2023 JC2 H2 Math Prelim P2 Marking Scheme

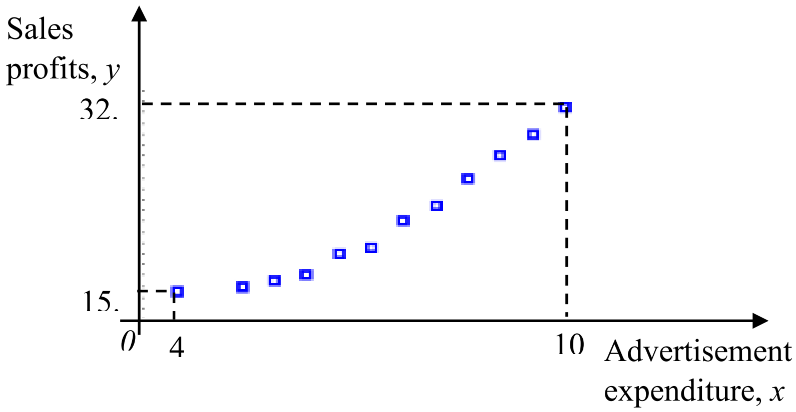
Qn	Solution	
1	$\int \frac{\sin 2x}{\cos^2 x+\cos x+2}\,\mathrm{d}x$ Let $u=\cos x$ $\frac{\mathrm{d}u}{\mathrm{d}x}=-\sin x$ $\int \frac{\sin 2x}{\cos^2 x+\cos x+2}\,\mathrm{d}x$ $=\int \frac{2\sin x\cos x}{\cos^2 x+\cos x+2}\,\mathrm{d}x$ $=-\int \frac{2\cos x}{\cos^2 x+\cos x+2}(-\sin x)\,\mathrm{d}x$ $=-\int \frac{2\cos x}{\cos^2 x+\cos x+2}\left(\frac{\mathrm{d}u}{\mathrm{d}x}\right)\mathrm{d}x$ $=-\int \frac{2u}{u^2+u+2}\,\mathrm{d}u$ $=-\int \frac{(2u+1)-1}{u^2+u+2}\,\mathrm{d}u$ $=-\int \frac{2u+1}{u^2+u+2}-\frac{1}{u^2+u+2}\,\mathrm{d}u$ $=-\int \frac{2u+1}{u^2+u+2}\,\mathrm{d}u+\int \frac{1}{(u+\frac{1}{2})^2+\frac{7}{4}}\,\mathrm{d}u$ $=-\ln\left\|u^2+u+2\right\|+\frac{2}{\sqrt{7}}\tan^{-1}\left(\frac{2u+1}{\sqrt{7}}\right)+c$ $=-\ln\left\|\cos^2 x+\cos x+2\right\|+\frac{2}{\sqrt{7}}\tan^{-1}\left(\frac{2\cos x+1}{\sqrt{7}}\right)+c$	
2(i)	$y=\cot(x+a)$ $\frac{\mathrm{d}y}{\mathrm{d}x}=-\operatorname{cosec}^2(x+a)=-\left(1+\cot^2(x+a)\right)=-\left(1+y^2\right)$ $\frac{\mathrm{d}y}{\mathrm{d}x}=-1-y^2$ $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-2y\frac{\mathrm{d}y}{\mathrm{d}x}=2y\left(1+y^2\right)=2y+2y^3$ $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=2\frac{\mathrm{d}y}{\mathrm{d}x}+6y^2\frac{\mathrm{d}y}{\mathrm{d}x}$ $=-2\left(1+y^2\right)-6y^2\left(1+y^2\right)$ $=-\left(6y^4+8y^2+2\right)$	
2(ii)	Since $\tan 0=0$, $\cot 0=\frac{1}{\tan 0}$ is undefined. Hence the Maclaurin series of $\cot(x+0)$ cannot be found.	

