YISHUN JUNIOR COLLEGE
2012 JC2 PRELIMINARY EXAMINATION

Higher 2 MATHEMATICS **9740/2**

Paper 2

17 AUGUST 2012
FRIDAY 0800h – 1100h

Additional materials:
- Answer paper
- Graph Paper
- List of Formulae (MF15)

TIME 3 hours

READ THESE INSTRUCTIONS FIRST

Write your name and CTG in the spaces provided on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphic calculator.
Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, write down the question number of the questions attempted, model of calculator used on the spaces provided on the cover page. Tie your cover page on top of the answer scripts before submission.
The number of marks is given in brackets [ ] at the end of each question or part question.

This paper consists of 6 printed pages.

Section A : Pure Mathematics [40 marks]

1. Two planes π_1 and π_2 have equations $x+2y+z=2$ and $2x+y-z=3$ respectively. The point A has coordinates (4, –1, 2).

(i) Find the acute angle between the planes π_1 and π_2. [2]

(ii) Let B be the foot of the perpendicular from A to π_1. Find the coordinates of B. [3]

(iii) If π_3 contains the line AB and is perpendicular to π_1 and π_2, find the position vector of the point of intersection of the 3 planes. [3]

2. The complex number z is given by $z=x+\mathrm{i}y$, where $x > 0$ and $y > 0$. Sketch an Argand diagram, with the origin O, showing the points P, R and Q representing z, 3iz and $(z+3\mathrm{i}z)$ respectively. [2]

Describe the geometrical relationship between O, P, Q and R. [1]

(a) Given that $y = 3x$, show that the point representing z^2 is collinear with the origin and the point Q. [2]

(b) Given that $|z|\le 3$ and $\tan^{-1}\left(\frac{1}{3}\right)\le \arg z\le \tan^{-1}(3)$, illustrate both of these relations on a single Argand diagram. Find the area of the region in which the point P can lie. [3]

3. The functions f and g are defined by

$$\mathrm{f}: x\mapsto 4-x^2-4\lambda x,\quad x\in\mathbb{R},\ \text{where } \lambda \text{ is a constant,}$$
$$\mathrm{g}: x\mapsto \ln(1-x),\quad x<1.$$

(i) Explain why function f does not have an inverse. [1]

(ii) The function f has an inverse if its domain is restricted to $x<k$. Find the largest value of k in terms of λ, and define f^{-1} corresponding to this domain of f. [4]

(iii) If $\lambda > -\frac{1}{2}$, show that the composite function gf^{-1} exists and find the range of gf^{-1} in terms of λ. [3]

4. A sequence $u_1,\ u_2,\ u_3,\ ....$ is such that $u_1=1$ and $3^{n+1}u_{n+1}=3^n u_n+2,$ for all $n\ge 1$.

(i) Find the values of u_2, u_3 and u_4, in the form of $\frac{k}{3^n}$, where k and n are positive integers. [1]

(ii) Give a conjecture for u_n in the form $\frac{\mathrm{f}(n)}{3^n}$, where $\mathrm{f}(n)$ is a linear expression in n. [1]

(iii) Prove your conjecture by induction. [4]

5. **(a)** A cylindrical container has a height of 200 cm. The container was initially filled with a liquid chemical but there was a leak from a hole in the base. When the leak was noticed, the container was half-full and the level of the chemical was decreasing at a rate of 1 cm per minute.

To model the situation, it is assumed that when the depth of the chemical remaining is x cm, the rate at which the level is decreasing is proportional to $\sqrt{x}$. By setting up and solving a differential equation, show that

$$2x^{\frac{1}{2}} = -\frac{1}{10}t + 2\sqrt{200} .$$

Hence, find, to the nearest minute, the duration for which the container has been leaking. [6]

(b) Using the substitution $v = x - y$, solve the equation $\frac{\mathrm{d}y}{\mathrm{d}x} + \left(1 + (x-y)^2\right)\cos x = 1$, expressing y in terms of x. [4]

Section B : Statistics [60 marks]

6. An employer has appointed an opinion polling company to survey 40 staff on the proposed changes in the shift work hours. The staff comprises 60 managers, 140 technicians and 300 factory workers.

(i) Give a disadvantage of using simple random sampling in this case. [1]

(ii) Describe how the sample could be chosen using stratified sampling and state one advantage of adopting this sampling method in this context. [3]

7. A manufacturer believes that its new coffee product will lower the low-density lipoprotein (LDL) level in the body. A group of 12 volunteers are randomly selected to consume this new coffee product every day, over a trial period of 10 weeks. The decrease in LDL level, denoted by X mg/dL, for each volunteer, is as follows:

32	44	52	59	37	15	18	21	28	51	40	23

The manufacturer will launch the new coffee product if the sample provides significant evidence that the mean decrease in LDL level is more than 25 mg/dL.

