Anglo - Chinese School
(Independent)

THE BEST IS YET TO BE

FINAL EXAMINATION 2017

YEAR 3 INTEGRATED PROGRAMME

CORE MATHEMATICS
PAPER 2

MONDAY **9th October 2017** **1 hour 30 minutes**

INSTRUCTIONS TO STUDENTS

Do not open this examination paper until instructed to do so.
A calculator is required for this paper.
Answer all the questions on the answer sheets provided.
At the end of the examination, fasten the answer sheets together.
Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures. Answers in degrees are to be given to one decimal place.

INFORMATION FOR STUDENTS

The maximum mark for this paper is 80.

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for correct method, provided this is shown by written working. You are therefore advised to show all working.

Answer all the questions on the answer sheets provided. Begin each question on a new page.

1 *[Maximum mark: 10]*

(a) It is given that $s = ut + \frac{1}{2}at^2$.

(i) Express a in terms of s, u and t.

[2]

(ii) Find the value of a when $s = 12.4$, $u = 0.256$ and $t = 0.25$, leaving your answer corrected to the nearest whole number.

[2]

(b) Solve the equation $\frac{4}{y-1} = \frac{y-1}{8}$, leaving your answers in the simplest surds.

[3]

(c) A man bought m pencils at r dollars per dozen. He sold them for k cents each. Find an expression, in terms of m, r and k, for the profit, in cents, that he made.

[3]

2 *[Maximum mark: 6]*

Find the value(s) of k for which the line $x + y = k$ is a tangent to the curve $x^2 - 3x + y^2 = 5$.

[6]

3 *[Maximum mark: 11]*

Solve the following equations:

(a) $2^{x+3} \times 8^{2x-3} = 64^{x+7}$

[3]

(b) $\log_2 x + \log_8 2x = 2\frac{1}{3}$

[4]

(c) Solve the equation $e^{1-t} + 2 = \frac{7}{e^t}$, giving your answer correct to 2 decimal places.

[4]

4 *[Maximum mark: 7]*

(a) The sum S of the first n integers is given by the formula $S = \frac{1}{2}n(n+1)$. What is the minimum number of integers required in order for the sum to exceed 325?

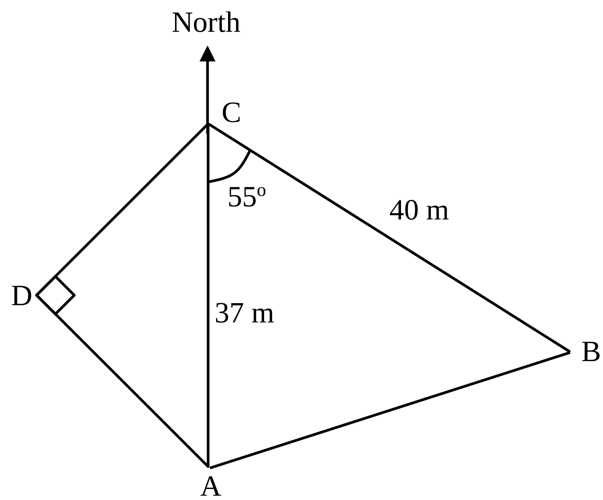
[3]

(b) Find the values of the integers a and b such that $\frac{a+b\sqrt{5}}{5+2\sqrt{5}} = \frac{5+2\sqrt{5}}{2+\sqrt{5}}$.

[4]

5 *[Maximum mark: 10]*

In the diagram, the points A, B, C and D are on level ground. The point D is equidistant from A and C and ∠ADC = 90º. C is due north of A, AC = 37 m, BC = 40 m and ∠BCA = 55º.



Calculate

(a) the area of $\triangle ABC$,

[2]

(b) $\measuredangle CBA$,

[4]

(c) the distance AD,

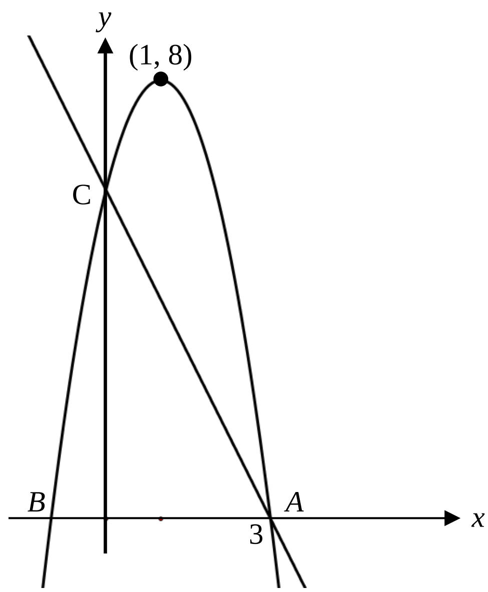
[2]

(d) the bearing of A from B.

[2]

6 *[Maximum mark: 12]*

The diagram below shows the curve of a quadratic function that cuts the *y*-axis at C and the *x*-axis at B and A where $x = 3$. It has a maximum point at (1, 8). A straight line passes through C and A.