2(iii)	$y=\cot\left(x+\frac{\pi}{2}\right)$ When $x=0$, $y=\frac{\cos\frac{\pi}{2}}{\sin\frac{\pi}{2}}=\frac{0}{1}=0$, $\frac{\mathrm{d}y}{\mathrm{d}x}=-1-0^2=-1$, $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=2(0)+2(0)^3=0$, $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=-\left(6(0)^4+8(0)^2+2\right)=-2$ $y\approx -x-\frac{2}{3!}x^3=-x-\frac{x^3}{3}$	
2(iv)	$\frac{y}{2+x}=y(2+x)^{-1}$ $=\frac{1}{2}y\left(1+\frac{x}{2}\right)^{-1}$ $\approx\frac{1}{2}\left(-x-\frac{1}{3}x^3\right)\left(1+(-1)\frac{x}{2}+\frac{(-1)(-2)}{2!}\left(\frac{x}{2}\right)^2\right)$ $=\frac{1}{2}\left(-x-\frac{1}{3}x^3\right)\left(1-\frac{1}{2}x+\frac{1}{4}x^2\right)$ $\approx\frac{1}{2}\left(-x-\frac{1}{3}x^3+\frac{1}{2}x^2-\frac{1}{4}x^3\right)$ $=-\frac{1}{2}x+\frac{1}{4}x^2-\frac{7}{24}x^3$	
3(i)	$\frac{\mathrm{d}v}{\mathrm{d}t}+\frac{6\pi\eta R}{m}v=g$ Substituting all the values, $\eta=1.3806, R=0.05, m=0.2, g=9.780$ gives $\frac{\mathrm{d}v}{\mathrm{d}t}+6.5059v=9.780$ $\Rightarrow\frac{\mathrm{d}v}{\mathrm{d}t}=9.780-6.5059v$ $\Rightarrow\int\frac{1}{9.780-6.5059v}\,\mathrm{d}v=t+c$ $\Rightarrow\frac{\ln\lvert 9.780-6.5059v\rvert}{-6.5059}=t+c$ $\Rightarrow 9.780-6.5059v=A\mathrm{e}^{-6.5059t}$, $A=\pm\mathrm{e}^{-6.5059c}$ At $t=0, v=0$, $9.780=A$ $\therefore 9.780-6.5059v=9.780\mathrm{e}^{-6.5059t}$ $\Rightarrow v=1.5032\left(1-\mathrm{e}^{-6.5059t}\right)$ $v=1.50\left(1-\mathrm{e}^{-6.51t}\right)$	
3(ii)	As $t\to\infty, \mathrm{e}^{-6.5059t}\to 0$, $v\to 1.5032$ The ball approaches a velocity of 1.50 ms^{-1} after a long time.	

3(iii)	$v=\dfrac{\mathrm{d}x}{\mathrm{d}t}$ $x=\int_0^{30} 1.5032\left(1-\mathrm{e}^{-6.5059t}\right)\mathrm{d}t$ (using GC) $=44.865\approx 44.9$ The distance covered by the ball after 30 seconds is 44.9 m. <u>Alternative Method</u> (not recommended as this is more tedious) $\dfrac{\mathrm{d}x}{\mathrm{d}t}=1.50\left(1-\mathrm{e}^{-6.51t}\right)$ $x=\int 1.50\left(1-\mathrm{e}^{-6.51t}\right)\mathrm{d}t$ $=1.5032\left[t+\dfrac{\mathrm{e}^{-6.5059t}}{6.5059}\right]+c$ When $t=0, x=0 \Rightarrow c=-\dfrac{1.5039}{6.5059}$ When $t = 30$, $x=1.5032\left[30+\dfrac{\mathrm{e}^{-6.5059(30)}}{6.5059}\right]-\dfrac{1.5032}{6.5059}$ $=44.9$ (3s.f.)	
4(a)(i)	$\dfrac{4}{4r^2+12r+5}=\dfrac{1}{2r+1}-\dfrac{1}{2r+5}$	
4(a)(ii)	$\sum_{r=2}^{n}\left(\dfrac{4}{4r^2+12r+5}\right)=\sum_{r=2}^{n}\left(\dfrac{1}{2r+1}-\dfrac{1}{2r+5}\right)$ $=\dfrac{1}{5}-\dfrac{1}{9}$ $+\dfrac{1}{7}-\dfrac{1}{11}$ $+\dfrac{1}{9}-\dfrac{1}{13}+\ldots$ $+\dfrac{1}{2n-3}-\dfrac{1}{2n+1}$ $+\dfrac{1}{2n-1}-\dfrac{1}{2n+3}$ $+\dfrac{1}{2n+1}-\dfrac{1}{2n+5}$ $=\dfrac{1}{5}+\dfrac{1}{7}-\dfrac{1}{2n+3}-\dfrac{1}{2n+5}$ $=\dfrac{12}{35}-\dfrac{1}{2n+3}-\dfrac{1}{2n+5}$	
4(a)(iii)	$u_r=u_{r-1}+\dfrac{4}{4r^2+12r+5}$ $u_r-u_{r-1}=\dfrac{4}{4r^2+12r+5}$	

