Topic 14
Current of Electricity
Content

- Electric current
- Potential difference
- Resistance and resistivity
- Electromotive force

Learning Outcomes

Candidates should be able to:

(a) show an understanding that electric current is the rate of flow of charge
(b) derive and use the equation $I = nAvq$ for a current-carrying conductor, where n is the number density of charge carriers and v is the drift velocity
(c) recall and solve problems using the equation $Q = It$
(d) recall and solve problems using the equation $V = W/Q$
(e) recall and solve problems using the equations $P = VI$, $P = I^2R$ and $P = V^2/R$
(f) recall and solve problems using the equation $V = IR$
(g) sketch and explain the I–V characteristics of various electrical components such as an ohmic resistor, a semiconductor diode, a filament lamp and a negative temperature coefficient (NTC) thermistor
(h) sketch the resistance-temperature characteristic of an NTC thermistor
(i) recall and solve problems using the equation $R = \rho l/A$
(j) distinguish between electromotive force (e.m.f.) and potential difference (p.d.) using energy considerations
(k) show an understanding of the effects of the internal resistance of a source of e.m.f. on the terminal potential difference and output power.

Introduction

Imagine a world without electricity. There would be no microwaves, no television, no hospitals or life-support systems, and worse for some, no wi-fi. In the classic science fiction movie, *The Day the Earth Stood Still*, an alien spacecraft neutralizes all electricity on Earth. As a result, society collapses, and technology is reduced to that of the 16th century.

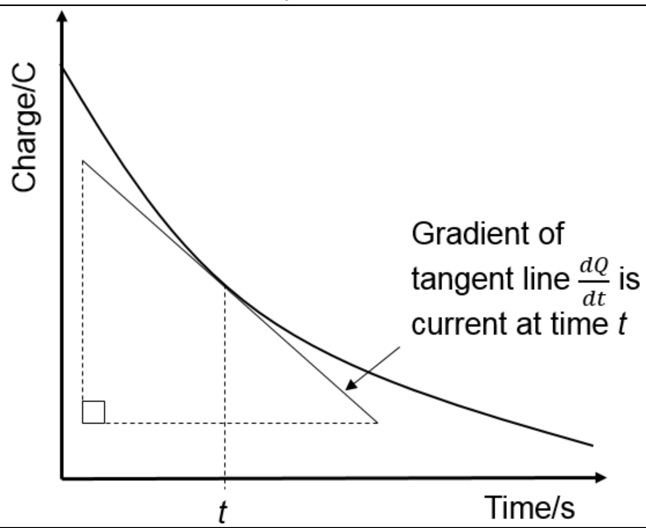
Electricity has become so vital in our daily life that we no longer notice its existence until we experience a power outage. In the future this influence can only grow. An understanding of the basic properties of electric current is important for its safe and effective use.

14.1 Electric Current & Charge

When an electrical conductor is conducting electricity, an electric current is said to flow through them. These electric currents are made up of a **net flow** of tiny charge carriers (or charged particles) such as electrons (–ve), protons (+ve) or ions (+ve or –ve) through the conductor. In this section, we will set the foundation right by defining electric current and electric charge formally for future discussions.

14.1.1 Electric Current

Electric current is one of the 7 base quantities. It is being introduced to quantify the rate of flow of charged particles. The following table gives a simple summary of what you need to know about electric current.

Physical Quantity	Electric Current
Definition	Electric current is the rate of flow of charge.
Type	Scalar
Symbol	I
SI Unit	Ampere (A)
Formula	In general $\boxed{I = \frac{dQ}{dt}}$ If current is constant, then $I = \frac{Q}{t}$
Graph	
Convention	Direction of electric current is taken to be the direction of flow of positively charged particles. It is usually denoted by an arrow.

Note to self

- It is important to note that current even though being given a direction and magnitude, is **NOT a vector** because it does not obey vector addition laws.

- Current is being chosen as one of the 7 base quantities in place of charge because it is easier to define the unit of electric current, which is linked to electric charge then to derive the coulomb from the current.

- Since = $\frac{dQ}{dt}$, current can be found by determining the gradient of a charge-time graph (*Q*-*t* graph).

14.1.1.1 (Conventional) Current Flow and Electron Current Flow

By convention, the direction of electric current is taken to be the direction of flow of positively charged particles (**conventional current**), which means flowing from a point of higher potential to a point of lower potential as shown in Figure 1. In metals, where the charge carriers are electrons (-ve), the electron flow is opposite to the direction of the conventional current. This type of current flow is known as the **electron current flow**.

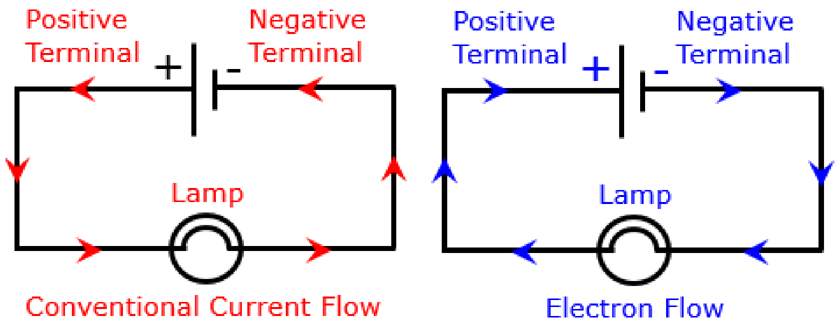
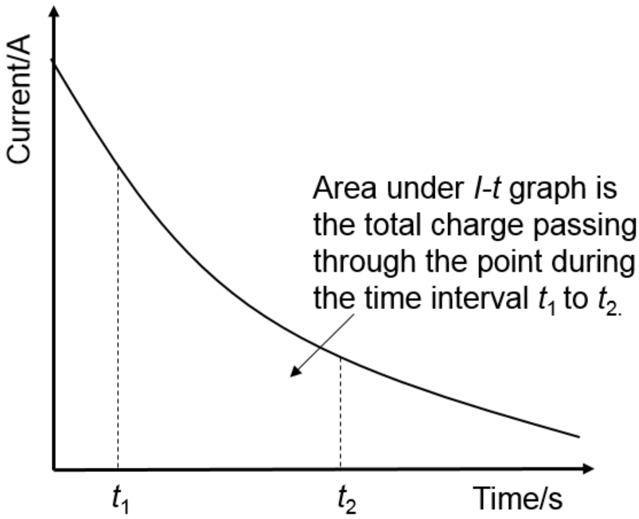


Fig. 1 Circuit diagram showing the direction of conventional current flow (left) and Electron Current Flow (right).

***Note that in physics, most of the problems are being discussed via adopting the direction of conventional current unless otherwise stated.*

14.1.2 Electric Charge

Electric charge is the physical quantity of matter that causes it to experience an electric force when it is in the vicinity of other electrically charged matter. However in this section, the focus is simply on the relation between charge and current.

Physical Quantity	Electric Charge
Definition	Charge is the property of a particle that causes an interaction of the particle with other charged particles and material of electrical nature.
Type	Scalar
Symbol	Q
SI Unit	Coulomb (C) One coulomb is the amount of charge which flows pass a point when a steady current of one ampere flows for one second.
Formula	In general $\boxed{Q = \int I\,dt}$ If current is constant, then $Q = It$
Graph	
Useful Formula	If *N* charge carriers each carrying charge *q* passes through a section of a circuit in time *t*, the average current can be expressed as: $I = \frac{total\ charge\ Q}{time\ t} = \frac{Nq}{t}$

Note to self

- A proton carries a charge of +1.60 × 10^{-19} C or e, where e denotes the **elementary positive charge** while an electron carries a charge of -1.60 × 10^{-19} C or –e; a helium nucleus carries a charge of 3.20 × 10^{-19} C or 2e, and so on.
- Any charge can only exist as an **integer multiple** of the elementary charge e.

- The amount of charge entering and leaving each junction is the same (or conserved) i.e. charges do not accumulate or disappear. This implies that the current remains constant along one single conductor. Similar to energy, charges obey the law of conservation: charges cannot be destroyed or created.

- Examples of charge carriers

Charge carriers	Conductor	Charge	Example
Ions	Molten ionic solids	Positive or negative	Melted sodium chloride solid
	Electrolyte	Positive or negative	Car battery
	Gas	Positive or negative	Fluorescent tube
Free electrons	metal	Negative	wires

Worked Example 1

A portable battery charger has a capacity of 7.49 × 10^4 C (20800 mAh). When it is being recharged, it draws a steady current of 1.5 A from the power supply.

(a) Determine the time required to fully recharge the portable battery charger.

Solution

Using $Q = It$,

$$t = \frac{7.49 \times 10^4}{1.5}$$
$$= 49933.33 \text{ s}$$
$$= 13.9 \text{ hr}$$

(b) Determine the number of electron required to carry 7.49 × 10^4 C of charge.

Solution

Each electron carries an elementary charge e = 1.60 × 10^{-19} C.

Using $Q = Ne$,

$$N = \frac{Q}{e} = \frac{7.49 \times 10^4}{1.60 \times 10^{-19}} = 4.68 \times 10^{23}$$

Therefore, 4.68 × 10^{23} electrons are needed to carry this amount of charge.

14.1.3 Deriving *I = nAvq* for a current-carrying conductor

When no potential difference is applied across a conductor, its mobile charge carriers undergo random motion similar to gas molecules in the air as shown in Fig. 2(a).

