2022 JC1 H2 MA Promo

1	Solution [5] System of linear equations	

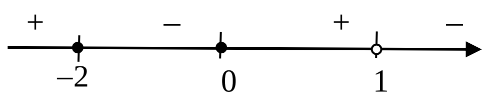
	Sub (1, 2):
	$a+b+c=2.....(1)$
	Sub (-1, 0):
	$-a+b+c=0$
	$a-b-c=0....(2)$
	$\frac{\mathrm{d}y}{\mathrm{d}x}=2bx-\frac{a}{x^2}$
	
	When $x=0.5,\ 2b(0.5)-\frac{a}{(0.25)}=-1$
	$b-4a=-1....(3)$
	Using GC, $a=1, b=3, c=-2$
	$\therefore\ y=\frac{1}{x}+3x^2-2$
		Generally well done.
		
		Some students make mistakes in arithmetic manipulation when sub values into the differential equation, and couldn't get eqn (3).

2	Solution [6] Transformation	
(i)	$y=\mathrm{f}(x)\to y=\mathrm{f}(2x)$ $(2a,0)\to(a,0)$ $\left(0,\frac{b}{2}\right)\to\left(0,\frac{b}{2}\right)$	A few mis-read the questions: - it is not about writing down whether it is possible or not - only required to write down the coordinates of the axial intercepts. Generally well done.
(ii)	$y=\mathrm{f}(x)\to y=\mathrm{f}(x+1)$ $(2a,0)\to(2a-1,0)$	Generally well done.
(iii)	$y=\mathrm{f}(x)\to y=\mathrm{f}(x+1)\to y=\mathrm{f}(2x+1)$ $(2a,0)\to(2a-1,0)\to\left(a-\frac{1}{2},0\right)$	Need to revise on the correct sequence of transformation required to effect the desired change.
(iv)	$y=\mathrm{f}(x)\to y=2\mathrm{f}(x)-1$ $\left(0,\frac{b}{2}\right)\to(0,b)\to(0,b-1)$	Generally well done.
(v)	$y=\mathrm{f}^{-1}(x)$ $(2a,0)\to(0,2a)$ $\left(0,\frac{b}{2}\right)\to\left(\frac{b}{2},0\right)$	Many are unaware if the point (x,y) lies on the curve of $y=\mathrm{f}(x)$, then the point (y,x) lies on the curve of $y=\mathrm{f}^{-1}(x)$. Some students mistook $y=\mathrm{f}^{-1}(x)$ as $y=\frac{1}{\mathrm{f}\ (x)}$ Note that $\frac{1}{\mathrm{f}\ (x)}=\left[\mathrm{f}\ (x)\right]^{-1}$

3	Solution [7] AP GP	
(i)	Let b and d be the first term and common difference of the AP respectively. $ar^{n-1} = b + 11d \cdots\cdots(1)$ $ar^{n} = b + 7d \cdots\cdots(2)$ $ar^{n+1} = b + 4d \cdots\cdots(3)$ Equation $(1)-(2)$ gives $ar^{n-1} - ar^{n} = 4d$ $\dfrac{ar^{n-1} - ar^{n}}{4} = d$ Equation $(2)-(3)$ gives $ar^{n} - ar^{n+1} = 3d$ $\dfrac{ar^{n} - ar^{n+1}}{3} = d$ Hence $\dfrac{ar^{n} - ar^{n+1}}{3} = \dfrac{ar^{n-1} - ar^{n}}{4}$ $4\left(ar^{n} - ar^{n+1}\right) = 3\left(ar^{n-1} - ar^{n}\right)$ $4ar^{n+1} - 7ar^{n} + 3ar^{n-1} = 0$ $ar^{n-1}\left(4r^2 - 7r + 3\right) = 0$ $(4r-3)(r-1) = 0$ since a and r are non-zero. $r = \dfrac{3}{4}$ or $r = 1$ Reject $r = 1$ since the GP a converging GP where $-1 < r < 1$. Therefore $r = \dfrac{3}{4}$ Alternatively $ar^{n-1} = b + 11d \cdots\cdots(1)$ $ar^{n} = b + 7d \cdots\cdots(2)$ $ar^{n+1} = b + 4d \cdots\cdots(3)$ $(2) \div (1)$: $\dfrac{ar^{n}}{ar^{n-1}} = \dfrac{b+7d}{b+11d} \Rightarrow r = \dfrac{b+7d}{b+11d}$ ---(4)	Many candidates are unable to set up (1), (2) and (3) and work with it meaningfully. Need to explain the reason for $r \neq 1$