	$\sum_{r=2}^{n}\left(u_r-u_{r-1}\right)=\sum_{r=2}^{n}\left(\frac{4}{4r^2+12r+5}\right)$ $u_2-u_1+u_3-u_4+\ldots+u_{n-1}-u_{n-2}+u_n-u_{n-1}=\frac{12}{35}-\frac{1}{2n+3}-\frac{1}{2n+5}$ $u_n-u_1=\frac{12}{35}-\frac{1}{2n+3}-\frac{1}{2n+5}$ $u_n=\frac{117}{35}-\frac{1}{2n+3}-\frac{1}{2n+5}$	
4(b)	$\lim_{n\to\infty}\sqrt[n]{\left\|\left(a_n\right)^n\right\|}$ $=\lim_{n\to\infty}\sqrt[n]{\left\|\left(\frac{-3n^3+nx}{5n^3+7}\right)^n\right\|}=\lim_{n\to\infty}\left\|\left(\frac{-3n^3+nx}{5n^3+7}\right)\right\|$ $=\lim_{n\to\infty}\left\|\left(\frac{n^3\left(-3+\frac{x}{n^2}\right)}{n^3\left(5+\frac{7}{n^3}\right)}\right)\right\|=\lim_{n\to\infty}\left\|\left(\frac{\left(-3+\frac{x}{n^2}\right)}{\left(5+\frac{7}{n^3}\right)}\right)\right\|$ $=\left\|-\frac{3}{5}\right\|=\frac{3}{5}<1$ as $n\to\infty,\ \frac{x}{n^2}\to 0,\frac{7}{n^3}\to 0.$ $\therefore$ By the root test, $\sum_{r=0}^{\infty}\left(\frac{-3r^3+rx}{5r^3+7}\right)^r$ converges for all values of x.	
5(i)	$x=\cos\theta-\frac{1}{\cos\theta},\ y=\cos\theta+\frac{1}{\cos\theta}$ $\frac{\mathrm{d}x}{\mathrm{d}\theta}=-\sin\theta-\frac{\sin\theta}{\cos^2\theta},\ \frac{\mathrm{d}y}{\mathrm{d}\theta}=-\sin\theta+\frac{\sin\theta}{\cos^2\theta}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-\sin\theta+\frac{\sin\theta}{\cos^2\theta}}{-\sin\theta-\frac{\sin\theta}{\cos^2\theta}}=\frac{-1+\frac{1}{\cos^2\theta}}{-1-\frac{1}{\cos^2\theta}}=\frac{\cos^2\theta-1}{\cos^2\theta+1}$	
5(ii)	At the point, $x=\cos p-\frac{1}{\cos p},\ y=\cos p+\frac{1}{\cos p}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\cos^2 p-1}{\cos^2 p+1}\Rightarrow$ gradient of normal $=\frac{1+\cos^2 p}{1-\cos^2 p}$ Equation of normal: $y-\left(\cos p+\frac{1}{\cos p}\right)=\frac{1+\cos^2 p}{1-\cos^2 p}\left(x-\left(\cos p-\frac{1}{\cos p}\right)\right)$ $y-\left(\frac{\cos^2 p+1}{\cos p}\right)=\frac{1+\cos^2 p}{1-\cos^2 p}\left(x-\left(\frac{\cos^2 p-1}{\cos p}\right)\right)$ $y-\left(\frac{\cos^2 p+1}{\cos p}\right)=\frac{1+\cos^2 p}{1-\cos^2 p}x+\frac{1+\cos^2 p}{\cos p}$ $y=\frac{1+\cos^2 p}{1-\cos^2 p}x+2\left(\frac{1+\cos^2 p}{\cos p}\right)$	