(i) Test, at the 5% level of significance, whether the mean decrease in LDL level is more than 25 mg/dL. [4]

(ii) State the assumption which is necessary for the above test to be valid. [1]

(iii) A senior analyst feels that a larger sample of size 100 should be taken to obtain more accurate results. Using the unbiased estimate of the population variance derived from the first sample, find the range of values of the mean decrease in LDL level of this sample which will result in the manufacturer not launching the new coffee product, at the 5% level of significance. [4]

8. **(a)** Three numbers are selected at random and without replacement from 1, 2, 3, 4, 5, 6, 7. The score is the largest of the three numbers. Calculate the probability that

(i) the score is 4, [1]

(ii) the score is 6, given that one of the numbers selected is 2. [2]

(b) Set A consists of integers between 0 and 1000 which can be formed using the following methods:

Method 1: Use digits chosen from 2, 4, 6 and no digits are repeated.
Method 2: Use digits chosen from 1, 3, 5, 7, where each digit may be repeated.

(i) Find the number of integers in set A. [3]

(ii) Two integers are selected at random and without replacement from set A. Find the probability that the sum of the two selected integers will be even. [3]

9. At a game stall, there are 4 balloons of equal size. In a game, a player is given 4 darts, and throws a dart at each balloon. The player wins a prize if all 4 balloons are burst. With each throw of a dart, a balloon has a probability p of bursting. Write down an expression for the probability that a player wins a prize in a particular game. [1]

(i) The game organiser can vary the value of p by varying the size of the balloons. Assuming that p is constant for each throw, find the range of values of p such that the probability of having at least one winner within the first ten games is at least 0.9. [3]

(ii) Assuming that $p = 0.7$ and 100 games are played daily at the game stall over a period of 60 days, find the probability that, on average, at most 24 prizes are given out per day. [4]

10. During a meteor shower, Jess observes meteors at an average rate of 1.3 per minute.

(i) State the conditions required for a Poisson distribution to be a suitable model for the number of meteors Jess observes during a randomly selected minute. [2]

You may assume that the conditions in part **(i)** are satisfied.

(ii) Find the probability that Jess observes more than 5 meteors in a period of 5 minutes. [2]

(iii) Use a suitable approximation to find the probability that Jess observes a total of at most 100 meteors over a period of one hour. [3]

(iv) Jess wishes to be at least 96% certain that she will observe more than 2 meteors. Find the minimum amount of time, to the nearest minute, that she needs to spend watching the meteors. [3]

11. The depth, x mm, at which seeds are sown, affects the percentage of seeds that germinate, y%. The table below shows corresponding values of x and y for an experiment in which seeds were sown at different depths.

Depth (x mm)	34	43	46	51	54	57	65	72
Percentage of seeds germinated (y)	22	α	50	57	49	58	59	61

(i) Given that the equation of the regression line of y on x is $y = 0.9978x - 4.0104$, find the value of α, giving your answer correct to the nearest integer. [3]

It is thought that the percentage of seeds germinated can also be modelled by the formula $y = a + b\ln x$.

(ii) Find the value of the product moment correlation coefficient of the regression line of

(a) y on x,

(b) y on $\ln x$. [2]

(iii) Using a scatter diagram and your answer in part **(ii)**, state with a reason, which of the models, $y = a + bx$ or $y = a + b\ln x$, is more appropriate. [2]

(iv) Use a suitable regression line to estimate the depth at which the seeds are sown when the percentage of seeds germinated is 55%. Comment on the reliability of your answer. [3]

12. A vending machine dispenses coke and lemon tea. The volumes, in ml, of coke and lemon tea dispensed into cups follow independent normal distributions with means and standard deviations as shown in the table. The drinks are dispensed into 500 ml cups.

	Mean	Standard deviation
Coke	μ	20
Lemon tea	450	30

(i) Find the largest value of μ such that less than 0.1% of the cups of coke will overflow, giving your answer correct to 2 decimal places. [3]

(ii) It is given that $\mu = 475$ ml and 120 cups of coke are sold on a particular day. Using a suitable approximation, find the probability that at least 10 cups of coke will overflow. [4]

(iii) Find the probability that the volume of 4 cups of coke is more than twice the volume of 2 cups of lemon tea. [3]

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