	$(3)\div(2)$: $\dfrac{ar^{n+1}}{ar^n}=\dfrac{b+4d}{b+7d}\Rightarrow r=\dfrac{b+4d}{b+7d}$ ---(5) $\dfrac{b+7d}{b+11d}=\dfrac{b+4d}{b+7d}$ $(b+7d)^2=(b+4d)(b+11d)$ $b^2+14bd+49d^2=b^2+15bd+44d^2$ $bd-5d^2=0$ $d(b-5d)=0$ $d=0$ or $b=5d$ Reject $d=0$ since $d=0\Rightarrow r=1$ but GP a converging GP where $-1<r<1$. Therefore $b=5d$ Sub $b=5d$ into (4): $r=\dfrac{b+7d}{b+11d}=\dfrac{5d+7d}{5d+11d}=\dfrac{3}{4}$	Need to explain the reason for $d\neq 0$. It is not just about getting $r=\dfrac{3}{4}$ For students who obtain $r=\frac{4}{3}$, they need to realize that $-1<r<1$.
(ii)	$\left\lvert\dfrac{a(1-r^n)}{1-r}-\dfrac{a}{1-r}\right\rvert<0.004\left(\dfrac{a}{1-r}\right)$ $-0.004\left(\dfrac{a}{1-r}\right)<\dfrac{a(1-r^n)}{1-r}-\dfrac{a}{1-r}<0.004\left(\dfrac{a}{1-r}\right)$ Note that $a>0$ and $-1<r<1$, then $\dfrac{a}{1-r}>0$ Divide throughout by $\dfrac{a}{1-r}$, $-0.004<(1-r^n)-1<0.004$ $-0.004<r^n<0.004$ $-0.004<\left(\dfrac{3}{4}\right)^n<0.004$ Note that $\left(\dfrac{3}{4}\right)^n>0>-0.004$ for all n. $\left(\dfrac{3}{4}\right)^n<0.004$ …….(*)	

	$n > \dfrac{\ln(0.004)}{\ln\left(\frac{3}{4}\right)}$ Note that $\ln\left(\frac{3}{4}\right) < 0$. $n > 19.19$ Hence least *n* is 20. Alternatively, $S_\infty - S_n < 0.004 S_\infty$ $\dfrac{a}{1-r} - \dfrac{a(1-r^n)}{1-r} < 0.004\left(\dfrac{a}{1-r}\right)$ (Since $S_\infty > S_n$ as a and r are positive.) $\left(\frac{3}{4}\right)^n < 0.004$ …….(*) $n > \dfrac{\ln(0.004)}{\ln\left(\frac{3}{4}\right)}$ Note that $\ln\left(\frac{3}{4}\right) < 0$. $n > 19.19$ Hence least *n* is 20. **Note:** For those who starts with $S_n - S_\infty < 0.004 S_\infty$ $\dfrac{a(1-r^n)}{1-r} - \dfrac{a}{1-r} < 0.004\left(\dfrac{a}{1-r}\right)$ This is **INCORRECT** because $S_\infty > S_n$ as a and r are positive.) Students who work with equality, $\dfrac{a}{1-r} - \dfrac{a(1-r^n)}{1-r} = 0.004\left(\dfrac{a}{1-r}\right)$ have to use a table of values to justify the minimum value of *n* is 20.	Many candidates are unaware $\ln\left(\frac{3}{4}\right) < 0$ and make erroneous manipulation. Many set up the incorrect relationship $\dfrac{a(1-r^n)}{1-r} - \dfrac{a}{1-r} < 0.004\left(\dfrac{a}{1-r}\right)$ It will end up with $\left(\frac{3}{4}\right)^n > -0.004$ which is a valid inequality for all *n*. Candidates should realize that the starting relationship is incorrect, when they arrive at $-0.004 < \left(\frac{3}{4}\right)^n$ instead, many went on to make incorrect manipulation from this point onwards.