5(iii)	$P: y=0 \Rightarrow 0=\frac{1+\cos^2 p}{1-\cos^2 p}x+2\left(\frac{1+\cos^2 p}{\cos p}\right)$ $x=-2\left(\frac{1+\cos^2 p}{\cos p}\right)\left(\frac{1-\cos^2 p}{1+\cos^2 p}\right)=2\left(\frac{\cos^2 p-1}{\cos p}\right)$ $Q: x=0 \Rightarrow y=2\left(\frac{1+\cos^2 p}{\cos p}\right)$ $\because \cos^2 p-1<0, \therefore OP=2\left(\frac{1-\cos^2 p}{\cos p}\right)$ Area OPQ = $\frac{1}{2}\left(2\left(\frac{1-\cos^2 p}{\cos p}\right)2\left(\frac{1+\cos^2 p}{\cos p}\right)\right)$ $=2\left(\frac{1-\cos^4 p}{\cos^2 p}\right)=2\left(\sec^2 p-\cos^2 p\right)$	
5(iv)	Let the area of OPQ be A. $A=2\left(\sec^2 p-\cos^2 p\right)$ $\frac{dA}{dt}=\frac{dA}{dp}\times\frac{dp}{dt}=2\left(2\sec^2 p\tan p+2\cos p\sin p\right)\frac{dp}{dt}$ When $p=\frac{\pi}{3}$, $\frac{dA}{dt}=4\left(\cos\frac{\pi}{3}\sin\frac{\pi}{3}+\sec^2\frac{\pi}{3}\tan\frac{\pi}{3}\right)(0.1)$ $=4\left(\frac{1}{2}\frac{\sqrt{3}}{2}+4\sqrt{3}\right)(0.1)$ $=1.7\sqrt{3}=2.94$ (3 s.f.)	
6(i)	Let Y be random variable "number of yellow chips in a box". $Y \sim B(36, 0.3)$ $P(Y\le 9)=0.32544$ $P(Y>4 \mid Y\le 9)=\frac{P(5\le Y\le 9)}{P(Y\le 9)}$ $=\frac{P(Y\le 9)-P(Y\le 4)}{P(Y\le 9)}$ $=0.978152\approx 0.978$ (3 sf.)	
6(ii)	Let W be random variable "number of boxes with at most 9 yellow chips". $W \sim B(59, 0.32544)$ Required probability = $P(W=13)\times 0.32544=0.00825$	
6(iii)	Let X be random variable "number of boxes containing at most 9 yellow chips in a carton". $X \sim B(75, 0.32544)$ $E(X)=75\times 0.32544=24.408$ $\mathrm{Var}(X)=75\times 0.32544\times(1-0.32544)=16.46466$	

	Since $n = 30$ is large, $\overline{X} \sim \mathrm{N}(24.408, \frac{16.46466}{30})$ approximately by Central Limit Theorem. $\mathrm{P}(\overline{X} > 25) = 0.21211 \approx 0.212$ (3 sf.)	
7(i)	The monthly sales profits is expected to increase by *m* thousands of dollars for every additional thousand dollars spent on the monthly advertisement expenditure.	
7(ii)	Substituting $\overline{x} = 7.208333$ and $\overline{y} = \frac{242+k}{12}$ into the least squares regression line of *y* on *x*: $\frac{242+k}{12} = 0.3604 + 3.0194 \times 7.208333$ $k = 22.12524066 \times 12 - 242$ $k = 23.503$ ≈ 23.5	
7(iii)		
7(iv)	The product moment correlation coefficient for Model A = 0.968 The product moment correlation coefficient for Model B = 0.982 Since the product moment correlation coefficient between *x* and ln *y* is closer to 1 as compared with the product moment correlation coefficient between *x* and *y*, it suggests that there is a stronger positive liner correlation between *x* and ln *y*. Furthermore, from the scatter diagram, <u>*y* is increasing at an increasing rate as *x* increases</u>, hence Model B will be the more appropriate model.	
7(v)	Using GC, the least squares regression line of ln *y* on *x* is $\ln y = 2.0867 + 0.13579x$ $\ln y = 2.09 + 0.136x$ When $x = 11$, $\ln y = 2.0867 + 0.13579 \times 11$ $y = 35.88754326$ Hence the estimate for the sales profit is $35,900 (or $35,888).	
7(vi)	Since $x = 11$ does not fall within the given data range $4 \le x \le 10$, the estimate obtained by extrapolation will not be reliable.	

