Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \dfrac{n!}{r!(n-r)!} = \dfrac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 **(i)** Differentiate $\ln(1+3x^2)^2$ with respect to x. [2]

(ii) Hence, find $\int \frac{6x}{1+3x^2}\,\mathrm{d}x$. [2]

2 **(i)** Given that $\dfrac{\sin(A-B)}{\sin(A+B)}=\dfrac{2}{3}$, prove that tan A = 5 tan B. [4]

(ii) Hence, or otherwise, solve the equation $3\sin(A-45°)=2\sin(A+45°)$ for $0° \le A \le 360°$. [3]

[Turn over

3 **(i)** Show that $x-2$ is a factor of x^3-2x^2+3x-6. [1]

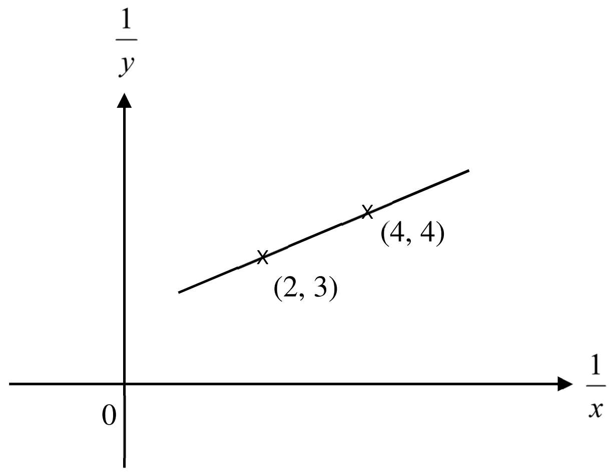
(ii) Express $\dfrac{11-2x}{x^3-2x^2+3x-6}$ as partial fractions. [6]

4 It is given that $\mathrm{f}(x)$ is such that $\mathrm{f}'(x) = \sin 2x + \cos 3x$. Given also that $\mathrm{f}\left(\frac{\pi}{6}\right) = 0$, show that $\mathrm{f}''(x) + 9\mathrm{f}(x) = -\frac{3}{4} - \frac{5}{2}\cos 2x$. [5]

5 **(a)** Simplify $\log_x 9 \times \log_{27} x$ **without using a calculator**. [3]

(b) Solve $\ln(2x + \mathrm{e}) = 1 + \dfrac{1}{\log_x \mathrm{e}}$. [4]

6



The diagram shows part of a straight line graph obtained by plotting $\frac{1}{y}$ against $\frac{1}{x}$.

Express y in terms of x. [4]

7 The equation of a circle is $x^2 + y^2 - 8x + 4y - 16 = 0$.

(i) Find the coordinates of the centre C of the circle and find the radius. [3]

(ii) The line $y = 2(x + 1)$ cuts the circle at two distinct points A and B.
Find the coordinates of A and of B. [5]

(iii) Find the equation of a **second** circle, with same centre C, and whose area is 4 times of the original circle. [2]

8 The mass, m mg, of a radioactive substance decreases with time, t hours.
Measured values of m and t are given in the table below.

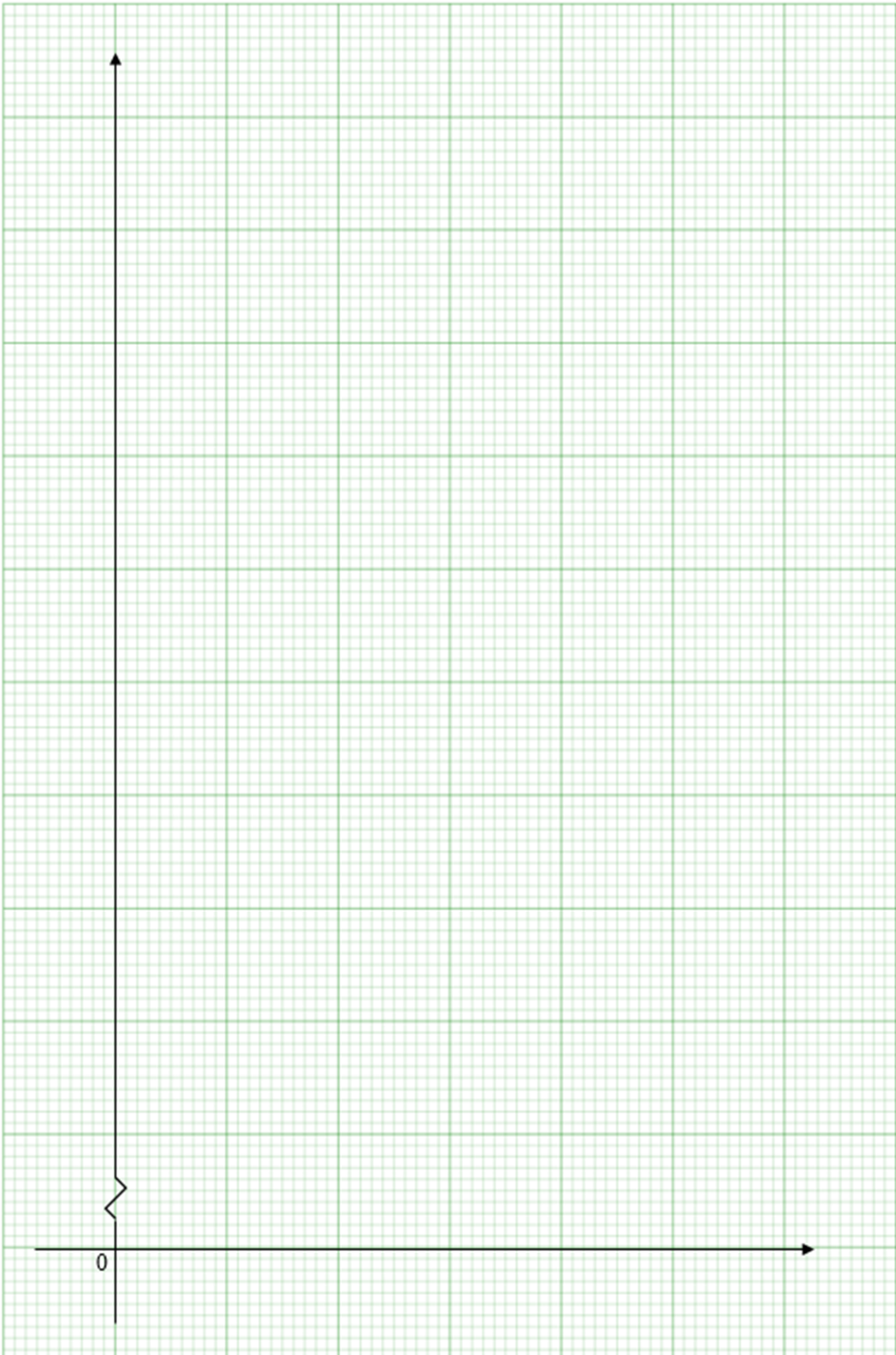
t (hours)	2	4	6	8	10
m (mg)	48.2	41.5	35.7	30.7	26.5