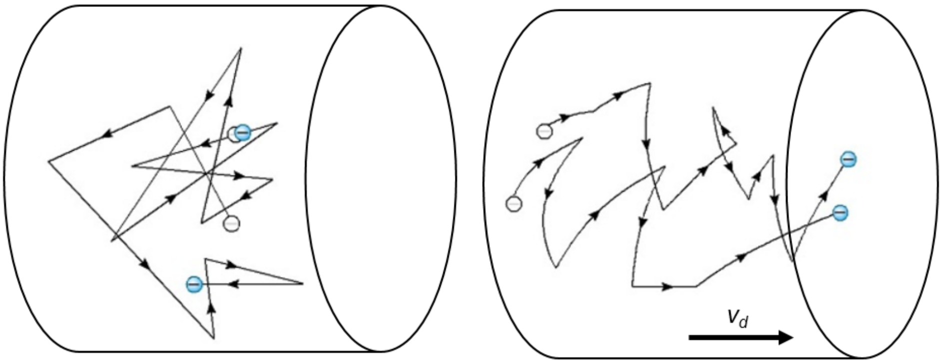


Fig. 2 (a) Charge carriers in random motion (left) and (b) Charge carriers in random motion with axial drift when a potential difference is applied (right)

Taking metal as an example, each atom contributes one or more free electrons to form the ionic bonds which hold the positive ions together in a metallic structure. The electrons are not attached to any particular ion but are mobile within the boundary of the lattice. As they are moving in random motion, they collide among themselves and with the positive ions. **The random speed of the charge carriers at room temperature is of the order 10^5 m s^{-1}.**

When a potential difference is being applied across the conductor, an electric field is set up at every point within the conductor. The mobile charge carriers will then experience a force directing them to the terminal that have an opposite charge to the charge they are carrying. As a result, the presence of a potential difference applied across the conductor causes the mobile charge carrier to drift axially along the conductor while it is moving randomly as shown in Fig. 2b. This rate of change of axial displacement of the mobile charge carriers with respect to time is known as **drift velocity, v_d.**

This drifting of mobile charge carriers causes a flow of charge and thus result in an electric current. **The typical drift velocity of electrons in metals is of the order of 10^{-4} m s^{-1}.**

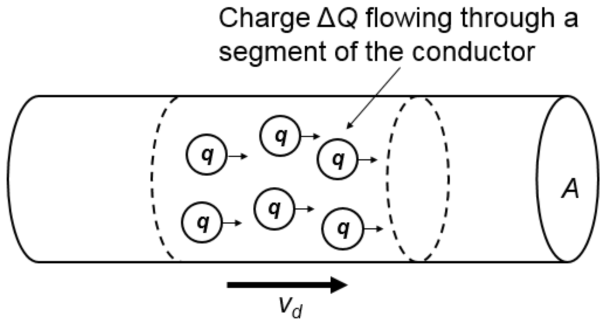
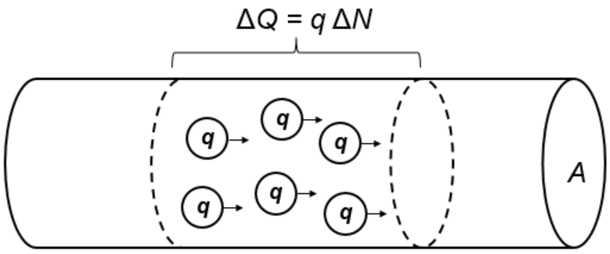
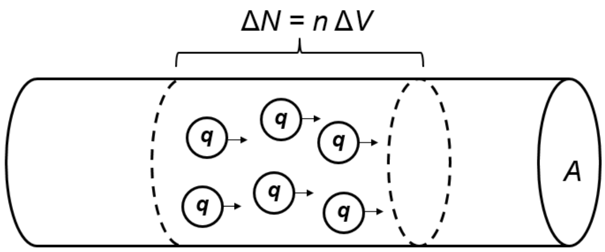
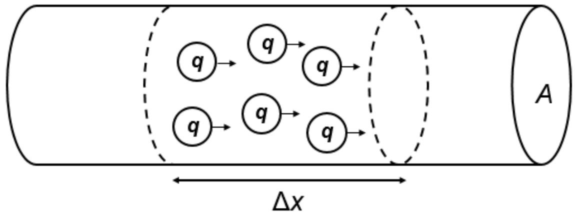
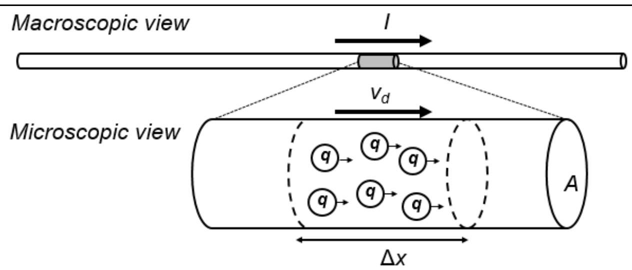
In short, the relationship between Current *I* and Drift velocity, v_d can be given as such:

$$\boxed{I = nAv_d q}$$

where *n* is the number density of mobile charge carriers (number of charge carriers per unit volume),
A is the cross-sectional area of the conductor,
V_d is the drift velocity, and
q is the charge carried by each mobile charge carrier.

The derivation of the above equation is presented in the following page and it is essential for you to know how to derive it.

<u>Derivation</u>

Step	Explanation	Mathematical steps	Diagram
1	To relate the current *I* to the flow of charges *q*, we consider a small flow of charge Δ*Q* through a small segment of conductor with volume Δ*V* in a time interval Δ*t*.	By definition, the current in this segment is given by $I = \frac{\Delta Q}{\Delta t}$	
2	The total charge, Δ*Q* flowing in the segment can be found by taking the product of the number of charge Δ*N* and the charge *q* carried by each mobile charge carrier. $\Delta Q = (\Delta N)q$	$I = \frac{(\Delta N)q}{\Delta t}$	
3	Let *n* be the number density of charge carriers (i.e. number of mobile charge carriers per unit volume). The number of charge carriers Δ*N* is the product of the number density of charge carriers *n* and the volume of the segment Δ*V.* $\Delta N = n(\Delta V)$	$I = \frac{(n\Delta V)q}{\Delta t}$	
4	The volume of this segment of conductor, Δ*V* is given by the product of its length Δ*x* and cross sectional area *A*. $\Delta V = A(\Delta x)$	$I = \frac{n(A\Delta x)q}{\Delta t}$	
5	Since $\frac{\Delta x}{\Delta t}$ is the average distance travelled by the charge carriers per unit time, it is also known as the **drift velocity, v_d.**	Finally, $I = nAv_dq$	

Worked Example 2

A copper wire carrying a 1.0 A current has a diameter of 0.500 mm. The number density of charge carriers in the wire is 8.5 × 10^{28} m^{-3}. Assuming the charge carriers in this wire are electrons, determine the drift velocity of the charge carriers.

Solution

Since the charge carriers are electrons, each electron will carry a charge of e = 1.6 × 10^{-19} C
Using $I = nAv_dq = n\left(\frac{\pi d^2}{4}\right)v_de$

$$1.0 = (8.5 \times 10^{28})\left(\frac{(\pi)(0.500 \times 10^{-3})^2}{4}\right)v_d(1.6 \times 10^{-19})$$
$$v_d = 3.74 \times 10^{-4}\ \text{m s}^{-1}$$

Note to self:

14.2 Electric Potential Energy and Power

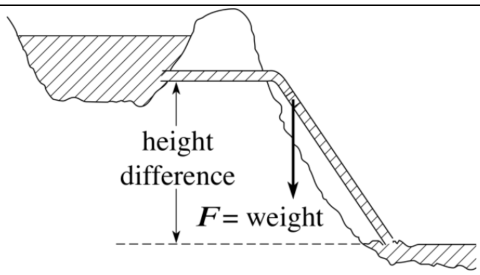
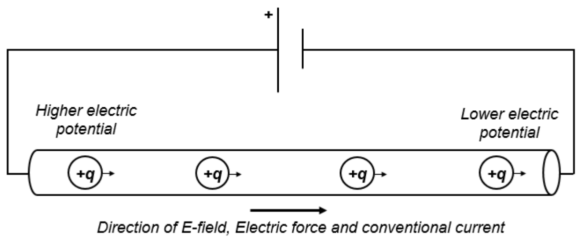
In a **closed circuit with current**, charge carriers possess electric potential energy (or electrical energy). The electric potential energy of a charge carrier may change as other forms of energy (such as heat or chemical energy) are converted from it or to it. This section deals with the energy considerations in various parts of a closed circuit.

In **an isolated wire** there is no net charge flow. The ions are in their lattice positions and their only movements are very small vibrations, the size (amplitude) of which depends upon the temperature of the wire. The charge carriers (e.g. free electrons) will also have a certain amount of kinetic energy associated with their temperature, but this thermal motion will be randomly directed like that of molecules in a gas and certainly won't cause a current. So how is it possible to arrange for a net movement of charge carriers (free electrons) through the wire?

The simplest answer follows from considering the energetics of the system: **Charge carriers (free electrons) will flow from A to B in a wire if, by so doing, the *potential energy* of the system is reduced.**

Hence, we see that an electric current flows through an electrical component (other than the e.m.f source) to reduce the potential energy of the system. In other words, current flows from a region of higher electric potential energy to a region of lower electric potential energy. This decrease in electric potential energy implies that electrical energy is being converted to other form of energies (heat, light, sound, mechanical etc.) when the current flows through the electrical component according to the principle of conservation of energy.

You may find the analogy between the flow of water in a pipe and the flow of a positive charge in a wire helpful in understanding the concept.

Analogy		
Situation	Flow of water down a pipe	Flow of charge carriers (free electrons) in a wire
Reasoning	Due to: Height difference → Difference in **gravitational potential energy** between the ends of pipe → A gravitational field is being set up → Gravitational force (weight of water) acts on water to push the water along the pipe from high to low potential	Due to: Circuit connected to terminal of different electric potential → Difference in **electric potential energy** across the wire → An electric field is being set up → Electric force acts on charge carriers to push it along the wire from high to low potential
Energy conversion	Water flowing down the pipes can be used to drive a turbine to do mechanical work. Gravitational potential energy → Mechanical energy	Current can be passed through a light bulb to light up a room. Electrical potential energy → Light energy

14.2.1 Electric Potential

When the terminals of a battery is connected to an electrical device (e.g. light bulb or buzzer), current (and hence charge) flow through the device from a point of higher electrical potential energy to a point of lower electrical potential energy in the direction of the electric field as shown in Fig. 3.