8(a)(i)	Number of ways = ${}^{26}C_6 \times {}^{3}C_1 \times 6! = 497296800$ **Alternative:** Number of ways = $26\times25\times24\times23\times22\times21\times {}^{3}C_1 = 497296800$	
8(a)(ii)	Case 1: 3 letters with two different colours each Number of ways = ${}^{26}C_3 \times \left({}^{3}C_2\right)^3 \times 6! = 50544000$ Case 2: 1 letter with 3 different colours and 1 letter with 2 colours and 1 letter with 1 colour Number of ways = ${}^{26}C_3 \times {}^{3}C_2 \times {}^{3}C_1 \times 3! \times 6! = 101088000$ OR ${}^{26}C_1 \times {}^{25}C_1 \times {}^{3}C_2 \times {}^{24}C_1 \times {}^{3}C_1 \times 6! = 101088000$ OR ${}^{26}C_1 \times {}^{25}C_2 \times {}^{2}C_1 \times {}^{3}C_1 \times {}^{3}C_2 \times 6! = 101088000$ Total number of ways = $50544000 + 101088000 = 151632000$	
8(b)(i)	$\frac{2+c}{41+c}\times\frac{1+c}{40+c}=\frac{1}{66} \quad ------(1)$ $66(2+c)(1+c)=(41+c)(40+c)$ $65c^2+117c-1508=0$ $c=4 \text{ or } -\frac{29}{5} \text{ (Rejected)}$ **Alternative:** $\frac{c}{41+c}\times\frac{c-1}{40+c}+\frac{2c}{41+c}\times\frac{1}{40+c}\times2+\frac{2}{41+c}\times\frac{1}{40+c}=\frac{1}{66}$ $65c^2+117c-1508=0$ $c=4 \text{ or } -\frac{29}{5} \text{ (Rejected)}$	
8(b)(ii)	**Method 1:** Case 1: red flower and non-red bead $\frac{6}{45}\times\frac{28}{44}\times2=\frac{28}{165}$ Case 2: red non-flower and non-red flower $\frac{11}{45}\times\frac{5}{44}\times2=\frac{1}{18}$ Required probability = $\frac{28}{165}+\frac{1}{18}=\frac{223}{990}$ **Method 2:** Case 1: red bead and non-red flower $\frac{17}{45}\times\frac{5}{44}\times2=\frac{17}{198}$ Case 2: red flower and non-red non-flower	

	$\frac{6}{45}\times\frac{23}{44}\times2=\frac{23}{165}$ Required probability = $\frac{17}{198}+\frac{23}{165}=\frac{223}{990}$ **Method 3:** Case 1: red flower and non-red flower $\frac{6}{45}\times\frac{5}{44}\times2=\frac{1}{33}$ Case 2: red flower and non-red non-flower $\frac{6}{45}\times\frac{23}{44}\times2=\frac{23}{165}$ Case 3: red non-flower and non-red flower $\frac{11}{45}\times\frac{5}{44}\times2=\frac{1}{18}$ Required probability = $\frac{1}{33}+\frac{23}{165}+\frac{1}{18}=\frac{223}{990}$ **Method 4:** Case 1: red bead and non-red bead $\frac{17}{45}\times\frac{28}{44}\times2=\frac{238}{495}$ Case 2: one red non-flower and non-red non-flower $\frac{11}{45}\times\frac{23}{44}\times2=\frac{23}{90}$ Required probability = $\frac{238}{495}-\frac{23}{90}=\frac{223}{990}$	
9(a)	$\frac{1}{3r+1}+\left(\frac{3r}{3r+1}\right)^2\times\frac{1}{3r+1}+\left(\frac{3r}{3r+1}\right)^4\times\frac{1}{3r+1}+\dots$ $=\frac{\frac{1}{3r+1}}{1-\left(\frac{3r}{3r+1}\right)^2}=\frac{3r+1}{6r+1}$	
9(b)(i)	$\mathrm{P}(T=0)=\frac{r}{3r+1}\times\frac{r-1}{3r}+\frac{2r}{3r+1}\times\frac{2r-1}{3r}=\frac{5r-3}{3(3r+1)}$ $\mathrm{P}(T=2)=\frac{2r}{3r+1}\times\frac{1}{3r}\times2!=\frac{4}{3(3r+1)}$ $\mathrm{P}(T=3)=\frac{2r}{3r+1}\times\frac{r}{3r}\times2!=\frac{4r}{3(3r+1)}$ $\mathrm{P}(T=5)=\frac{r}{3r+1}\times\frac{1}{3r}\times2!=\frac{2}{3(3r+1)}$	