4	Solution [7] Inequality	
(i)	$\dfrac{2x^2+6x+1}{1-x}-(x+1)\geq 0$ $\dfrac{2x^2+6x+1-(x+1)(1-x)}{1-x}\geq 0$ $\dfrac{2x^2+6x+1-x+x^2-1+x}{1-x}\geq 0$ $\dfrac{3x^2+6x}{1-x}\geq 0....(*)$ $\dfrac{x(x+2)}{1-x}\geq 0$  $\therefore x\leq -2$ or $0\leq x<1$	**Comments:** 1. Other than some careless algebraic mistakes made by some students, this part of the question is generally well done. 2. Several students solved the inequality in (*) of the workings by considering cases. Even though this is commendable and extremely useful for tougher order thinking questions, they should consider going for efficient method (number line) especially for this standard question. **Common Mistakes:** 1. Should not cross-multiply the denominator $(1-x)$; can only be done if it is definitely positive which we are uncertain till x is solved. 2. Some students do not know how to combine into a single fraction with the correct denominator, giving the following: $\dfrac{2x^2+6x+1}{1-x}-(x+1)\geq 0$ $\Rightarrow \dfrac{2x^2+6x+1}{(1-x)(x+1)}-\dfrac{(1-x)(x+1)}{(1-x)(x+1)}\geq 0$
(ii)	$\lvert x\rvert+1\leq \dfrac{2x^2+6\lvert x\rvert+1}{1-\lvert x\rvert}$ Replace x with $\lvert x\rvert$	**Comments:** 1. Majority of students were able to see that this type of question requires the use of substitution. Unfortunately, many were

	$\therefore \lvert x \rvert \le -2$ (No Solution) or $0 \le \lvert x \rvert < 1$ Taking intersection $-1 < x < 1$	not able to solve it properly subsequently. **Common Mistakes:** 1. Students made mistakes in solving $\lvert x \rvert \le -2$, writing out solution such as: (a) $x \ge 0$ (b) $-2 \le x \le 2$ Students needs to learn that for special inequality expression such as: (a) for $\lvert x \rvert \le -2$; then there's no solution since $\lvert x \rvert$ is never lesser than a negative value for all real values of *x*. Other examples are such as: (b) for $\lvert x \rvert \ge -2$; then $x \in \mathbb{R}$. (c) for $e^x \ge -2$; then $x \in \mathbb{R}$, etc. 2. Similarly, when solving $0 \le \lvert x \rvert < 1$, students could consider writing it as: $0 \le \lvert x \rvert$ and $\lvert x \rvert < 1$ $x \in \mathbb{R}$ and $-1 < x < 1$ Finding intersection, $-1 < x < 1$

5	Solution [9] Summation	
(i)	$\dfrac{4r+2}{r(r+1)(r+2)}=\dfrac{1}{r}+\dfrac{2}{r+1}-\dfrac{3}{r+2}$	**Comments:** Generally, well done. Though several students used a rather long approach to solve for this 1-mark question. Do consider using substituting special values of r at the stage where: $4r+2=$ $A(r+1)(r+2)+Br(r+2)+Cr(r+1)$ so that the constants A, B and C can be solved faster, as opposed to comparing coefficients (which is generally more time-consuming).
(ii)	$\sum_{r=2}^{n}\dfrac{4r+2}{r(r+1)(r+2)}$ $=\sum_{r=2}^{n}\left(\dfrac{1}{r}+\dfrac{2}{r+1}-\dfrac{3}{r+2}\right)$ $=\Big[\dfrac{1}{2}+\dfrac{2}{3}-\dfrac{3}{4}$ $+\dfrac{1}{3}+\dfrac{2}{4}-\dfrac{3}{5}$ $+\dfrac{1}{4}+\dfrac{2}{5}-\dfrac{3}{6}$ $+\quad\vdots$ $+\dfrac{1}{n-2}+\dfrac{2}{n-1}-\dfrac{3}{n}$ $+\dfrac{1}{n-1}+\dfrac{2}{n}-\dfrac{3}{n+1}$ $+\dfrac{1}{n}+\dfrac{2}{n+1}-\dfrac{3}{n+2}\Big]$ $=\dfrac{1}{2}+\dfrac{2}{3}+\dfrac{1}{3}-\dfrac{3}{n+1}+\dfrac{2}{n+1}-\dfrac{3}{n+2}$ $=\dfrac{3}{2}-\dfrac{1}{n+1}-\dfrac{3}{n+2}$	**Comments:** Most students knew that this is a question on Method of Differences. Other than some arithmetic error brought over from (i), many students presented it well enough. However, several students rewrite $\sum_{r=2}^{n}\dfrac{4r+2}{r(r+1)(r+2)}$ $=\sum_{r=1}^{n}\dfrac{4r+2}{r(r+1)(r+2)}-\dfrac{4(1)+2}{(1)(2)(3)}$ before starting the M.O.D. process, which is really unnecessary. Students who did such workings probably thought that M.O.D. must start with r=1, which is NOT TRUE.
(iii)	As $n\to\infty,\dfrac{1}{n+1}\to 0,\dfrac{3}{n+2}\to 0$	**Comments:** 1. It is important that