(a) Find the equation of the curve.

[4]

(b) Find the equation of the perpendicular bisector of AC.

[4]

(c) The perpendicular bisector of AC intersects the line AC at D and the *y*-axis at E. Find the area of triangle CDE.

[4]

7 *[Maximum mark: 7]*

Given that the equation $x^2 + 4x + h = 0$ has two real roots,

(i) find the range of possible values of h.

[2]

(ii) find the value of h if the two roots are α and β where $\alpha - \beta = 4\sqrt{3}$.

[5]

8 *[Maximum mark: 13]*

A closed rectangular box of height h cm has a horizontal rectangular base of sides $3x$ cm and $2x$ cm, where $1 \le x \le 6$.

(a) If the volume of the box is 200 cm^3, express h in terms of x and show that the total surface area, A cm^2, of the box is given by $A = 12x^2 + \dfrac{1000}{3x}$.

[4]

The table below shows some values of x & the corresponding values of A (correct to the nearest integer).

x	1	2	3	4	5	6
A	345	215	a	275	367	488

(b) Calculate the value of a.

[1]

(c) Taking 2 cm to represent 1 unit on the horizontal axis and 2 cm to represent 50 units on the vertical axis, draw the graph of $A = 12x^2 + \dfrac{1000}{3x}$ for $1 \le x \le 6$.

[4]

(d) From the graph, estimate
(i) the minimum value of A, and the corresponding height of the box,
(ii) the range of values of x for which $A \le 300$.

[4]

9 *[Maximum mark: 4]*

The roots, α and β, of the equation $ax^2 + bx + c = 0$ are in the ratio 1 : n. Show that $(1+n)^2 ac = nb^2$.

[4]

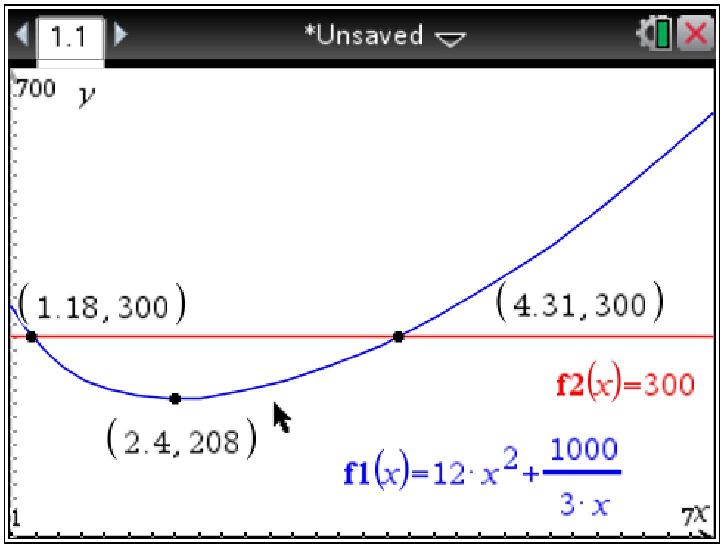
---END OF PAPER--

1ai)	$s = ut + \frac{1}{2}at^2$ $s - ut = \frac{1}{2}at^2$ $2s - 2ut = at^2$ $a = \frac{2s - 2ut}{t^2} = \frac{2(s - ut)}{t^2}$	
1aii)	$a = \frac{2(12.4) - 2(0.256)(0.25)}{0.25^2}$ $a = 395$	
1b)	$\frac{4}{y-1} = \frac{y-1}{8}$ $(y-1)^2 = 32$ $y - 1 = \pm 4\sqrt{2}$ $y = 1 \pm 4\sqrt{2}$	
1c)	Cost of each pencil = $\frac{mr100}{12}$ Selling price = mk Profit = $mk - \frac{mr100}{12} = m(k - \frac{25}{3}r)$	
2)	$x = k - y$ $(k-y)^2 - 3(k-y) + y^2 = 5$ $k^2 - 2ky + y^2 - 3k + 3y + y^2 - 5 = 0$ $2y^2 + 3y - 2ky - 5 - 3k + k^2 = 0$ $2y^2 + (3 - 2k)y - 5 - 3k + k^2 = 0$ $(3 - 2k)^2 - 4(2)(k^2 - 3k - 5) = 0$ $9 - 12k + 4k^2 - 8k^2 + 24k + 40 = 0$ $-4k^2 + 12k + 49 = 0$ $4k^2 - 12k - 49 = 0$ $k = 5.31$ or $k = -2.31$	
3a)	$2^{x+3} \times 2^{6x-9} = 2^{6x+42}$ $2^{7x-6} = 2^{6x+42}$	