t	0	2	3	5
$\mathrm{P}(T=t)$	$\frac{5r-3}{3(3r+1)}$	$\frac{4}{3(3r+1)}$	$\frac{4r}{3(3r+1)}$	$\frac{2}{3(3r+1)}$

9(b)(ii)

$$E(T)=\sum tP(T=t)$$
$$=0\times\frac{10r-4}{3(3r+1)}+2\times\frac{4}{3(3r+1)}+3\times\frac{4r}{3(3r+1)}+5\times\frac{2}{3(3r+1)}$$
$$=\frac{8+12r+10}{3(3r+1)}=\frac{2(2r+3)}{3r+1}$$

9(c)

x	-0.25	0.10	0.15
$P(X=x)$	$\frac{r}{3r+1}$	$\frac{2r}{3r+1}$	$\frac{1}{3r+1}$

$$P(X_1>X_2)=P(X_1=0.15,X_2=-0.25)$$
$$+P(X_1=0.15,X_2=0.10)$$
$$+P(X_1=0.10,X_2=-0.25)$$

$$\frac{2r^2+3r}{(3r+1)^2}=\frac{27}{112}$$

$$19r^2-174r+27=0$$

$r=\frac{3}{19}\approx 0.15789$ (rejected) $\quad \therefore r=9$

$$E(X)=\sum xP(X=x)$$
$$=-0.25\times\frac{9}{28}+0.10\times\frac{18}{28}+0.15\times\frac{1}{28}$$
$$=-0.010714\approx-0.0107$$

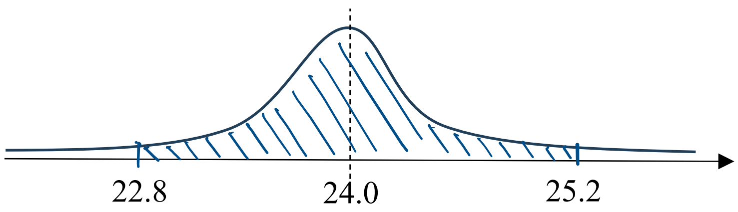
Since $E(x)<0$, the game is not fair for a player, as he/she will be expected to lose 1.07 cents every time he/she plays.

10(a)(i)

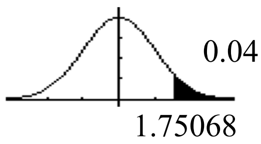
Let B be random variable "diameter of football".

$$B\sim N(\mu,0.4^2)$$
$$B-1.1F\sim N(\mu-1.1\times22,0.4^2+1.1^2\times0.3^2)$$
$$B-1.1F\sim N(\mu-24.2,0.2689)$$
$$P(B>110\%\times F)=0.35$$
$$P(B-1.1F>0)=0.35$$
$$P\left(Z>\frac{0-(\mu-24.2)}{\sqrt{0.2689}}\right)=0.35$$
$$\Rightarrow\frac{0-(\mu-24.2)}{\sqrt{0.2689}}=0.38532$$
$$\therefore \mu=24.2-0.38532\times\sqrt{0.2689}\approx24.0$$

10(a)(ii)