	$\therefore \frac{3}{2}-\frac{1}{n+1}-\frac{3}{n+2} \to \frac{3}{2}$ $\therefore$ the series converges and the value of the sum to infinity is $\frac{3}{2}$.	students realized the question instructed to "Give a reason…", so it is necessary to explain a bit more as oppose to simply saying that $\sum_{r=2}^{\infty} \frac{4r+2}{r(r+1)(r+2)} \to \frac{3}{2}$. **Common Mistakes:** 1. Some students set $r \to \infty$, and then $\frac{4r+2}{r(r+1)(r+2)} \to 0$ Students who did such steps need to know that $\frac{4r+2}{r(r+1)(r+2)}$ is the expression for the general term (sequence) and by doing so, only proves that the sequence has a limit. On the other hand, the **series** has a **result** given as $\frac{3}{2}-\frac{1}{n+1}-\frac{3}{n+2}$, and we are trying to prove that it is actually the series that converges. 2. The phrase "sum to infinity" is NOT exclusive for the Geometric Progression only. Some students applied the formula $S_\infty = \frac{a}{1-r}$ or commented that $\lvert r \rvert < 1$. Students need to know that these results are only to be used when there is actually a common ratio between consecutive terms of the

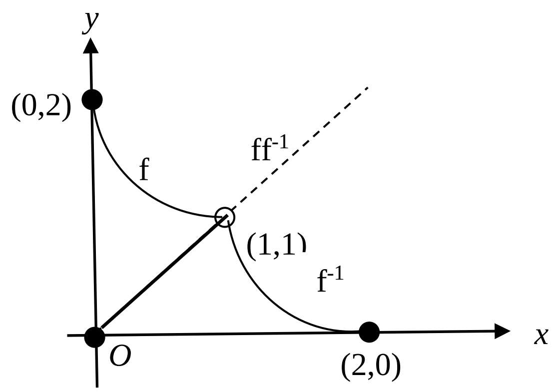
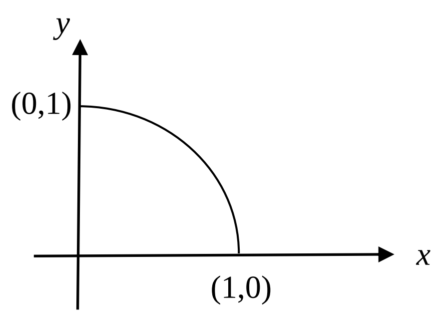
		sequence/series. In fact, by listing out: $\sum_{r=2}^{\infty}\frac{4r+2}{r(r+1)(r+2)}$ $=\frac{6}{2(3)(4)}+\frac{10}{3(4)(5)}+\frac{14}{4(5)(6)}+...$ students could actually see that there's no common ratio and hence it's not a GP.
(iv)	$\sum_{r=2}^{n}\frac{4r+6}{(r+1)(r+2)(r+3)}$ Replace *r* with $k-1$ $\sum_{k-1=2}^{k-1=n}\frac{2(k-1)+6}{((k-1)+1)((k-1)+2)((k-1)+3)}$ $=\sum_{r=3}^{n+1}\frac{4k+2}{k(k+1)(k+2)}$ $=\sum_{r=2}^{n+1}\frac{4k+2}{k(k+1)(k+2)}-2nd\ term\ ...\ (*)$ $=\frac{3}{2}-\frac{1}{(n+1)+1}-\frac{3}{(n+1)+2}-\frac{4(2)+2}{(2)(2+1)(2+2)}$ $=\frac{3}{2}-\frac{1}{n+2}-\frac{3}{n+3}-\frac{10}{24}$ $=\frac{13}{12}-\frac{1}{n+2}-\frac{3}{n+3}$	**Comments:** Most students knew that this question involves replacing the variable *r*. More importantly, quite a number of students did not read the question carefully to see that the instruction is by "Using your result in part (ii),…", and went on performing another Method of Differences and thus will be marked down. Such instruction restricts students to use only the result, and not the method from previous part of the question. **Common Mistakes:** 1. For students who identified using replacement of variable, they weren't able to correctly work out the procedure, writing out mistakes such as: $\sum_{k=1}^{n-1}\frac{4k+2}{k(k+1)(k+2)}$, etc.