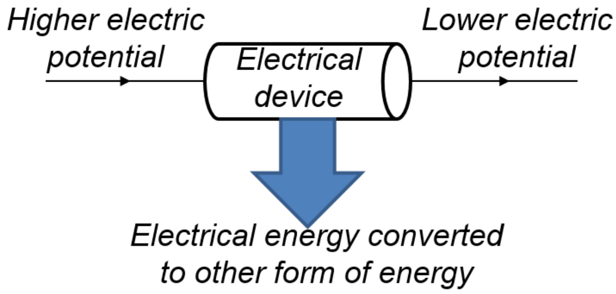


Fig. 3 Current flow from a point of high electric potential to a point of low electric potential through a device, converting electrical energy to other forms of energy

Analogous to the definition of gravitational potential which give the gravitational potential energy per unit mass at any point in space we can also describe **electric potential** which gives the electrical potential energy per unit charge at any point in a circuit. This will thus help us in setting up a "reference ruler" to determine the change in electric potential energy.

With regards to the reference zero potential, it can be chosen at any point in the circuit (more on zero point selection in topic 15) just like how the zero reference point can be chosen for gravitational potential (at infinity or at sea level depending on situation). This is because the difference in potential is the quantity that is physically meaningful, not the potential itself.

Note to self:

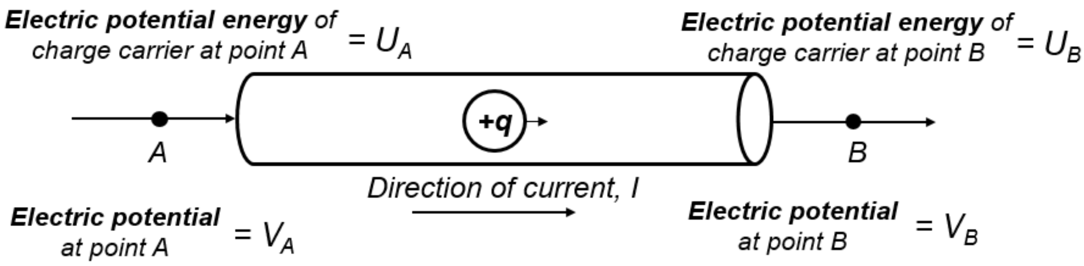
- Higher does not always means positive, lower does not always means negative. They are just references being made to a point.
- For convenience, the point where the circuit is earthed will be set as the zero potential. If the circuit is not earthed, the negative terminal is usually set as the zero potential.

14.2.2 Electric Potential Difference (p.d)

In a close circuit, when the current passes through an electrical component (e.g. a resistor, bulb or buzzer), electrical energy is converted into other forms of energy such as heat light or sound. This results in a decrease in the electric potential energy of the charge carrier after passing through the component.

It is also known that the change in the electric potential energy of a charge carrier as it moves within an electric field is proportional to the total charge of that charge carrier. In other words:

Change in Electrical Potential Energy ∝ Total Charge Q of charge carrier



$$\boxed{\begin{gathered}\Delta U = q\Delta V \\ (U_B - U_A) = q(V_B - V_A)\end{gathered}}$$

[More on this in topic of electric field]

Fig. 4 Change in electric potential energy across a device is proportional to charge carried by charge carrier

Thus, if we divide the change in a charge carrier's electric potential energy by its total charge we get a new quantity, the change in electric potential energy per unit charge, that is independent of the charge carrier's charge and therefore tells us more about the electrical environment (i.e. the electric field) in which the charge carrier is located. Furthermore, it is important to note that in this case, the electric potential energy carried by the charge carrier is being converted to other form of energy. With this, we are now ready to define the term **Electric Potential Difference or potential difference (p.d.).**

Physical Quantity	Electric Potential Difference or potential difference (p.d.)
Definition	The potential difference across a device is the electrical energy converted to other forms of energy, per unit charge passing through the device.
Type	Scalar
Symbol	*V*
SI Unit	Volt (V) One volt is the potential difference across a device when the amount of electrical energy converted to other form of energy per unit charge passing through the device is one joule per coulomb.
Formula	In general $\boxed{V = \frac{W}{Q}}$ Where *W* is the amount of energy converted **from electric potential energy to other forms of energy** (in joule) *Q* is the amount of charge passing through the device (in coulomb) Therefore SI unit can also be expressed as J C^{-1}

Worked Example 3

An electric iron is connected to a 230 V voltage supply for 15 minutes. During this 15 minutes, it produced 720 kJ of heat.

(a) Determine the amount of charge that flow through the heating element.

Solution

Using $V = \frac{W}{Q}$

$$Q = \frac{W}{V} = \frac{(720 \times 10^3)}{230} = 3130 \text{ C}$$

(b) State an assumption that you made in your calculations.

Solution

All electrical energy are being converted to heat. The efficiency of the electric iron is 100%.

Note to self

14.2.3 Electromotive Force (e.m.f)

In section 14.2.2, we discussed about the electrical energy being converted into other forms of energy. However, it must be noted that for the charge carriers to possess electrical potential energy, there must be an energy source that provides for these electrical energy. In this section, we are going to discuss about these sources of energy and quantify them.

The battery/dry cell is a simple and common source of electrical energy. When conventional current flows out of a battery, the charge carriers +Q moves from the positive terminal to the negative terminal of the battery. When these charge carriers return to the battery at the negative terminal, they have used up all the electrical potential energy it originally possessed. Inside the battery, they are driven by the battery internally to the positive terminal. When this happens, the battery is said to have done work on the charge carriers. The energy to do work on the charge carrier to increase their electrical potential energy comes from the chemical energy of the battery. (Go read up on 9729 H2 Chemistry Topic 12 Electrochemistry).

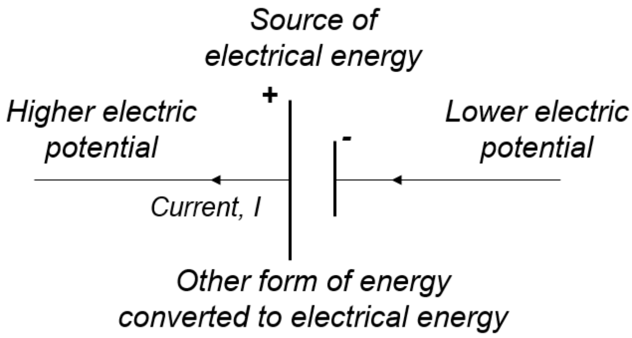


Fig. 5 An e.m.f. source will raise the electrical potential energy of a charge carrier when the charge carrier passes through the source.

Physical Quantity	Electromotive Force (e.m.f.)
Definition	The electromotive force of a source is the electrical energy, converted from other forms of energy per unit charge, transferred by the source in driving a unit charge round a complete circuit.
Type	Scalar
Symbol	ε
SI Unit	Volt (V)
Formula	In general $\boxed{\varepsilon = \frac{W}{Q}}$ Where ε is the electromotive force (in volt) W is the amount of energy converted **from other forms of energy to electric potential energy** (in joule) Q is the amount of charge passing through the device (in coulomb) Therefore SI unit can also be expressed as J C^{-1}

Note to self:

- Electromotive force is not a force. It is a scalar quantity and it exists whether current flows or not.

Common sources of e.m.f.

Primary source	Principle	Examples	Application
Light	Photoelectric effect	Solar cell	Satellite power, calculator, light sensing devices
Chemical energy	Chemical reaction between electrodes and an electrolyte	Dry cell, battery	Car battery, all portable electronic equipment
Heat	Seebeck effect – e.m.f created between 2 junctions of different metals at different temperatures.	Thermocouple, thermophile	Heat sensing device, thermocouple thermometer
Electromagnetic induction	Induced e.m.f – by changing the magnetic flux of a conductor.	a.c. generator, dynamo	Power station, dynamo inside a car for recharging battery.

14.2.4 Electric Power

In this section, we will discuss about **electric power produced by electrical devices** and **electrical power provided by a power source**. It is important for us to distinguish between the two so that we know exactly what we are substituting into the formula to calculate the power that we are interested in.

Electrical appliances are devices that allow us to harness electrical energy. This is mainly done by converting electrical energy to other forms of energy. The more electrical energy a device can convert per unit time, the more power it produces.

Physical Quantity	Electric Power
Definition	Power is the rate of work done
Type	Scalar
Symbol	*P*
SI Unit	Watt (W)
Formula	In general $\boxed{P = \frac{dW}{dt} = \mathbb{V}I = I^2R = \frac{\mathbb{V}^2}{R}}$ Where *W* is the amount of energy converted from electric potential energy to other forms of energy (in joules). $\mathbb{V}$ represents either potential difference across device or e.m.f provided by electric source (in volt) *I* is the current flowing through the device or electric source (in ampere) *R* is the resistance of the electrical device (in ohm)

Power emitted by electrical devices	Power supplied by electric source
Substitute $\mathbb{V}$ as p.d. applied across the device *V* to determine the power emitted: $P_{emitted\ by\ device} = VI = I^2R = \frac{V^2}{R}$	Substitute $\mathbb{V}$ as e.m.f. provided by the source, ε to determine the power provided: $P_{provided\ by\ source} = \varepsilon I$

Worked Example 4

An electric kettle operates at 230 V and draws a current of 8 A. It requires 5 minutes to bring a full kettle of water from room temperature to its boiling point.

(a) Determine the power rating of the electric kettle.

Solution

Using $P = IV$

$$Power = (8)(230) = 1840 \text{ W}$$

(b) Determine the thermal energy gained by the kettle of water.

Solution

Using $P = \frac{E}{t}$

$$E = Pt = (1840)(5 \times 60) = 5.52 \times 10^5 \text{ J}$$

14.2.5 Distinguishing between p.d. and e.m.f.

Taking a broader view on what we have just discussed, it is not difficult for one to realise that the terms voltage and volts are being used interchangeably to discuss about potential difference (p.d.) and electromotive force (e.m.f).