It is known that m and t are related by the equation $m = m_0 e^{-kt}$, where m_0 and k are constants.

(i) On the grid on the next page, draw a straight line graph of $\ln m$ against t, using a scale of 2 cm for 0.1 unit on the ln m-axis, **starting from 3.2**, and a scale of 1 cm for 1 unit on the t-axis. [3]

(ii) Use your graph to estimate the value of k and of m_0. [5]

(iii) Use your graph to estimate the number of hours for the mass of the substance to be reduced by 20%. [2]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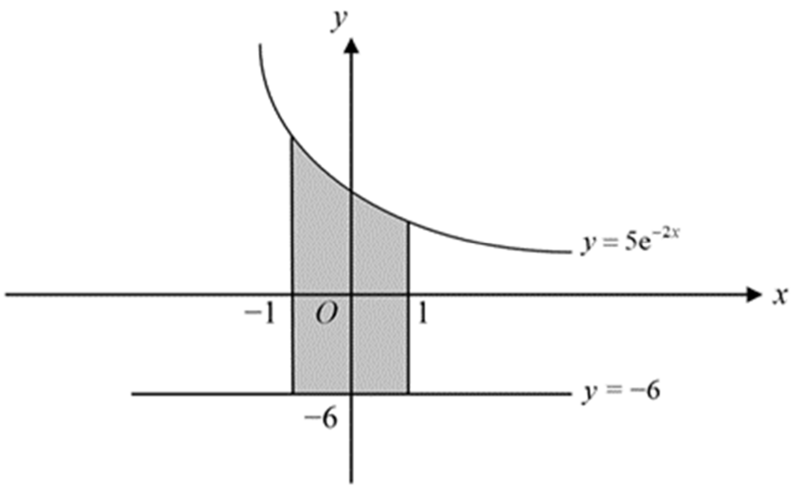
9 **(i)** Find $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{\ln x}{4x}\right)$. [2]

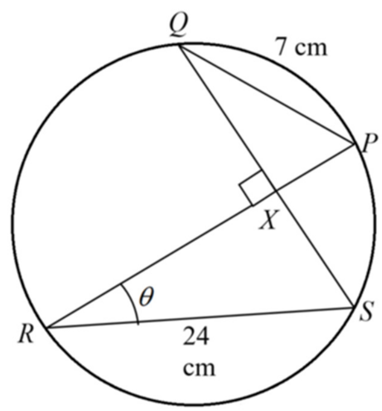
(ii) Hence, determine the range of values of x for which the curve $y = \dfrac{\ln x}{4x}$ is a decreasing function. Leave your answer in terms of e. [3]

10



Find the area of the shaded region. The figure above is not drawn to scale. [5]

11 The diagram below shows a circle with chords *PQ* and *RS* intersect at the point *X*. *P*, *Q*, *R* and *S* are points on the circle. Given that *PQ* = 7 cm, *RS* = 24 cm, angle $PXQ = 90°$ and angle $PRS = \theta$, where θ varies.



(i) Prove that $PR = 24\cos\theta + 7\sin\theta$. [3]

(ii) Express *PR* in the form $R\cos(\theta - \alpha)$. [3]

(iii) Hence, find the value of θ for which PR = 15 cm. [3]

(iv) Find the maximum length of PR and the corresponding value of θ. [3]

12 **(a)** Solve $0.5\sin 2x = 3\sin x$ for $0 < x < 2\pi$. [4]

(b) Solve, for values of θ between 0° and 180°, [3]

$$\operatorname{cosec}\left(\frac{\theta}{2} - 40^{\circ}\right) + 5 = 0.$$

[Turn over

13 **(a)** Find $\int \left(\frac{3}{\sqrt{2+7x}} + \sin 5x \right) \mathrm{d}x$. [3]

(b) If $\mathrm{f}'(x) = 2x$ and $\mathrm{g}'(x) = 2x$, can we conclude that $\mathrm{f}(x) = \mathrm{g}(x)$? Give a reason for your answer. [2]

(c) Given that $V = 20 - 3\mathrm{e}^{x}$, explain why the value of V can never be 20. [2]

End of Paper