6	Solution [7] Maclaurin Series	
(i)	Given that $y=\mathrm{e}^{-x}\sin x$, then $\frac{\mathrm{d}y}{\mathrm{d}x}=\mathrm{e}^{-x}\cos x+(\sin x)(-\mathrm{e}^{-x})=\mathrm{e}^{-x}\cos x-y$ $\frac{\mathrm{d}y}{\mathrm{d}x}+y=\mathrm{e}^{-x}\cos x -----(1)$ Then differentiating (1) wrt x again: $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\frac{\mathrm{d}y}{\mathrm{d}x}=\mathrm{e}^{-x}(-\sin x)+(\cos x)(-\mathrm{e}^{-x})=-y-\left(y+\frac{\mathrm{d}y}{\mathrm{d}x}\right)$ Hence, we have $2y+2\frac{\mathrm{d}y}{\mathrm{d}x}+\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=0-----(2)$ (shown)	**Comments:** Many students were able to do this part properly. In fact, many students found the first and second derivative and used the method of LHS =… = RHS to show the necessary result. However, note that this particular method might not work well at times and Implicit Differentiation should then be considered.
(ii)	Differentiating (2) wrt x again: We have $2\frac{\mathrm{d}y}{\mathrm{d}x}+2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=0-----(3)$ Then we subs $x=0$ in $y=\mathrm{e}^{-x}\sin x$, to get $y=\mathrm{e}^0\sin 0=0$ Next sub $x=0,\ y=0$ into (1) to get $\frac{\mathrm{d}y}{\mathrm{d}x}=-0+\mathrm{e}^0\cos 0=1$ Then for (2) $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-2y-2\frac{\mathrm{d}y}{\mathrm{d}x}=-2(0)-2(1)=-2$ And for (3) $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=-2\frac{\mathrm{d}y}{\mathrm{d}x}-2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-2(1)-2(-2)=2$ Thus, we have $\mathrm{f}(0)=0,\ \mathrm{f}'(0)=1,\ \mathrm{f}''(0)=-2,\ \mathrm{f}'''(0)=2$ for Maclaurin series for y. So, we have $y=\mathrm{e}^{-x}\sin x=$ $=f(0)+\frac{f'(0)}{1!}x+\frac{f''(0)}{2!}x^2+\frac{f'''(0)}{3!}x^3+.....$ $y=0+1x+\frac{(-2)}{2!}x^2+\frac{2}{3!}x^3=x-x^2+\frac{1}{3}x^3$ (up to x^3 term)	**Comments:** Interestingly, many students used the standard series of e^{-x} and $\sin x$ to solve the question, which is fine. Do take note that some instructions or other questions force students to use the general series $f(0)+\frac{f'(0)}{1!}x+\frac{f''(0)}{2!}x^2+\frac{f'''(0)}{3!}x^3+.....$ instead, so do watch out. A handful of students have no idea how to start this part of the question, suggesting the need to review this part of the topic again.
(iii)	For the series in part (ii), if we replace ‘x’ by ‘$-x$’:	**Comments:** Again, the instruction given is by “Using the result in part (ii)…” to solve for the power series of

	$y = \mathrm{e}^{-(-x)}\sin(-x) = \left((-x)-(-x)^2+\frac{1}{3}(-x)^3\right)$ $\Rightarrow -\mathrm{e}^x \sin x = \left(-x - x^2 - \frac{1}{3}x^3\right)$ since $\sin(-x) = -\sin x$ $\Rightarrow \mathrm{e}^x \sin x = x + x^2 + \frac{1}{3}x^3$ (up to x^3 term) <u>Student can check the correctness using available formulae in MF26.</u> Using standard series in MF26 We have $\mathrm{e}^x \sin x =$ $= \left(1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + ..\right)\left(x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 + ...\right)$ $= x + x^2 + \frac{1}{2}x^3 - \frac{1}{6}x^3 + ...$ $= x + x^2 + \frac{1}{3}x^3$ (up to x^3 term)	$y = e^x \sin x$. Hence, any workings that do not involve the use of (ii)'s result – NOT method - will be marked down.