It is therefore very tempting for students to naively think that since voltage is being used interchangeably to describe p.d. and e.m.f, we can also use the terms p.d. and e.m.f interchangeably as well. <u>This is a very wrong misconception!!!!</u>

This is because, p.d. and e.m.f refers to 2 different physical quantities and talks about different parts of an electrical circuit (if they are the same, reading section 14.2.2 and 14.2.3 will be completely a waste of your time). Even though they both refer to the amount of work done per unit charge and share the same unit, they are still fundamentally different in their energy considerations.

The following table gives a quick comparison between p.d. and e.m.f.

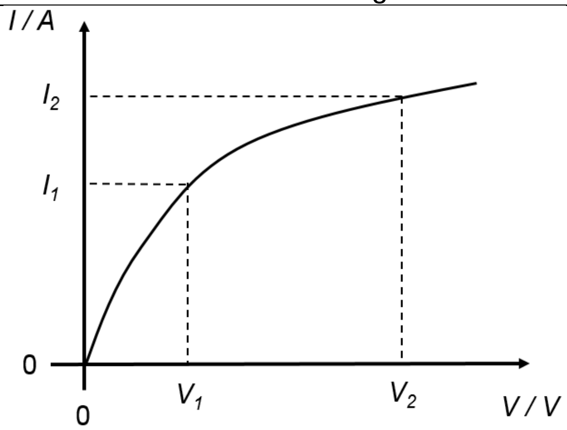
Potential Difference (p.d.)	Electromotive Force (e.m.f.)
Compared across 2 points in the circuit	All about the source
Electrical energy → other form of energy Per unit charge Passing through the device	Other form of energy → Electrical energy Per unit charge To drive a unit charge round a complete circuit
$V = \frac{W}{Q}$ *W* is the amount of electric potential energy converted **<u>to</u>** other form of energy.	$\varepsilon = \frac{W}{Q}$ *W* is the amount of electric potential energy converted **<u>from</u>** other form of energy.
p.d. is zero when no current if flowing $V = IR = 0$	e.m.f. is independent on current flowing in circuit or the total resistance of the circuit.
To determine the power emitted by a device with resistance *R* when a p.d. of *V* is applied across it. $P_{emitted\ by\ device} = VI = I^2R = \frac{V^2}{R}$	To determine the power supplied by the electrical source when current is flowing in the circuit. $P_{provided\ by\ source} = \varepsilon I$

14.3 Resistance and Resistivity

14.3.1 Resistance

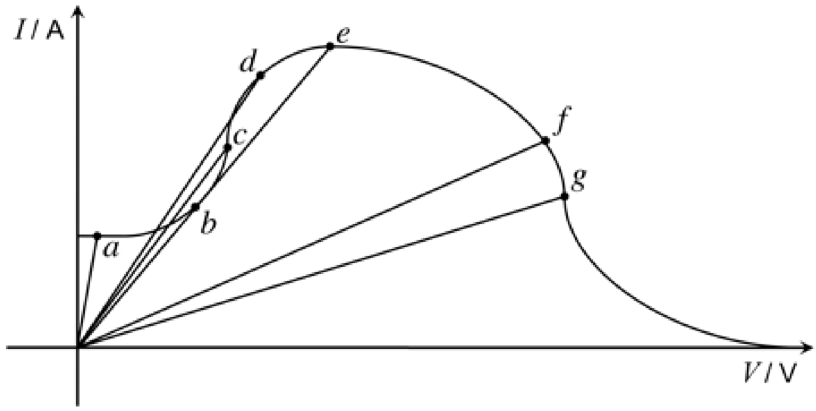
Resistance is the property of a conductor which limits the current flow. Microscopically, it is due to frequent collisions of the drifting charge carriers by
(i) Vibrating lattice and
(ii) Crystal defects in lattice e.g. impurity atoms and dislocations.

When accelerated by an electric field, free moving charge carriers collide with the vibrating ions in the lattice. The electron transfer part of their energy to the ions and hence slow down. The frequency of collision can be increased by increasing temperature and by the presence of crystal defects in the lattice, resulting in greater resistance.

Physical Quantity	Resistance
Definition	The resistance of a conductor is defined as the **ratio** of the potential difference across the conductor to the current flowing through it.
Type	Scalar
Symbol	*R*
SI Unit	Ohm (Ω) One ohm is defined as the resistance of a conductor when the potential difference across it is one volt per ampere of current flowing through it.
Formula	In general $\boxed{R = \frac{V}{I}}$ Where *R* is the resistance (in ohms), *V* is the potential difference across the conductor (in volts), *I* is the current through the conductor (in ampere)
Graph	 Note: The gradient of the *I*-V graph does not carry any physical meaning because resistance is defined as the ratio of p.d. against current, not rate of change of p.d. with current. Hence, it is conceptually incorrect to use the gradient of the *I-V* graph to determine the resistance of the conductor. Where $R_1 = \frac{V_1}{I_1}$ and $R_2 = \frac{V_2}{I_2}$.

Worked Example 5

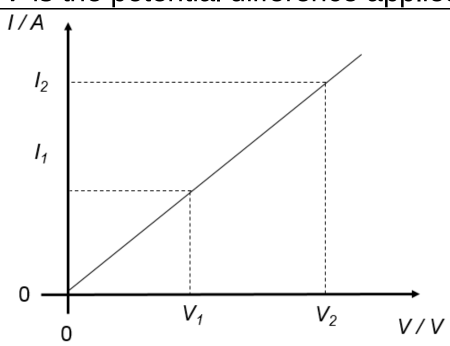
The diagram below shows the *I*-*V* graph of an electrical component. List the points a-g in descending order of resistance.



Solution
To determine *R* from *I*-*V* graph,
1) Draw a line **from the origin** to the point of interest on the *I*-*V* graph.
2) The gradient of this line (<u>not</u> the tangent at that point) gives the ratio $\frac{I}{V}$.
3) Calculate the inverse of $\frac{I}{V}$ to determine *R.*

The gradient of each line gives the ratio $\frac{I}{V}$ which is the inverse of *I.* Thus the steepest gradient indicates the lowest resistance. Therefore, the **points in descending order of resistance are g, f, (e =b), c, d, and a.**

14.3.2 Ohm's Law

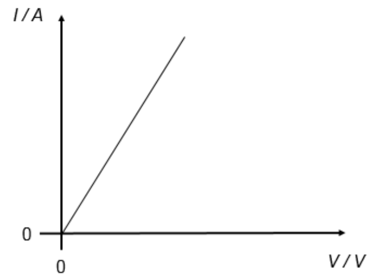
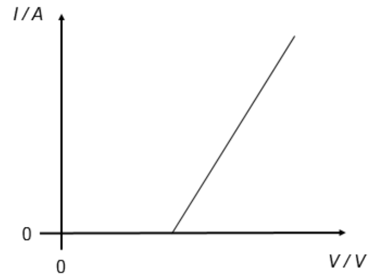
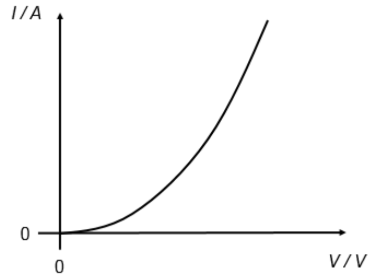
Definition	Ohm's Law states that the current, *I* through a metallic conductor is **directly proportional** to the potential difference, *V* across it, **provided the temperature and other physical conditions are kept constant.**
Mathematical representation	$$I \propto V$$ where *I* is the current flowing through the material in ampere (A) *V* is the potential difference applied across the conductor in volts (V)
Graphical	 <u>Features of graph:</u> 1) Linear graph (straight line) 2) Passes through the origin <u>Inferences</u> - Current increases with increasing p.d. applied proportionally. - Resistance remains constant with varying p.d.

Whether a conductor obeys Ohm Law or not is empirically determined by conducting an experiment described later in section 14.3.3. For most conductors, *I* is not directly proportional to *V*. In other words, *R* is not constant. Even for a pure metallic conductor, when current is increased with an increase in p.d., the temperature increases and thus increasing the resistance. The current in the metallic conductor would only be directly proportional to the p.d. when temperature and other physical conditions are kept constant. E.g. by immersing the specimen in a constant temperature heat bath.

The "million dollar question": Since resistance is a ratio of p.d. against current, why not analyse a V – I graph instead of an I – V graph?

Answer: Experimentally, it is often easier to control the p.d. applied across a device than to control the amount of current passing through it (more on this during practicals). Hence, this make p.d. (*V*) the independent variable which is normally plotted on the *x* –axis.

Worked Example 6

Case A	Case B	Case C
		
Is the conductor ohmic?		
Ohmic	**Non-Ohmic**	**Non-Ohmic**
How resistance varies with increasing p.d. ?		
Constant	**Decreases**	**Decreases**

14.3.3 *I-V* Characteristic Curve

To investigate the variation of resistance of different electrical components/device, it is useful for us to plot an *I-V* characteristic curve for the device of interest. To do so, the following circuit (potential divider) as shown in Fig. 6 can be set up.

By varying the p.d. applied across the device using the above set up (a potential divider), we can obtain the current flowing across the device. An *I-V* characteristic curve of the device can then be drawn to investigate the resistance across the device at different p.d.

Due to the fact that not all devices are ohmic conductors, certain devices possess unique electrical properties that gives rise to unique characteristic *I-V* curves. These *I-V* curves helps to show how their resistance varies with increasing p.d. across them.