(iii)	$B-\overline{F} \sim \mathrm{N}(24-22, 0.4^2+\frac{0.3^2}{10})$ $B-\overline{F} \sim \mathrm{N}(2, 0.169)$ $\mathrm{P}\left(\left\|B-\overline{F}\right\|<1.5\right)=0.11194 \approx 0.112$	
10(b) (i)	Let A be event "player scores on his first attempt" and B be event "player scores on his second attempt". $\mathrm{P}(A)=\frac{5}{11} \qquad \mathrm{P}(B)=\frac{5}{8}$ $\mathrm{P}(A\cap B)=\mathrm{P}(A)+\mathrm{P}(B)-\mathrm{P}(A\cup B)$ $=\frac{5}{11}+\frac{5}{8}-\mathrm{P}(A\cup B)$ Since $\mathrm{P}(A\cup B)\le 1 \quad \therefore \mathrm{P}(A\cap B)\ge \frac{7}{88}$	
10(b) (ii)	$\mathrm{P}(A\cap B')=\frac{15}{88}$ $\mathrm{P}(A\cup B)=\mathrm{P}(A\cap B')+\mathrm{P}(B)=\frac{15}{88}+\frac{5}{8}=\frac{35}{44}$ $\mathrm{P}(A)+\mathrm{P}(B)-\mathrm{P}(A\cap B)=\frac{35}{44}$ $\mathrm{P}(A\cap B)=\frac{5}{11}+\frac{5}{8}-\frac{35}{44}=\frac{25}{88}$ Since $\mathrm{P}(A)\times\mathrm{P}(B)=\frac{5}{11}\times\frac{5}{8}=\frac{25}{88}=\mathrm{P}(A\cap B)$, the events are independent.	
11(i)	$\bar{x}=\frac{\sum x}{n}=\frac{10446}{30}=348.2$ (exact) $s^2=\frac{1}{n-1}\left(\sum x^2-\frac{\left(\sum x\right)^2}{n}\right)=\frac{1}{29}\left(3638000-\frac{(10446)^2}{30}\right)=24.23448$ Let μ be the population mean number of calories in Perfect Protein bars. Null hypothesis $H_o: \mu = 350$ Alternative hypothesis $H_1: \mu < 350$ Perform 1-tail test at 3 % significance level. Under H_0, $\overline{X} \sim \mathrm{N}\left(350, \frac{24.23448}{30}\right)$ by Central Limit Theorem since n = 30 is large enough. From GC, $z=\frac{\bar{x}-\mu_0}{\sqrt{\frac{s^2}{n}}}=\frac{348.2-350}{\sqrt{\frac{24.23448}{30}}}=-2.0027$ p-value $=P\left(Z<-2.0027\right)=0.0226$ p-value < 0.03 $\therefore$ Reject H_o There is sufficient evidence at 3% level of significance to conclude that the mean number of calories in Perfect Protein bars is less than 350 cal.	

11(ii)	The test would be inadmissible if the nutritionist had taken a random sample of 15 energy bars as the distribution of the population is unknown (and Central Limit Theorem cannot be applied). The nutritionist will need to assume that the number of calories in Perfect Protein bars follow a normal distribution.	
11(iii)	Null hypothesis $\mathrm{H_o}: \mu = 350$ Alternative hypothesis $\mathrm{H_1}: \mu \neq 350$ Perform 2-tail test at α % significance level. p-value $= 2 \times P(Z < -2.0027) = 0.0452$ To reject $\mathrm{H_o}$, p-value = 0.045209 < α% $\therefore$ smallest significance level is 4.52%.	
(iv)	Let μ be the population mean number of calories in Dyanmic Protein bars. Null hypothesis $H_o: \mu = 350$ Alternative hypothesis $H_1: \mu > 350$ At 4% level of significance, we do not reject H_o. For p-value > 0.04 or $z < 1.750686$, we do not reject H_o. $z = \dfrac{\overline{x} - \mu_0}{\sqrt{\dfrac{\sigma^2}{n}}} = \dfrac{352.2 - 350}{\sqrt{\dfrac{\sigma^2}{45}}} < 1.750686$  $1.750686\sqrt{\dfrac{\sigma^2}{45}} > 2.2$ $\sigma^2 > 45\left(\dfrac{2.2}{1.750686}\right)^2$ $\sigma^2 > 71.0626$ $\therefore \sigma^2 > 71.1$ (3 sf.)	