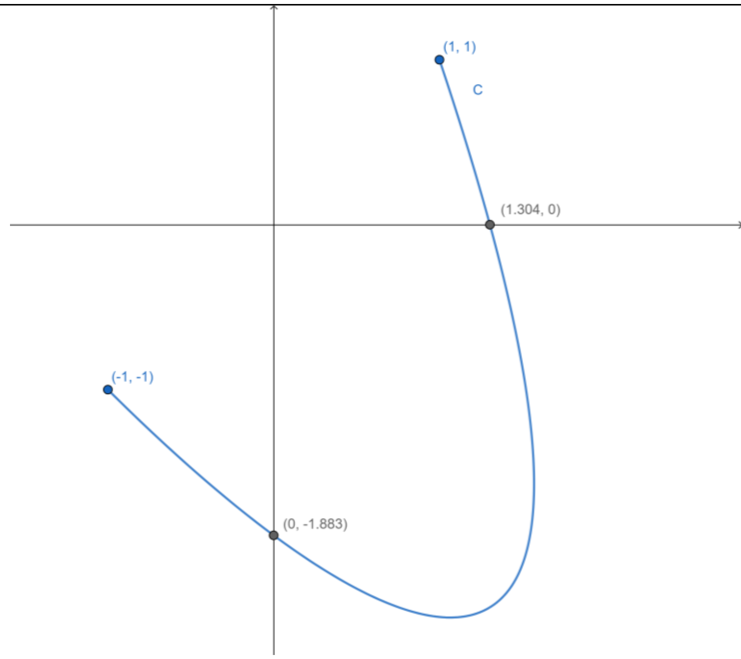
7	Solution [8] Graphing	
(i)	Given lines $x=-1$ and $y=x+2$ are asymptotes $y=x+2+\frac{k}{x+1}$ $=\frac{(x+2)(x+1)+k}{x+1}$ $=\frac{x^2+3x+(2+k)}{x+1}$ Comparing, $a=1, b=3$	Generally well done for students who are able to construct an expression for the function with the help of observing the asymptotes. Many students who did my long division method were not able to solve this question due to confusion of unknowns.
(ii)	$\frac{\mathrm{d}y}{\mathrm{d}x}=1+\frac{(x+1)(0)-k}{(x+1)^2}$ *Let* $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ $\Rightarrow 1-\frac{k}{(x+1)^2}=0$ when $x=\sqrt{2}-1$ $\therefore \frac{k}{2}=1$ $k=2$ Since $c=k+2$ $c=4$	Generally badly done. Students were not able to differentiate the given expression correctly due to the unknowns or have applied quotient rule incorrectly.
(iii)	From graph, *A* is the point (-1, 1) Sub (-1, 1) into equation: $RHS = m(-1)+(1+m)=1=LHS$ $m\leq 1$ Note: m is the gradient of the line $y=mx+(1+m)$. When $m=1$, the line $y=mx+(1+m)$ is $y=x+2$ which is the oblique asymptote of the curve, the line $y=mx+(1+m)$ which passes through the point (-1, 1)needs to be - either more gentle (with a positive gradient) or - takes on a non-positive gradient if it is not to intersects the graph again. Therefore $m\leq 1$	First part is generally well done; students were able to identify point *A* and prove that it lies on the given line. Range of values of *m* part was badly done even though similar tutorial question was given. Many failed to realize that values of *m* have to be smaller or equal to 1 rather than the other way round.

8	Solution [7] Integration	
(i)	$\int \cos 6x \cos 4x \, dx$ $= \frac{1}{2}\int (\cos 10x + \cos 2x) \, dx$ $= \frac{1}{2}\left[\frac{1}{10}\sin 10x + \frac{1}{2}\sin 2x\right] + c$	Generally averagely done. Many students were not able to use MF26 to help them to break down the given expression using factor formulae.
(ii)	$\int x\cos 4x \cos 6x \, dx$ Let $u = x, \frac{dv}{dx} = \cos 4x \cos 6x$ $\frac{du}{dx} = 1 \qquad v = \int \cos 4x \cos 6x \, dx$ $= \frac{1}{2}\left[\frac{1}{10}\sin 10x + \frac{1}{2}\sin 2x\right]$ $\therefore \int x\cos 4x \cos 6x \, dx$ $= \frac{x}{2}\left[\frac{1}{10}\sin 10x + \frac{1}{2}\sin 2x\right] - \int \frac{1}{2}\left[\frac{1}{10}\sin 10x + \frac{1}{2}\sin 2x\right] dx$...(*) $= \frac{x}{2}\left[\frac{1}{10}\sin 10x + \frac{1}{2}\sin 2x\right] - \left[-\frac{1}{200}\cos 10x - \frac{1}{8}\cos 2x\right] + c$ $= \frac{x}{2}\left[\frac{1}{10}\sin 10x + \frac{1}{2}\sin 2x\right] + \frac{1}{200}\cos 10x + \frac{1}{8}\cos 2x + c$	Badly done; students were not able to identify that this is a question to be solved using by parts. Many were not able to do the first part and hence many left this part empty. Some students who attempted this question were not able to simplify what is cos (-2x) for instance.