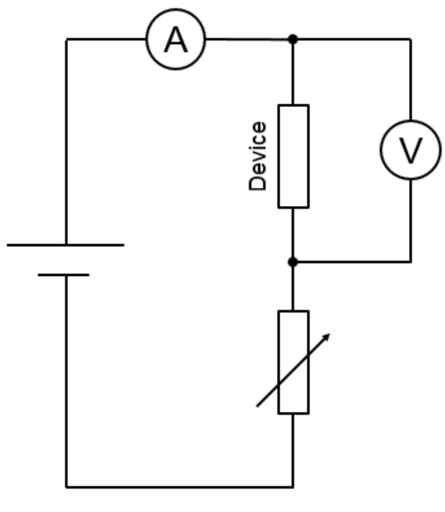
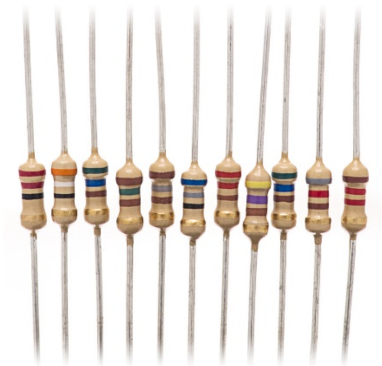
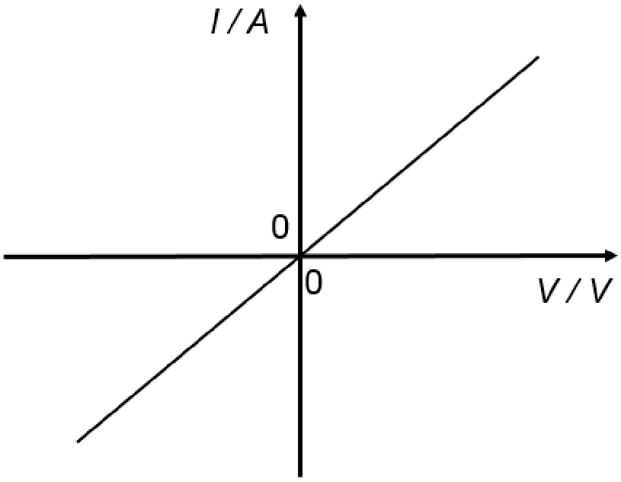


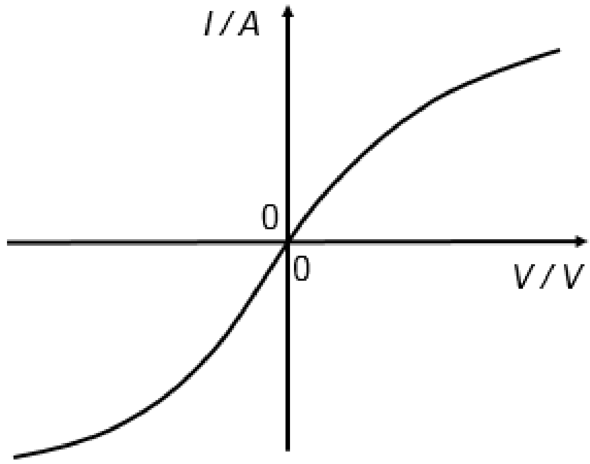
Fig. 6 A potential divider circuit used to obtain a *I-V* graph of the device

In this section, we are going to discuss and explain for the characteristic *I-V* curve for 4 different devices: **1) A metallic conductor** at constant temperature**, 2) a filament lamp, 3) a semiconductor diode, and 4) a Negative Temperature Coefficient (NTC) thermistor.**

14.3.3.1 Metallic Conductor at Constant Temperature		
Descriptions & Examples	 Coils of wires	 Resistors with different resistance.
Graph		
Characteristics	The *I*-*V* characteristics curve of a metallic conductor is a **straight line passing through the origin.** Current *I* increases proportionally with potential difference *V* and the ratio $\frac{V}{I}$ is a constant indicating that **resistance *R* remains constant** throughout. A metallic conductor at constant temperature is **Ohmic.**	
Explanation	While p.d. is being varied, the temperature of the metallic conductor is being kept constant. Since the mobility of the charge carriers, which depends on the temperature of the metallic conductor, is being kept constant, the resistance of the metallic conductor remains constant.	

Note to self:

- The standard resistor, if heat up due to external factors does changes its resistance, but not as significant as a thermistor. Under normal usage, the temperature of a resistor remains constant and thus remains ohmic.

<table>
<tr><td colspan="2">14.3.3.2 Filament Lamp</td></tr>
<tr><td>Descriptions & Examples</td><td>The filament is a non-ohmic conductor that changes temperature significantly when the current through it varies. The change in temperature (a physical condition) of the filament results in a change in its resistance.
</td></tr>
<tr><td>Graph</td><td></td></tr>
<tr><td>Characteristics</td><td>The I-V characteristic curve of a filament lamp is a characteristic S-shape curve that passes through the origin.

Current I increases at a decreasing rate with potential difference V. Hence, the ratio $\frac{V}{I}$ is increasing, indicating that resistance R increases with increasing p.d. applied across.

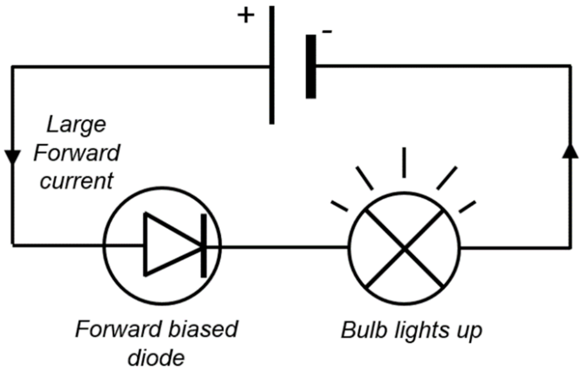
A filament lamp is a non-ohmic conductor.</td></tr>
<tr><td>Explanation</td><td>As p.d. applied across the filament lamp is increased, the steady-state temperature of the filament increases. This is because an increase in p.d. will result in a larger current flowing in the filament and hence more frequent collision between the conducting electrons with the lattice ions. This causes the lattice ions to gain more kinetic energy, vibrates more vigorously and attain a higher temperature.

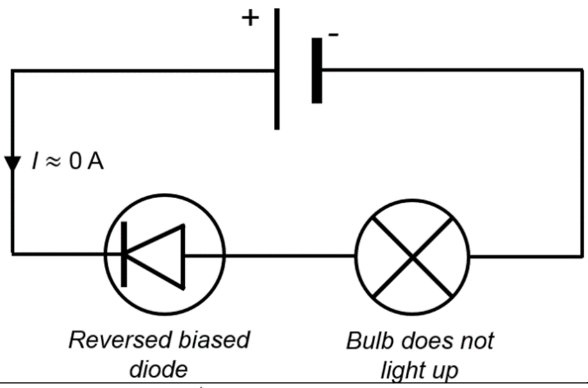
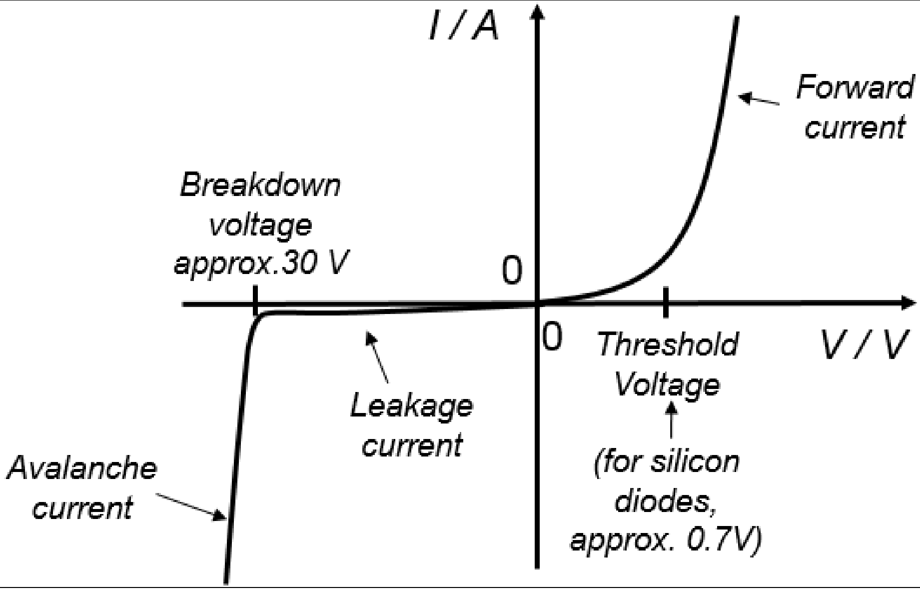
As a result of this temperature increase, the conducting electrons will collide even more frequently with the lattice ions in the wire which further reduces the mobility of the conducting electrons. Since the mobility of the charge carriers decreases, the resistance of the filament increases with increasing p.d.</td></tr>
</table>

Note to self:

<table>
<tr><td colspan="2">14.3.3.3 Semiconductor Diode</td></tr>
<tr><td>Descriptions & Examples</td><td>A semiconductor diode is an electrical component that conducts electric current in only one direction.

When it does so, we call this direction of current conduction the forward-bias direction. However, when it is connected in the opposite direction, it does not allow current to flow through and we call this the reverse-bias direction.

