TAMPINES MERIDIAN JUNIOR COLLEGE

JC2 PRELIMINARY EXAMINATION

CANDIDATE NAME: ____

CIVICS GROUP: ____

H2 MATHEMATICS 9758/02

Paper 2

19 SEPTEMBER 2023

3 hours

Candidates answer on the question paper.

Additional material: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your name and Civics Group on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the question paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

For Examiners' Use	
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Total	

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

This document consists of 6 printed pages and 0 blank pages.

[Turn Over

Section A: Pure Mathematics [40 marks]

1 The position vectors of points A, B and P with respect to an origin O are **a**, **b** and **p** respectively, where $\mathbf{p} = 3\mathbf{a} - 2\mathbf{b}$.

(a) Show that A, B and P are collinear and hence find the ratio $BP : AP$. [4]

(b) It is given that **b** is a unit vector, $|\mathbf{a}| = \frac{4}{3}$ and the angle between **a** and **b** is $\frac{\pi}{3}$.

The point N is a point on line BP which is closest to the origin. Find the value of $\frac{BN}{BA}$. [5]

2 **(a)** Logarithmic functions are useful as measurement units for phenomena with large scale of values, such as decibels (for sound) or Richter scale (for earthquakes). Logarithmic functions make the measurement units smaller and easier to work with. An example of a logarithmic function is given as

$$y = \ln\left(1 + 2x + 3x^2\right).$$

(i) By using differentiation, find the Maclaurin series for y, up to and including the term in x^2. [4]

(ii) By using the standard series in MF26, verify that the answer obtained in part **(i)** is correct. [2]

(b) Find the series expansion of $\frac{x}{\sqrt{4+x}}$, up to and including the term in x^3. [4]

3 **(a)** Given that $z_1 = r_1(\cos\theta_1 + \mathrm{i}\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + \mathrm{i}\sin\theta_2)$, prove, using a trigonometric method, that $\dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}\left[\cos(\theta_1 - \theta_2) + \mathrm{i}\sin(\theta_1 - \theta_2)\right]$.

Hence state $\arg\left(\dfrac{z_1}{z_2}\right)$ in terms of θ_1 and θ_2. [3]

Let $w_r = r + \mathrm{i}$, where r is a real constant, $r \geq 0$.

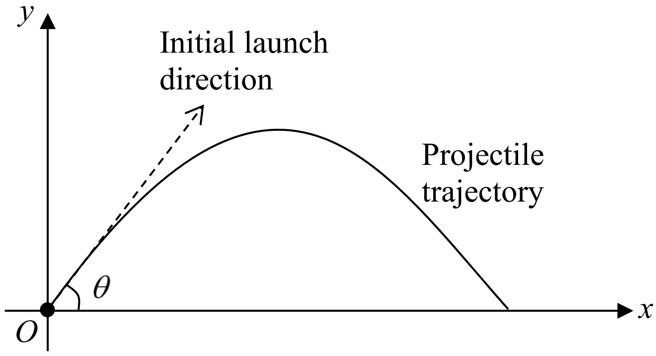
(b) Use the method of differences to express $\displaystyle\sum_{r=1}^{n}\left(\arg(w_{r-1}) - \arg(w_r)\right)$ in the form $k - \arg(n + \mathrm{i})$, where k is an exact real constant to be determined. [3]

(c) Explain why $\displaystyle\sum_{r=1}^{\infty}\left(\arg(w_{r-1}) - \arg(w_r)\right)$ converges and state its limit. [2]

(d) Hence, or otherwise, find the exact value of

$$\arg\left(\frac{1+\mathrm{i}}{2+\mathrm{i}}\right)^3 + \arg\left(\frac{2+\mathrm{i}}{3+\mathrm{i}}\right)^3 + \arg\left(\frac{3+\mathrm{i}}{4+\mathrm{i}}\right)^3 + \ldots.$$ [4]

4 Projectile motion is the motion of a launched object, subject to only the constant acceleration of gravity, g ms^{-2}. The object is called a projectile, and its path is called a trajectory.



A projectile is launched from the origin with an initial velocity of u ms^{-1} at an angle θ radian above the horizontal, where $0 < \theta < \frac{\pi}{2}$, as shown in the diagram above.

