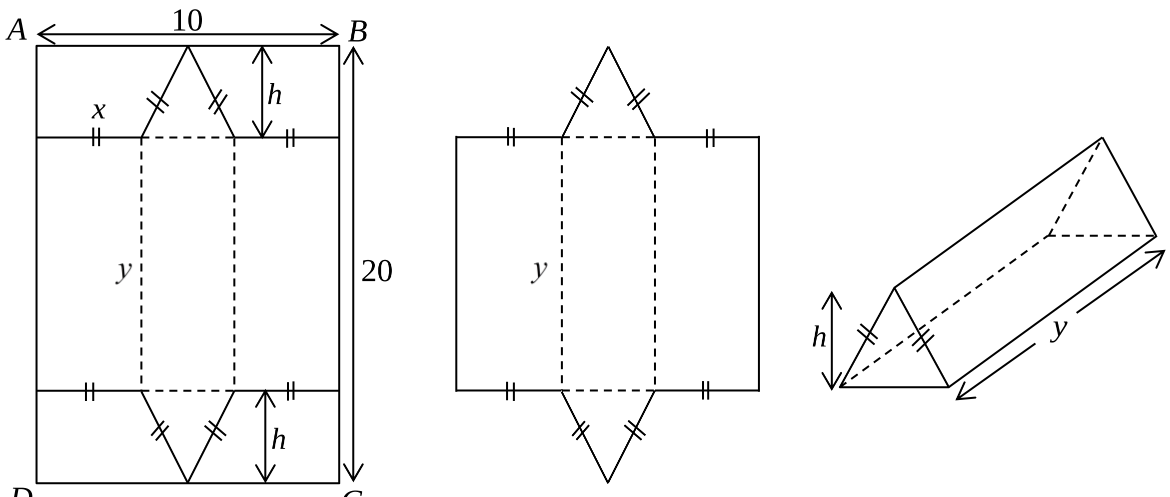

Fig. 1 **Fig. 2** **Fig. 3**

Fig. 1 shows a rectangular piece of cardboard *ABCD* of sides 10 cm and 20 cm. A trapezium shape is cut out from each corner, to give the shape shown in Fig. 2. This shape consists of 2 isosceles triangles and 3 rectangles of different sizes. The remaining cardboard shown in Fig. 2 is folded along the dotted lines, to form a closed triangular prism shown in Fig. 3.

(i) Show that the volume V cm^3 of the closed triangular prism is given by $V = \frac{1}{5}\left(h^4 - 10h^3 - 25h^2 + 250h\right)$. [4]

(ii) Use differentiation to find the maximum value of V, proving that it is a maximum. [5]

3 **(a)** It is given that $1+2\mathrm{i}$ is a root of $x^3+ax^2+bx-5=0$, where a and b are real. Find the values of a and b and the other roots. [4]

(b) **(i)** Find the possible values of z such that $\left|z^4\right|=2$ and z^4 is a negative real number. [3]

Let z_1, z_2, z_3 and z_4 be the values of z found in part **(i)** such that

$$-\pi < \arg(z_1) < \arg(z_2) < \arg(z_3) < \arg(z_4) \leq \pi .$$

(ii) Write down a complex number w such that $z_2 = wz_1$. [1]

(iii) Hence, or otherwise, find the exact value of $\left|wz_3 - w^* z_3\right|^2$, where w^* is the conjugate of w. [2]

[Turn Over

4 The planes p and q have equations $\mathbf{r}\cdot\begin{pmatrix}1\\1\\1\end{pmatrix}=1$ and $\mathbf{r}\cdot\begin{pmatrix}1\\0\\1\end{pmatrix}=2$ respectively, and meet in the line l.

(i) Find the acute angle between p and q. [2]

(ii) Find a vector equation for l. [2]

Let P be the set of planes $p_1, p_2, p_3, \ldots$, such that the cartesian equation of p_n is given by

$$x+u_n y+z=S_n,$$

where u_n is the n^{th} term of a geometric progression with first term 1 and common ratio $\frac{1}{2}$, and S_n is the sum of the first n terms of the geometric progression.

(iii) Write down a cartesian equation of p_k in terms of k, where $k \in \mathbb{Z}^{+}$. Hence state the limit of the acute angle between p_1 and p_k as $k \to \infty$. [3]

(iv) Show that l lies in all planes in P. [2]

Let π be a plane with equation $\mathbf{r}\cdot\begin{pmatrix}a\\b\\c\end{pmatrix}=d$.

(v) If $a \neq c$, describe the relationship between π and two randomly chosen planes in P. [1]

(vi) If $a=c=d=1$ and $b=\frac{1}{2}$, find the perpendicular distance from the point $(2,-1,0)$ to π. [3]

Section B: Statistics [60 marks]

5 A group of nine people consists of five men and four women.

(a) The nine people are to form a queue. Find the number of different arrangements if there are no two people of the same gender standing next to each other. [2]

(b) The group of nine people finds a round table with eight seats. Assuming only eight people are to be seated, find the number of different arrangements if

(i) there are no restrictions, [2]

(ii) two particular woman are seated at the round table but are not next to each other. [2]

6 It is known that p% of college students in Singapore score distinction for their Mathematics final year examination.

(a) Given the probability that at least 2 students out of 10 randomly chosen students score distinction for their Mathematics final year examination is 0.95, form an equation in terms of p and hence find the value p. [3]

(b) 50 samples of 10 students each are randomly chosen. Given that $p = 40$, find the probability that the mean number of students per sample who score distinction for their Mathematics final year examination is less than 3.5. [3]

[Turn Over

7 The number of people arriving at a particular bus stop in a period of 5 minutes is a random variable with the distribution $\text{Po}(2)$.