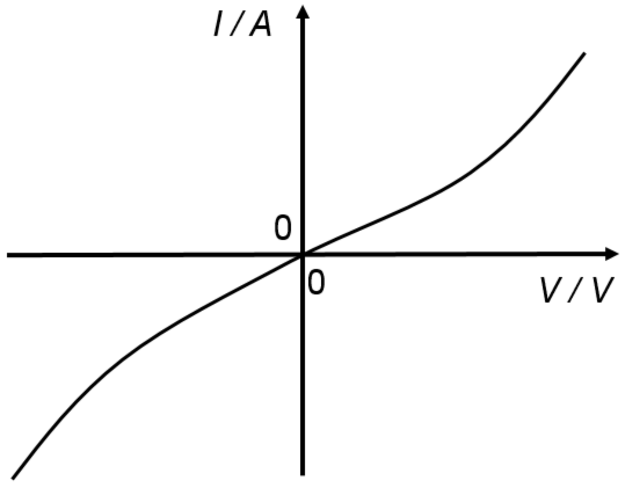
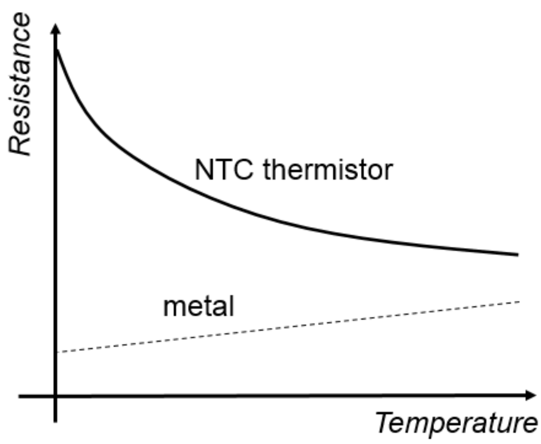
The purpose of semiconductor diode in a circuit is to act as a “one-way valve” for current flow. A smart combination of them can rectify an A.C. current to a D.C. current. (More on this in topic on alternating current)</td></tr>
<tr><td colspan="2">Circuit Diagrams



</td></tr>
<tr><td>Graph</td><td>
</td></tr>
<tr><td>Characteristics</td><td>The I-V characteristic curve of a semiconductor diode is a characteristic asymmetrical curve.

In the forward-bias direction, the graph is an exponential curve that begins shortly after the origin when p.d. applied is above the threshold voltage. Beyond the threshold voltage, I increases exponentially with increasing V. Hence, the resistance of the diode can be said to be insignificant.</td></tr>
</table>

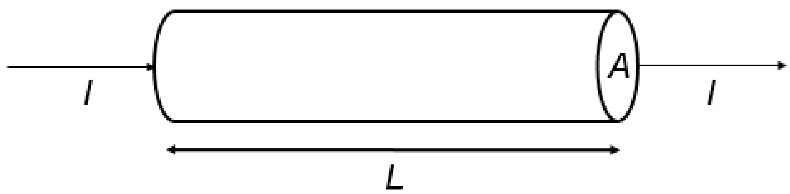
	However, in the reverse bias direction, the graph is almost a horizontal line from the origin followed by a near-vertical line. This shows that there is still a small current flowing in the reverse direction, but the magnitude is insignificant. We call this the **leakage current**. Since I is rather insignificant before reaching the breakdown voltage, the diode's resistance in this region is very high. Current I increases at a decreasing rate with potential difference V. Hence, the ratio $\frac{V}{I}$ is not constant. A semiconductor diode is a **non-ohmic** conductor.
Explanation	When the diode is in forward biased direction, its resistance is very low when p.d. is above the threshold voltage. Hence, it conducts electric current more easily than in reverse biased direction where the resistance in such case is very high. More explanation on why a semiconductor diode works this way requires knowledge on p-n junction which will not be elaborated here as it is out of syllabus.

Note to self:

14.3.3.4 Thermistor	
Descriptions & Examples	Thermistor as the name suggest is a type of resistor (-stor) whose resistance varies with temperature (Thermi-) more significantly as compared to standard resistors. There are 2 type of thermistor: 1) Negative Temperature Coefficient (NTC) (in syllabus): - Resistance decreases as temperature increases and vice versa. 2) Positive Temperature Coefficient (PTC) (not in syllabus) - Resistance increases as temerpature increases and vice versa. 
Graph	
Characteristics	The *I-V* characteristic curve of a NTC thermistor is a characteristic **reverse-S-shape curve** that **passes through the origin.** Current *I* increases at an increasing rate with potential difference *V*. Hence, the ratio $\frac{V}{I}$ is decreasing, indicating that **resistance *R* decreases with increasing p.d. applied across.** A NTC thermistor is a **non-ohmic** conductor.
Explanation	A thermistor is made of a semiconductor material such as silicon and germanium. For a semiconductor, an increase in temperature would increase the thermal agitation of the atoms and set free more electrons and holes (a type of positive charge carrier) in the crystal lattice. This increase in concentration of charge carriers offsets the increase in resistance due to increase in frequency of collisions with the lattice ions. Therefore the resistance of a NTC thermistor decreases as temperature increases. 

14.3.4 Resistivity

The resistance of a conductor can also be affected by its own dimension. Hence, it makes it difficult for us to just simply use resistance to quantify how well a material resists current flow. Hence, to break free from the dependence of the conductor's dimension when comparing how well different material resist current flow, we need to introduce the term **resistivity**.

Physical Quantity	Resistivity
Definition	Resistivity is a measure of how strongly a material opposes electric current. It characterises the resistance of materials at a fixed temperature regardless of the materials' dimension (shape and sizes)
Type	Scalar
Symbol	ρ
SI Unit	Ohm metre (Ω m)
Resistance & Resistivity	The resistance *R* of a uniform conductor of a given material at a given temperature is directly proportional to its length *L* and inversely proportional to its cross sectional area *A*. summarised by the following formula: $\boxed{R = \frac{\rho L}{A}}$ Hence, mathematically speaking, we can say that resistivity ρ behaves like a proportionality constant helping us to relate resistance *R* to the dimension of the wire. 

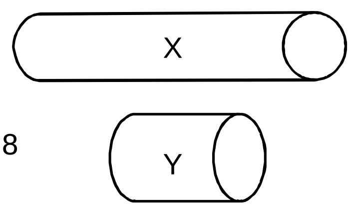
Some extra knowledge and terminology (not in syllabus):

- Conductivity, σ is the reciprocal of resistivity ➔ $\sigma = \frac{1}{\rho}$. The unit is Ω^{-1} m^{-1}.
- Conductance, *G* is the reciprocal of resistance ➔ $G = \frac{1}{R}$. the unit is Siemens (S)

Worked Example 7

Given two wires X and Y are of the same material but wire X is twice the length and half the diameter of wire Y. What is the ration of the resistances $\frac{R_X}{R_Y}$?

Solution



Using $R = \frac{\rho L}{A}$, $\rho_X = \rho_Y$ (same material) and $A = \pi\left(\frac{d}{2}\right)^2$,

$$\frac{R_X}{R_Y} = \frac{\left(\frac{\rho_X L_X}{A_X}\right)}{\left(\frac{\rho_Y L_Y}{A_Y}\right)} = \frac{L_X}{L_Y}\left(\frac{d_Y}{d_X}\right)^2 = (2)(2)^2 = 8$$

14.3.5 Internal Resistance of an e.m.f source

14.3.5.1 What is internal resistance?

For all the discussions above, we see that the e.m.f. source is a power source that provides electrical energy to all part of the circuit except for itself. In practice, this ideal situation is not possible because of **internal resistance**. Any source of e.m.f. has an internal resistance. For example, in the a.c. generator, it is due to the resistance of the coils. Inside a dry cell, it is mainly due to resistance of the electrolyte and polarisation around the electrodes. The internal resistance of a dry cell is approximately 1 Ω.

As the internal resistance is an "inherent resistance" present in a practical e.m.f. source, it is impossible for us to eliminate it. To continue our work of analysing a practical circuit, we can **consider a practical cell as equivalent to a pure source of e.m.f. ε connected in series with its internal resistance r (drawn in a dashed-line box)** as shown in **Fig. 7**.

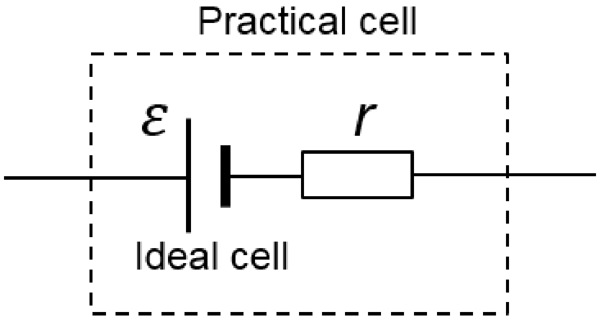


Fig. 7 A practical cell can be conceptualised as an ideal cell connected in series to an internal resistor

14.3.5.2 Effects of internal resistance

When an e.m.f. source supplies a current to the circuit, part of the energy is used to overcome the internal resistance of the cell. Hence, the p.d. across the terminals of the cell drops. To find out about the p.d. across the terminals also known as terminal p.d., we can consider the following circuits shown in Fig. 8.

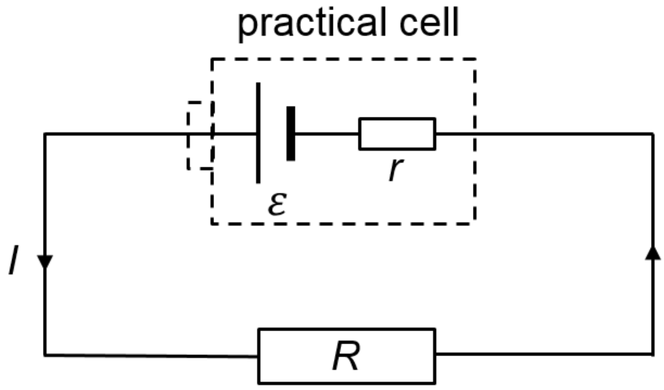


Fig. 8 A practical cell connected in series to a resistor with resistance R

In this circuit, a cell of e.m.f. ε and internal resistance *r* is connected in series with an external resistance *R*. As a result, the total resistance in the circuit becomes (*r* + *R*).

By the Principle of Conservation of Energy,

Power supplied by e.m.f. source = Power output in *R* + Power loss through *r*

$$P_\varepsilon = P_R + P_r$$
$$I\varepsilon = IV_R + IV_r$$
$$I\varepsilon = I^2R + I^2r$$
$$\varepsilon = IR + Ir$$

Using $V_T = IR$ and rearranging the equation, we arrive at

$$\boxed{V_T = \varepsilon - Ir}$$

where V_T is the terminal p.d.