9	Solution [9] Functions	
(i)	$y=(x-1)^2+1 \Rightarrow x=1\pm\sqrt{y-1}$ Since $0\le x<1$, $x=1-\sqrt{y-1}$. Thus, $f^{-1}: x\mapsto 1-\sqrt{x-1}$, $D_{f^{-1}}=R_f=(1,2]$	Badly done. Many students were not able to identify the correct expression for the inverse function and taking into consideration of the range of values of *x*. Domain for inverse function is well done by many students.
(ii)	Note that $D_{f^{-1}f}=D_f=[0,1)$.  	Badly done; many students were not able to indicate the endpoints of the question and hence it caused marks to be deducted. The line *y=x* graph to be drawn was badly done as many students did not consider the domain for the graph.
(iii)	 Since $y=g(x)$ is symmetrical about $y=x$, $g=g^{-1}$. (or g is a self-inverse function.)	Generally well done for the graph though marks were not deducted for students who fail to recognize that the end points are excluded. Some students did not connect the points to the axes and hence was penalized. Reasoning given was generally well done; many used the term ‘same’, which was not accepted mathematically. They need to understand that this is a self-inverse function.
(iv)	$R_g=(0,1)$, $D_f=[0,1)$ Since $R_g\subseteq D_f$, fg exists. $(0,1)\xrightarrow{g}(0,1)\xrightarrow{f}(1,2)$	Many fail to recognize the condition to prove for existence of composite function. Please remember

		the correct method; Range of composite function was averagely done, definitely an improvement from common test. Many students were not able to use the mapping method; they need to revise on the method to obtain the solution.

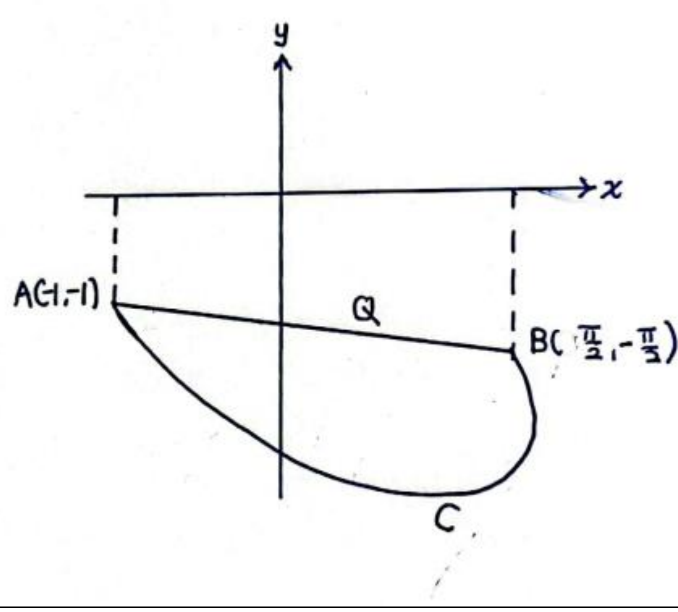
<table>
<tr><td>10</td><td>Solution [11] Integration</td><td></td></tr>
<tr><td>(a)</td><td>$\int (\cos x - x\sin x)(x\cos x)\ \mathrm{d}x$
$= \int u\ \mathrm{d}u$ $u = x\cos x$
$= \frac{1}{2}u^2 + c$ $\frac{\mathrm{d}u}{\mathrm{d}x} = \cos x - x\sin x$
$= \frac{1}{2}x^2\cos^2 x + c$</td><td>Generally well done.

Common errors:
- Not able to simplify the substitution
- Not expressing the final answer in terms of x
- Omitting the constant of integration</td></tr>
<tr><td>(b)
(i)</td><td></td><td>Generally well done

Common errors:
- Incorrect domain for t.
- Not labelling coordinates</td></tr>
<tr><td>(b)
(ii)</td><td>$x = \cos t + t\sin t \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}t} = t\cos t$
$y = \cos t - t\sin t \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}t} = -2\sin t - t\cos t$
$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-2\sin t - t\cos t}{t\cos t}$
$\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right\|_{t=\pi} = \frac{-2\sin\pi - \pi\cos\pi}{\pi\cos\pi} = -1$
Therefore, gradient of Normal to C at P = 1

When $t = \pi$, $P(-1,-1)$
Therefore, equation of Normal to C at P:</td><td>Generally well done, but students were weak in algebraic manipulation and differentiation.

Some candidates were able to use their GC to find the derivative, to their benefit.</td></tr>
</table>