The trajectory of a projectile depends on its motion in two dimensions – horizontal (x) and vertical (y) components. The position of the projectile at time t seconds after it is launched can be defined by

$$x = ut\cos\theta \qquad \text{and} \qquad y = ut\sin\theta - \frac{1}{2}gt^2,$$

where u and g are positive real constants.

(a) When $y = 0$, find the value of t in terms of θ, g and u, where $t \neq 0$. State what this value of t means in the context of the question. [3]

The range of the projectile is defined as the horizontal distance the projectile travels from the time it is launched to the time it returns to the same height at which it is launched.

(b) Show that the range of the projectile can be expressed as $x = \frac{u^2}{g}\sin(2\theta)$. [1]

(c) Use differentiation to find the value of θ that gives the maximum range of the projectile. Hence find the maximum range of the projectile in terms of g and u. [5]

Section B: Probability and Statistics [60 marks]

5 For events A and B, it is given that $\mathrm{P}(B') = 0.51$ and $\mathrm{P}(A' \cap B') = 0.15$.

(a) Find $\mathrm{P}(A \cap B')$. [2]

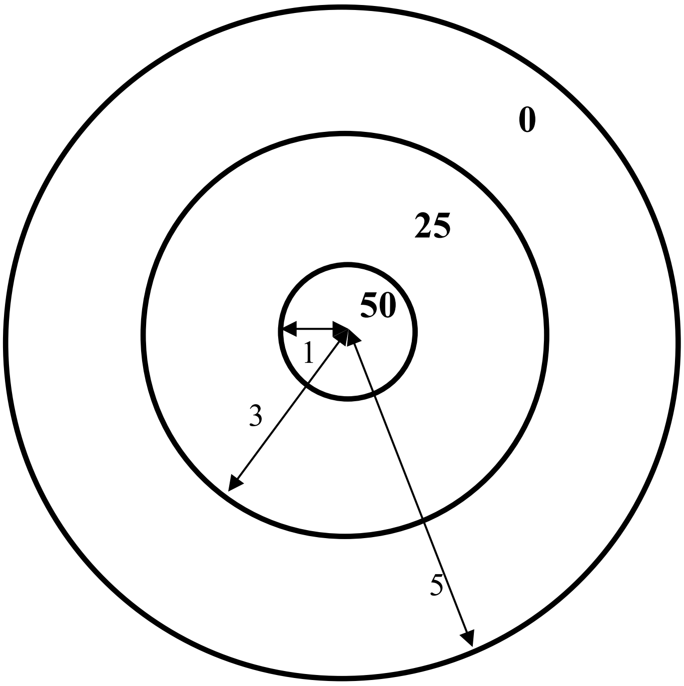
It is also given that $\mathrm{P}(B \mid A) = \dfrac{11}{29}$.

(b) Find $\mathrm{P}(A)$. [3]

For a third event C, it is given that A and C are independent of each other, and B and C are mutually exclusive.

(c) Find the maximum value of $\mathrm{P}(C)$. [3]

6 At a particular game stall, players throw darts at a target board to earn points for redeeming prizes.



The target board can be modelled by three concentric circles with radii one unit, three units and five units respectively. A player will get a score of 50, 25 or 0 points if the dart lands on the respective regions labelled **50**, **25** or **0** as shown in the diagram above.

The probability that a dart lands on any region labelled **50**, **25** or **0** is proportional to the area of the region. It can be assumed that the borders between the regions have negligible thickness and the dart will only land on the target board, which has a total area of 25π units^2.

In a game, a player throws two darts at the target board, one after the other. The first dart will be removed before the next dart is thrown. Let S be the combined score that the player achieves from the two throws.

(a) Show that $\mathrm{P}(S = 75) = \dfrac{16}{625}$. [3]

(b) Find the probability distribution of S. [4]

(c) Find the expectation and variance of S. [2]

7 A researcher is investigating the relationship between the students' test marks and their average daily duration of sleep one week leading up to their test. The following table gives the average daily duration of sleep, t hours, of 9 students and their test marks, x, out of 100.

t	5.5	5.8	6.1	6.5	7.2	7.4	8.0	8.2	9.1
x	39	43	52	60	66	68	77	78	80

(a) Explain whether the product moment correlation coefficient between t and x would differ if the researcher converted the average daily duration of sleep to minutes. [1]

The researcher decides to fit a model of the form $\ln(100-x) = a + bt$ to the data.