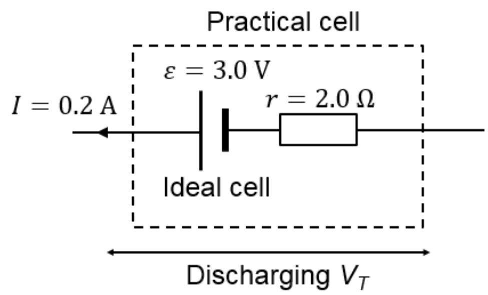
Note that : The p.d. across the resistor *R* is equal to the p.d. supplied at the terminal, i.e. $V_T = IR = V_R$

As seen from the above, the p.d. across the terminals of the practical cell, known as the **terminal potential difference**, is less than the e.m.f. ε of the cell. The 'lost' voltage appears as the potential difference across the internal resistance of the cell.

Thus, for a practical source of e.m.f., the total electrical power generated is not fully available to the external load; part of the total electrical power is spend in overcoming the internal resistance of the source. This represents the power loss as heat in the e.m.f. source. (Also the reason why your phone battery heats up!!!)

Worked Example 8

(a) A 3.0 V cell of internal resistance 2.0 Ω releases a current of 0.2 A, determine the terminal p.d. across the cell.
(b) A 3.0 V cell of internal resistance 2.0 Ω is being charged by a current of 0.5 A, determine the terminal p.d. across the cell.



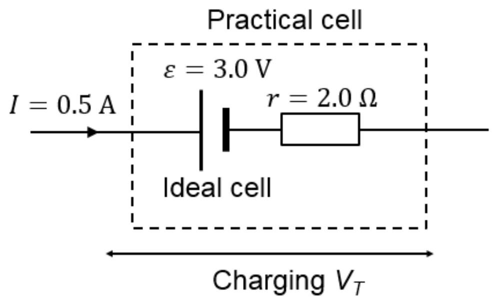
Solution

(a) During discharging,

$$V_T = \varepsilon - Ir$$
$$= 3.0 - (0.2)(2.0)$$
$$= 2.6 \text{ V}$$

(b) During charging, we need to supply a terminal p.d. larger than the e.m.f. plus the "lost voltage" so as to compensate for the energy lost due to internal resistance.

$$V_T = \varepsilon + Ir$$
$$= 3.0 + (0.5)(2.0)$$
$$= 4.0 \text{ V}$$



14.3.5.3 Determining internal resistance *r*

As the internal resistance of an e.m.f. source is an inherent resistance, there is no way of physically separating the resistance from the source to measure its resistance. We can only perform the measurement indirectly via the following set up in Fig. 9.

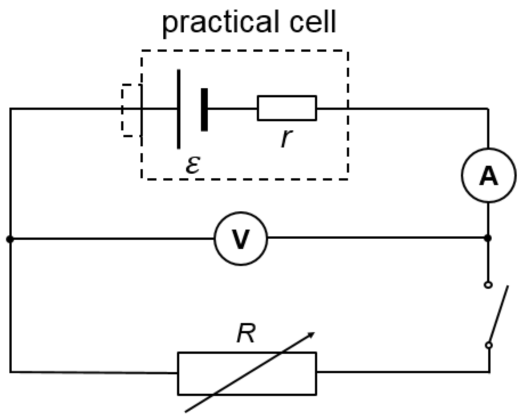


Fig. 9 Set up to determine the internal resistance of an e.m.f. source

In the circuit shown in Fig. 9, When the switch is open, the resistance in the circuit is infinite (ideal voltmeter) and the current in the circuit is zero. Therefore, the voltmeter gives the reading of the e.m.f. ε of the source.

When the switch is closed, the resistance in the external circuit becomes (R + r). By varying R, the voltmeter will give us a set of readings for the terminal p.d. V_T across the practical cell, while the ammeter will give us the corresponding current I flowing in the circuit.

A graph of terminal p.d., V_T against current in circuit, I can be plotted as shown in Fig. 10 and the internal resistance r can be determined via calculating the gradient of the graph. The y-intercept can then be used to verify the e.m.f. ε of the cell.

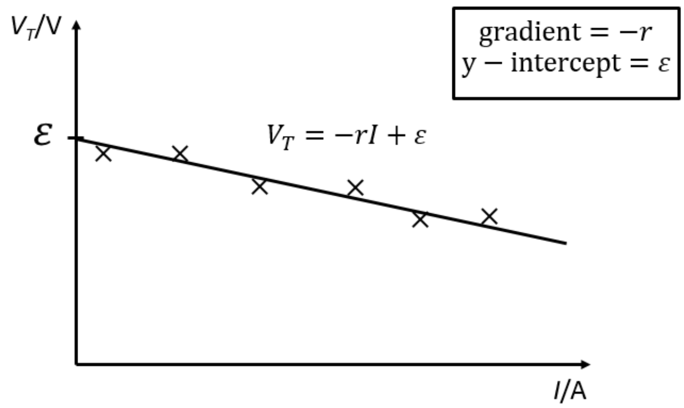
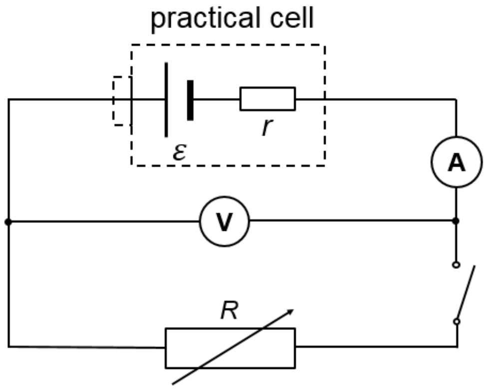


Fig. 10 Graph of V_T against I

In summary

- Ideal source → internal resistance, r = 0 Ω → e.m.f. of source, ε = terminal p.d., V_T
- Practical source → Internal resistance > 0 Ω → e.m.f. of source > terminal p.d, V_T
- Note: When no current is flowing, e.m.f. of source, ε = terminal p.d., V_T (No energy loss to overcome internal resistance.)

Worked Example 9



A battery is connected to a variable resistor and a voltmeter is connected across its terminals as shown on the left. When the variable resistor has resistance of 6.0 Ω, the voltmeter reading is 4.0 V. When the resistance is 10 Ω the voltmeter reading is 4.4 V. Determine the e.m.f. and the internal resistance of the battery.

Solution

When R = 6.0 Ω, V_T = 4.0 V → $I = \frac{V_T}{R} = \frac{4.0}{6.0}$ = 0.67 A
Applying $V_T = \varepsilon - Ir$ we get,

$$\varepsilon = 4.0 + (0.67)r \text{ ------ (1)}$$

When R = 10.0 Ω, V_T = 4.4 V → $I = \frac{V_T}{R} = \frac{4.4}{10.0}$ = 0.44 A
Applying $V_T = \varepsilon - Ir$ we get,

$$\varepsilon = 4.4 + (0.44)r \text{ ------ (2)}$$

Solving (1) and (2),

$$\varepsilon = 5.2 \text{ V and } r = 1.7\ \Omega$$

14.3.5.2 Maximum Power Transfer Theorem

In practice, when an energy source is being used to supply energy, we would want it to give as much useful energy output as possible at the external load i.e. Maximum efficiency (100% if ideal). However, the presence of internal energy in practice means that the amount of useful energy output that we can get from the source is being reduced because some of the input energy is being lost via energy dissipation through internal resistance in the e.m.f source itself.

Aim: Find out the optimal resistance of the external load, *R* such that the useful energy output at the external load can be maximised.

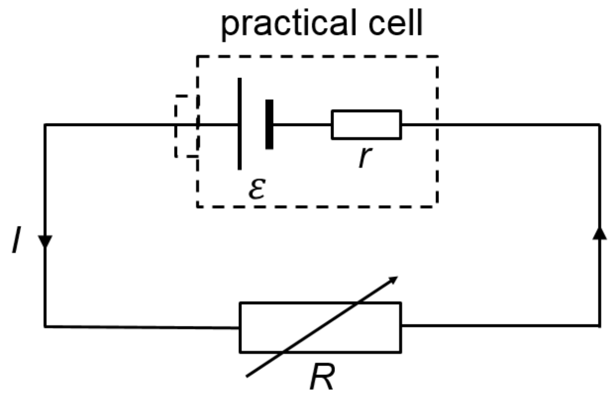


Fig. 11 Set up to investigate maximum power theorem

Given variables:
1) External load: a variable resistor with resistance: *R*
2) Internal resistance of source: *r*
3) E.m.f. of source: ε

Assumptions:
1) Power supplied by the source is constant.
2) Energy is only being lost at the external load and internal resistance of the e.m.f source (connecting wires have zero resistance).

To determine the power transferred to the external load, P_R we use the following formula:

$$P_R = I^2 R \text{ ------- (1)}$$

However, as we are not intending to express our final expression in terms of current *I* flowing across the external load, we will need to find a way to replace variable *I* using variables: *R, r* and ε.

To do so, we can consider a circuit as shown in Fig. 11.
Using conservation of energy,

$$P_{source} = P_R + P_r$$

where P_r is power loss due to internal resistance of source.

Using $P_{source} = I\varepsilon$, $P_R = I^2 R$ and $P_r = I^2 r$, we arrive at

$$I\varepsilon = I^2 R + I^2 r$$

which can be further simplified into:

$$\Rightarrow \varepsilon = IR + Ir$$
$$\therefore I = \frac{\varepsilon}{R + r}$$

Substituting this expression into (1), we get:

$$P_R = \left(\frac{\varepsilon}{R + r}\right)^2 R$$

$$\Longrightarrow P_R = \varepsilon^2 \frac{R}{(R+r)^2} \text{ ------------ (2)}$$

To determine the value of *R* such that maximum power, P_R can be delivered to the external load, we are actually solving an optimization problem. In other words, find the turning point of the P_R against *R* graph in Fig. 12.