	$7x-6=6x+42$ $x=48$	
3b)	$\log_2 x+\log_8 2x=2\frac{1}{3}$ $\log_8 x^3+\log_8 2x=2\frac{1}{3}$ $\log_8 2x^4=2\frac{1}{3}$ $2x^4=128$ $x^4=64$ $x=2.83$	
3c)	$e^{1-t}+2=\frac{7}{e^t}$ $\frac{e}{e^t}+2=\frac{7}{e^t}$ $e+2e^t=7$ $e^t=\frac{7-e}{2}$ $\ln e^t=\ln\left(\frac{7-e}{2}\right)$ $t=0.761$	
4a)	$\frac{1}{2}n(n+1)>325$ $n^2+n-650>0$ $(n-25)(n+26)>0$ $n<-26$ or $n>25$ Hence, minimum number of integers = 26	
4b)	$\frac{a+b\sqrt{5}}{5+2\sqrt{5}}=\frac{5+2\sqrt{5}}{2+\sqrt{5}}$ $a+b\sqrt{5}=\frac{(5+2\sqrt{5})^2}{2+\sqrt{5}}$ $a+b\sqrt{5}=\frac{45+20\sqrt{5}}{2+\sqrt{5}}$	

	$a+b\sqrt{5}=\frac{45+20\sqrt{5}}{2+\sqrt{5}}\times\frac{2-\sqrt{5}}{2-\sqrt{5}}$ $a+b\sqrt{5}=\frac{90-45\sqrt{5}+40\sqrt{5}-100}{-1}$ $a+b\sqrt{5}=10+5\sqrt{5}$	
5a)	Area of triangle = $\frac{1}{2}ab\sin C$ Area of triangle = $\frac{1}{2}(37)(40)\sin 55$ Area of triangle = 606	
5b)	$AB^2=37^2+40^2-2(37)(40)\cos 55$ $AB^2=1271.213$ $AB=35.65$ $\frac{\sin 55}{35.65}=\frac{\sin\measuredangle CBA}{37}$ $\sin\measuredangle CBA=0.85017$ $\measuredangle CBA=58.2^o$	
5c)	$\cos 45^o=\frac{AD}{37}$ $AD=26.2$	
5d)	Alternate Angle = 55^o Bearing = $360-55+58.2$ Bearing = 246.8^o	
7a)	$y=a(x-1)^2+8$ When $x=3$ and $y=0$, $0=a(3-1)^2+8$ $-8=4a$ $a=-2$ $y=-2(x-1)^2+8$	
7b)	When $x=0$, $y=6$ Coordinate of C(0, 6)	

	Midpoint of AC = $\left(\frac{0+3}{2}, \frac{6+0}{2}\right)$ Midpoint of AC = $\left(\frac{3}{2}, 3\right)$ Gradient of AC = $-\frac{6}{3}$ Gradient of AC = -2 Gradient of perpendicular bisector = $\frac{1}{2}$ Equation is: $y - 3 = \frac{1}{2}(x - \frac{3}{2})$ $y = \frac{1}{2}x + \frac{9}{4}$	
7c)	D $\left(\frac{3}{2}, 3\right)$ When $x = 0$, $y = \frac{9}{4}$ Area of triangle CDE = $\frac{1}{2}(6 - \frac{9}{4})(\frac{3}{2})$ Area of triangle CDE = 2.81	
8ai)	$4^2 - 4(1)(h) > 0$ $16 - 4h > 0$ $-4h > -16$ $h < 4$	
8aii)	$\alpha + \beta = -4$ $\alpha\beta = h$ $\alpha - \beta = \sqrt{(\alpha + \beta)^2 -}$ $4\sqrt{3} = \sqrt{(-4)^2 - 4h}$ $16(3) = 16 - 4h$ $4h = -32$ $h = -8$ $\alpha + \beta = -4$ $\alpha^2 + \beta^2 + 2\alpha\beta = 16$ -------(1) $\alpha - \beta = 4\sqrt{3}$ $\alpha^2 + \beta^2 - 2\alpha\beta = 48$ ----- (2) (1) $-$ (2): $4\alpha\beta = -32$ $\alpha\beta = -8$	
9a)	$3x \times 2x \times h = 200$ $h = \frac{200}{6x^2}$ Area = $2(3x)(2x) + 2(2x)(h) + 2(3x)(h)$	

	Area = $12x^2 + 10xh$ Area = $12x^2 + 10x\left(\frac{200}{6x^2}\right)$ Area = $12x^2 + \frac{1000}{3x}$	
9b)	a = $12(3)^2 + \frac{1000}{3(3)}$ a = 219	
9c)		
9di)	Minimum Volume = 208 Corresponding height = $\frac{200}{6(2.4)^2}$ Corresponding height = 5.79	
9dii)	$1.18 \leq x \leq 4.31$	
10)	The roots are α and $n\alpha$ $\alpha + n\alpha = -\frac{b}{a}$ $\alpha(1+n) = -\frac{b}{a}$ $\alpha = -\frac{b}{a(1+n)}$ $(\alpha)(n\alpha) = \frac{c}{a}$ $n\alpha^2 = \frac{c}{a}$	

	$n\left(-\frac{b}{a(1+n)}\right)^2 = \frac{c}{a}$ $\frac{nb^2}{a^2(1+n)^2} = \frac{c}{a}$ $\frac{nb^2}{a(1+n)^2} = c$ $nb^2 = ac(1+n)^2$	