(b) Find the product moment correlation coefficient between t and $\ln(100-x)$. What conclusion can the researcher reach about the relationship between t and $\ln(100-x)$? Justify your answer. [2]

(c) Draw a scatter diagram for $\ln(100-x)$ against t. Draw, by eye, a line of best fit on your scatter diagram. [2]

(d) Use your calculator to find the equation of the least squares regression line of $\ln(100-x)$ on t. [1]

(e) Use the equation in part **(d)** to estimate the test marks for a student who has an average daily duration of sleep of 10 hours. Explain whether you would expect this estimate to be reliable. [3]

8 A grocery store sells two types of apples, Brand A and Brand S. The masses (in grams) of the apples for each brand can be modelled by independent normal distributions with means and standard deviations as shown in the table.

	Mean	Standard Deviation
Brand A	82.2	2.2
Brand S	78.8	3.1

You may assume that the following samples of apples are randomly chosen.

(a) Find the probability that the mass of a Brand A apple is between 80 grams and 84 grams. [1]

(b) Find the probability that the total mass of five Brand A apples is more than 408 grams. [3]

(c) The mass of a **peeled** apple is 90% of its original mass. Find the probability that the difference in mass between a Brand S apple and a **peeled** Brand A apple exceeds one gram. [5]

9 Tim is a student in a school with a large student population. As part of his project, Tim would like to interview some of his schoolmates to obtain some feedback about the school.

(a) It can be assumed that the school has equal number of students in Year 1 and Year 2 and there are equal numbers of male and female students in each year. If Tim would like to interview 80 students from the school, explain how he should select the 80 students such that he obtains a fair and representative sample. [2]

While in school, Tim overheard a group of teachers claiming that students in the school spend an average of 3 hours each day on their homework. Tim believes that students are spending more time on their homework. Tim interviewed 80 randomly chosen students and summarised the amount of time, x hours, that the students spend on homework each day as shown below.

$$\sum(x-3)=16 \text{ and } \sum(x-3)^2=164.$$

(b) Explain why Tim should carry out a one-tailed test. [1]

(c) Calculate unbiased estimates of the population mean and variance. [2]

(d) Conduct a hypothesis test at a suitable level of significance to determine if Tim's belief is accurate. [4]

(e) Tim interviews another random sample of 10 students and found that the mean time spent each day on homework for these 10 students is 3.2 hours. You may assume that the population standard deviation of the time spent each day on homework is known to be σ hours. Given that the teachers' claim was rejected using a two-tail hypothesis test at 2% level of significance, determine the range of values of σ.
In your working, write down clearly an assumption needed for the calculations to be valid. [4]

10 Serene plays a simple online game which moves a counter one step at a time along a straight line. The counter starts from point S. In each turn, the counter moves one step forward with probability $\frac{3}{5}$ or one step backward with probability $\frac{2}{5}$. The online game ends after 10 turns. Let X be the number of turns, out of 10, where the counter moves one step forward.

(a) Explain why $X = 8$ means the counter ends up 6 steps ahead of point S at the end of the online game.

Hence find that probability that at the end of the online game, the counter ends up 6 steps ahead of point S. [3]

A player wins the online game if at the end of the online game, the counter ends up **at least** 6 steps ahead of point S.

(b) Show that the probability that Serene wins the online game is 0.167, rounded to 3 significant figures. [3]

(c) Serene plays the online game 20 times. Find the probability that she wins exactly 15% of these 20 online games. [2]

(d) Serene also plays a word puzzle game. The probability that Serene wins any word puzzle game is 0.4. Between the online game and the word puzzle game, on average, Serene chooses to play the online game 30% of time and the word puzzle game 70% of the time. Serene randomly chooses a game to play. Given that she wins a game, determine the probability that she plays the word puzzle game. [4]

End of Paper