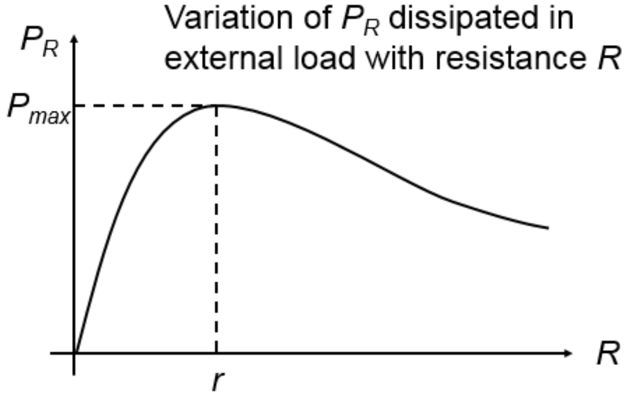


Fig. 12 Graph of P_R against *R*

Mathematically, it means that we are solving for

$$\frac{dP_R}{dR} = 0 \text{ ---------- } (3)$$

Differentiating equation (2) with respect to *R* on the LHS of equation (3)

$$\frac{d}{dR}\left[\varepsilon^2\left(\frac{R}{(R+r)^2}\right)\right] = 0$$

$$\Rightarrow \frac{d}{dR}[R(R+r)^{-2}] = 0 \text{ (as } \varepsilon \text{ is a constant)}$$

Applying product rule,

$$(R+r)^{-2} + (R)(-2)(R+r)^{-3} = 0$$

Solving, we will arrive at

$$R = r$$

Mathematically, to show that this stationary point is a maximum point, we need to prove that $\frac{d^2P_R}{dR^2} < 0$. This shall be left as an exercise for the reader to try.

Hence, in conclusion, for power transferred to the external load from the source to be a maximum, the external load must have the same resistance as the internal resistance of the battery. This is known as the **Maximum Power Transfer Theorem**.

Appendix A: Deriving the equation for electric power

From the definition of power,

$$P = \frac{dW}{dt}$$

Where *W* is the amount of energy in units of joules converted **from electric potential energy to other form of energy.** (in other words, we are talking about the power consumed by an electrical device).

Since the electrical device does work to move a quantity of charge *Q* across a potential difference *V*, from the definition of p.d. $V = \frac{W}{Q}$.

$$P = \frac{VQ}{t}$$
$$= V\left(\frac{Q}{t}\right)$$
$$= VI$$

From the definition of electric current $I = \frac{Q}{t}$ at constant *V*,

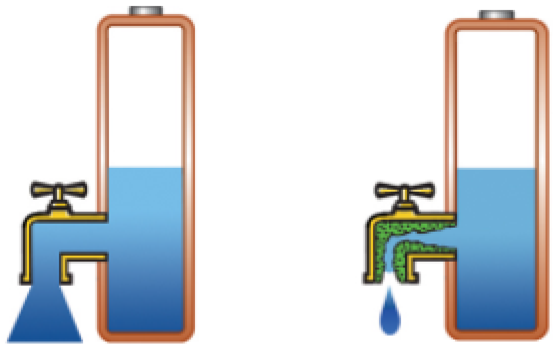
$$P = VI$$

Since the p.d. *V* across the device and the current *I* in the corresponding section of the circuit is proportional to its resistance *R*, i.e. $R = \frac{V}{I}$,

$$\boxed{P = VI = I^2R = \frac{V^2}{R}}$$

Appendix B: Internal resistance of handphone batteries

Capacity alone is of limited use if the pack cannot deliver the stored energy effectively; a battery also needs low internal resistance. Measured in milliohms (mΩ), resistance is the gatekeeper of the battery; the lower the resistance, the less restriction the pack encounters. This is especially important in heavy loads such as power tools and electric powertrains. High resistance causes the battery to heat up and the voltage to drop under load, triggering an early shutdown. Figure 1 illustrates a battery with low internal resistance in the form of a free-flowing tap against a battery with elevated resistance in which the tap is restricted.



Low resistance, delivers high current on demand; battery stays cool.

High resistance, current is restricted, voltage drops on load; battery heats up.

Figure 1: Effects of internal battery resistance.

A battery with low internal resistance delivers high current on demand. High resistance causes the battery to heat up and the voltage to drop. The equipment cuts off, leaving energy behind.

Courtesy of Cadex

Lead acid has a very low internal resistance and the battery responds well to high current bursts that last for a few seconds. Due to inherent sluggishness, however, lead acid does not perform well on a sustained high current discharge; the battery soon gets tired and needs a rest to recover. Some sluggishness is apparent in all batteries at different degrees but it is especially pronounced with lead acid. This hints that power delivery is not based on internal resistance alone but also on the responsiveness of the chemistry, as well as temperature. In this respect, nickel- and lithium-based technologies are more responsive than lead acid.

Sulfation and grid corrosion are the main contributors to the rise of the internal resistance with lead acid. Temperature also affects the resistance; heat lowers it and cold raises it. Heating the battery will momentarily lower the internal resistance to provide extra runtime. This, however, does not restore the battery and will add momentary stress.

Crystalline formation, also known as "memory," contributes to the internal resistance in nickel-based batteries. This can often be reversed with deep-cycling. The internal resistance of Li-ion also increases with use and aging but improvements have been made with electrolyte additives to keep the buildup of films on the electrodes under control. (See BU-808b: What causes Li-ion to Die?) With all batteries, SoC affects the internal resistance. Li-ion has higher resistance at full charge and at end of discharge with a big flat low resistance area in the middle.

Alkaline, carbon-zinc and most primary batteries have a relatively high internal resistance, and this limits their use to low-current applications such as flashlights, remote controls, portable entertainment devices and kitchen clocks. As these batteries deplete, the resistance increases further. This explains the relative short runtime when using ordinary alkaline cells in digital cameras.

Two methods are used to read the internal resistance of a battery: Direct current (DC) by measuring the voltage drop at a given current, and alternating current (AC), which takes reactance into account. When measuring a reactive device such as a battery, the resistance values vary greatly between the DC and AC test methods, but neither reading is right or wrong. The DC reading looks at pure resistance (R) and provides true results for a DC load such as a heating element. The AC reading includes reactive components and provides impedance (Z). Impedance provides realistic results on a digital load such as a mobile phone or an inductive motor. (See BU-902: How to Measure Internal Resistance)

The internal resistance of a battery does not consist of the cells alone but also includes the interconnection, fuses, protection circuits and wiring. In most cases these peripherals more than double the internal resistance and can falsify rapid-test methods. Typical readings of a single cell pack for a mobile phone and a multi-cell battery for a power tool are shown below.

Internal Resistance of a Mobile Phone Battery

Cell, single, high capacity prismatic	50mΩ	subject to increase with age
Connection, welded	1mΩ	
PTC, welded to cable, cell	25mΩ	18–30 mΩ according to spec
Protection circuit, PCB	50mΩ	
Total internal resistance	**ca. 130mΩ**	

Internal Resistance of a Power Pack for Power Tools

Cells 2P4S at 2Ah/cell,	18mΩ	subject to increase with age
Connection, welded, each	0.1mΩ	
Protection circuit, PCB	10mΩ	
Total internal resistance	**ca. 80mΩ**	

Source: Siemens AG (2015, München)

Figures 2 and 3 reflect the runtime of three batteries with similar Ah and capacities but different internal resistance when discharged at 1C, 2C and 3C. The graphs demonstrate the importance of maintaining low internal resistance, especially at higher discharge currents. The NiCd test battery comes in at 155mΩ, NiMH has 778mΩ and Li-ion has 320mΩ. These are typical resistive readings on aged but still functional batteries. (See BU-208: Cycling Performance) that demonstrates the relationship of capacity, internal resistance and self-discharge.)

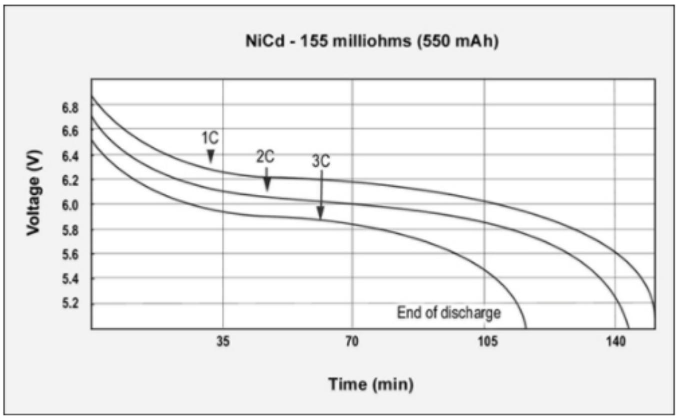


Figure 2: GSM discharge pulses at 1, 2, and 3C with resulting talk-time
The capacity of the NiCd battery is 113%; the internal resistance is 155mΩ. 7.2V pack.

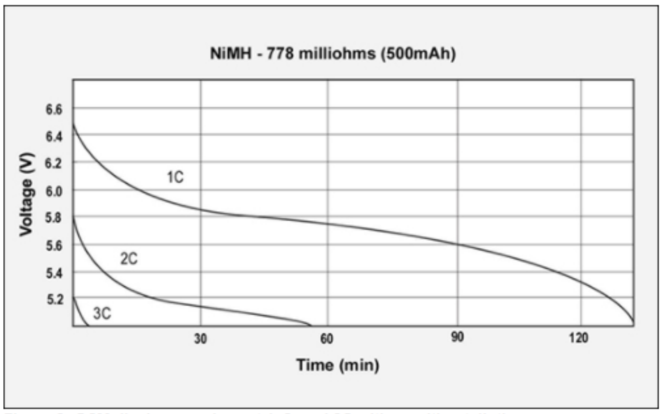


Figure 3: GSM discharge pulses at 1, 2, and 3C with resulting talk-time
The capacity of the NiMH battery is 94%, the internal resistance is 778mΩ. 7.2V pack

The above article is extracted from (http://batteryuniversity.com/learn/article/rising_internal_resistance)