Name:		Index Number:		Class:	

DUNMAN HIGH SCHOOL
Quiz 1
Year 5

MATHEMATICS (Higher 2) 9758

22 August 2024

40 minutes

Additional Material: Printed Answer Booklet

READ THESE INSTRUCTIONS FIRST

Answer **all** the questions.
Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you must present the mathematical steps using mathematical notations and not calculator commands.
You must show all necessary working clearly.

The number of marks is given in brackets [ ] at the end of each question or part question.

1 The first three terms of an infinite series are 16, x, 9.

(i) Find the value(s) of x, if the series is

(a) geometric,

(b) arithmetic. [3]

(ii) If all the terms in part **(i)(a)** are positive, calculate its sum to infinity. [2]

(iii) Given that the sum of the first n terms in part **(i)(b)** is less than -64, find the least value of n. [3]

2 **(i)** Find $\sum_{r=1}^{n}\left(\frac{1}{2}\right)^r$ in terms of n. [2]

A sequence is such that $u_0 = 3$ and $u_r - u_{r-1} = \left(\frac{1}{2}\right)^r$ for $r \geq 1$.

(ii) Show that $\sum_{r=1}^{n}\left(\frac{1}{2}\right)^r = u_n - u_0$. Hence find u_n in terms of n. [2]

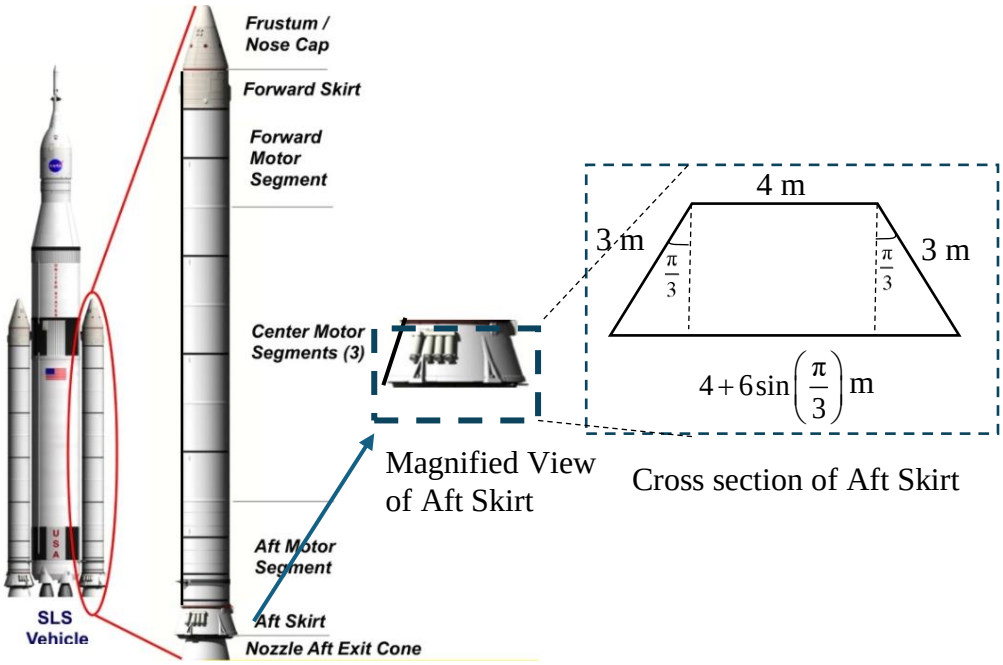
(iii) Show that $S = 4n - 2 + 2\left(\frac{1}{2}\right)^n$, where S denotes the sum of the first n terms of the sequence. [2]

(iv) Determine, with reason, whether

(a) u_n converges, [1]

(b) S converges. [1]

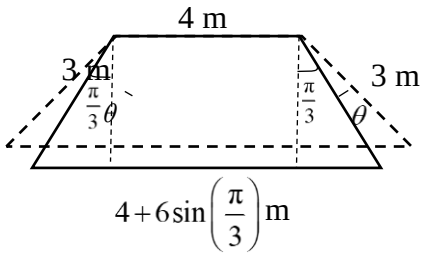
3 A famous entrepreneur, Elon Tusk, has designed a new rocket booster for his company SpaceY. A rocket booster consists of the following parts as shown in the following diagram.



Rocket Booster

The design of the Aft Skirt can be viewed using a vertical cross sectional view with the stated dimensions (see dotted box above).

Due to the amount of heat and pressure generated from the thrust of the rocket during lift off, the sides of the Aft Skirt will tilt **outward** by a **very small angle**, θ, while keeping the slant length at 3 m (see diagram below).



Before the lift off, the diameter of the circular base of the Aft Skirt is given by $\left(4+6\sin\left(\frac{\pi}{3}\right)\right)$ m. In order for the booster to function properly during the lift off, the diameter at the bottom of the Aft Skirt must be less than 9.197 m.

Given that θ is a sufficiently small angle, show that θ satisfies the inequality $a\theta^2+b\theta+c<0$ where a, b and c are constants to be determined. Hence find the range of values for θ, giving your answers correct to 6 decimal places. [4]