(i) Given that the probability that more than 9 people arrive at that particular bus stop in a period of t minutes is less than 0.759, show that the greatest possible integer value of t is 30. [2]

(ii) The number of people leaving that particular bus stop in a period of 5 minutes is a random variable with the distribution $\text{Po}(2.5)$. At 0800 on a particular day, there are 5 people at the bus stop. Using a suitable approximation, estimate the probability that there are more than 15 people at the bus stop at 0900. (You may assume that there are always people at that particular bus stop during this period. You should assume also that people arriving and leaving that particular bus stop are independent of each other.) [5]

(iii) Explain why a Poisson model would probably not be valid if applied to a time period of several hours. [1]

8 The time required by a student to complete the homework given in a week is a normally distributed continuous random variable X. Over a long period, it is known that the mean time required is 16 hours. The students are encouraged to switch off their mobile phones while doing their homework, and afterwards the time required, x hours, is measured for a random sample of 14 students. The results are summarised as follows.

$$\sum(x-16) = -23.8 \qquad \sum(x-16)^2 = 149.127$$

(i) Find unbiased estimates of the population mean and variance. Test, at the 5% significance level, whether there has been a change in the mean time required by a student to complete the homework given in a week. [6]

(ii) It is given that the standard deviation of X is 3 hours and the mean time taken for another 14 randomly chosen students to complete the homework is $\bar{x}$. Find the set of values of $\bar{x}$ for which the result of the test in part **(i)** would be to not reject the null hypothesis. [3]

9 **(a)** Sketch a diagram that might be expected when x and y are related approximately by $y = a + b\ln x$ in each of the cases **(i)** and **(ii)** below. In each case your diagram should include 6 points, approximately equally spaced with respect to x, and with all x- and y-values positive.

(i) a and b are both positive. [1]

(ii) a is positive and b is negative. [1]

(b) In a chemical reaction, the yield of a chemical compound, m grams, at different temperature t ^{o}C are given in the table below.

t	10	15	22	27	31	35	40	45
m	1.83	3.02	4.67	13.96	5.67	8.07	8.35	8.60

(i) Draw the scatter diagram for these values, labelling the axes clearly. [2]

(ii) One of the pairs of data appears to be recorded wrongly. Indicate the corresponding point on your diagram by labelling it P, and explain why the scatter diagram for the remaining points may be consistent with a linear model. [2]

For the rest of the questions, omit P in your calculation.

(iii) Calculate the equation of the least squares regression line of m on t. [1]

(iv) When the data $t = 50$ and $m = k$ was obtained and added to the seven pairs of data above, the new regression line is $m = 0.16934t + 0.72035$ correct to 5 decimal places. Find the value of k, giving your answer in 2 decimal places. [3]

[Turn Over

10 A manufacturer produces chocolate bars and candy bars. The masses, in grams, of the chocolate bars and candy bars are modelled as having independent normal distributions with means and standard deviations as shown in the table.

	Mean mass	Standard deviation
Chocolate bars	25	0.3
Candy bars	32	0.5

(i) Find the probability that the mean mass of 2 randomly chosen chocolate bars and 3 randomly chosen candy bars is less than 29 grams. [3]

(ii) To ensure that the chocolate bars produced are of similar quality, a quality control test was conducted and it was found that for 95% of the time, the difference in masses between any 2 randomly chosen chocolate bars is less than k grams. Find the value of k. [4]

The chocolate bars and candy bars are sold by weight at \$0.08 per gram and \$0.03 per gram respectively.

(iii) Find the probability that the total selling price of a randomly chosen chocolate bar and a randomly chosen candy bar is more than \$3. [3]

11 **(a)** A bag contains 25 balls that are indistinguishable apart from their colours. 15 of the balls are red and the rest are blue. 8 balls are drawn at random from the bag, without replacement. The number of red balls drawn is denoted by R.

(i) Show that $\mathrm{P}(R=3)=0.106$ correct to 3 significant figures. [2]

(ii) The most probable number of red balls drawn is denoted by r. By using the fact that $\mathrm{P}(R=r)>\mathrm{P}(R=r+1)$, show that r satisfies the inequality $(r+1)!(14-r)!(7-r)!(3+r)!>r!(15-r)!(8-r)!(2+r)!$ and use this inequality to find the value of r. [5]

(b) Kaylyn has an option of 2 routes to travel to school everyday. The probability that she chooses the first route is denoted by p. There is a 90% chance that she gets to school early using the first route. If she chooses the second route, there is a 85% chance that she will be early. For a general value of p such that $0\leq p\leq 1$, the probability that Kaylyn chooses the first route when she gets to school early is denoted by $\mathrm{f}(p)$. Show that $\mathrm{f}(p)=\dfrac{18p}{17+p}$. Prove by differentiation that as p increases, $\mathrm{f}(p)$ increases at a decreasing rate. [4]

END OF PAPER