	$y-(-1)=(1)(x-(-1))$ $y=x$	
(b) (iii)	Given that $\begin{matrix} x=\cos t+t\sin t \\ y=\cos t-t\sin t \end{matrix}$, $A(-1,-1)$ and $B\left(\frac{\pi}{2},\frac{-\pi}{2}\right)$. When $t=\pi$ $x=\cos\pi+\pi\sin\pi=(-1)+\pi(0)=-1$ $y=\cos\pi-\pi\sin\pi=(-1)-\pi(0)=-1$ Therefore $A(-1,-1)$. (Verified) When $t=\frac{\pi}{2}$ $x=\cos\frac{\pi}{2}+\frac{\pi}{2}\sin\frac{\pi}{2}=(0)+\frac{\pi}{2}(1)=\frac{\pi}{2}$ $y=\cos\frac{\pi}{2}-\frac{\pi}{2}\sin\frac{\pi}{2}=(0)-\frac{\pi}{2}\frac{\pi}{2}(1)=-\frac{\pi}{2}$ Therefore $B\left(\frac{\pi}{2},\frac{-\pi}{2}\right)$ (Verified)	Moderately well done, but few excellent answers. Students seemed confused when asked to "verify". Many attempted to show the stated t value with the given x and y values, which was accepted, but lead to a lot of unnecessary working. Some student attempted this, but only used the x value. This did not earn the mark. Some students failed to show sufficient working or just asserted "by GC". This is not sufficient for a verify question.
(b) (iv)		Poorly done. Students struggled to form the correct expression for the area. Many attempted to solve the question by hand, rather than using their GC.

	Area $= \int_{-1}^{\frac{\pi}{2}} \left(\frac{(2-\pi)x - 2\pi}{2+\pi} \right) \mathrm{d}x - \int_{-1}^{\frac{\pi}{2}} y \ \mathrm{d}x$ $= \int_{-1}^{\frac{\pi}{2}} \left(\frac{(2-\pi)x - 2\pi}{2+\pi} \right) \mathrm{d}x - \int_{\pi}^{\frac{\pi}{2}} (\cos t - t\sin t)(t\cos t) \ \mathrm{d}t$ $= -3.304496877 - (-4.934802201)$ $= \ 1.6303 \text{ units}^2$	

11	Solution [12] Max/Min	
(i)	Floor area $xy = 40 \Rightarrow y = \frac{40}{x}$ Area $A = 2(4x) + 2x\sqrt{\left(0.01y^2\right)^2 + \left(\frac{y}{2}\right)^2}$ ....(*) $= 8x + 2x\sqrt{\left(\frac{1}{100}\cdot\frac{40^2}{x^2}\right)^2 + \left(\frac{40}{2x}\right)^2}$ $= 8x + 2x\sqrt{\frac{256}{x^4} + \frac{400}{x^2}}$ ....(**) $= 8x + 2x\sqrt{\left(\frac{400}{x^2}\right)\left(1 + \frac{16}{25x^2}\right)}$ $= 8x + 40\sqrt{\frac{16}{25x^2} + 1}$	Moderately well done. Most students were able to see the relationship between x and y and form an expression for area. However, there were many mistakes in the algebraic manipulation. Students should be reminded to <u>neatly</u> present all their working, especially for "show" questions.
(ii)	$A = 8x + \sqrt{\frac{16}{25x^2} + 1}$ $\frac{\mathrm{d}A}{\mathrm{d}x} = 8 + 40\left(\frac{1}{2}\right)\left(\frac{16}{25x^2} + 1\right)^{-1/2}\left(-\frac{32}{25x^3}\right)$ $\frac{\mathrm{d}A}{\mathrm{d}x} = 8 - \frac{128}{5x^3\sqrt{\frac{16}{25x^2} + 1}}$ Let $\frac{\mathrm{d}A}{\mathrm{d}x} = 0$ $8 - \frac{128}{5x^3\sqrt{\frac{16}{25x^2} + 1}} = 0$ $\sqrt{\frac{16}{25x^2} + 1} = \frac{16}{5x^3}$	Moderately well done. Most students were able to correctly differentiate the expression, although there were careless errors in doing so. Most were able to identify that the derivative was zero and attempted to solve. However, there were many mistakes in the subsequent algebraic manipulation.

<table>
<tr>
<td></td>
<td>$\frac{16}{25x^2}+1=\frac{256}{25x^6}$
$25x^6+16x^4-256=0$ (Shown)</td>
<td></td>
</tr>
<tr>
<td>(iii)</td>
<td>Solving $25x^6+16x^4-256=0$ using GC
Rejecting all negative and complex roots.
$x=1.406342039$

Method 1:
<table>
<tr><td>x</td><td>1.356</td><td>1.4063</td><td>1.456</td></tr>
<tr><td>$\frac{dA}{dx}$</td><td>-0.8431</td><td>0</td><td>0.7311</td></tr>
<tr><td></td><td>\</td><td>–</td><td>/</td></tr>
</table>

Method 2:
Let $A(x)=8x+(40)\sqrt{\frac{16}{25x^2}+1}$
Using GC, $\frac{d^2A}{dx^2}=15.67>0$ when x = 140.6342
Hence A is minimum when $x=1.406342039$
Minimum Cost = $(3.10)\cdot A(1.406342039)$
= $177.53</td>
<td>This question was a mixed bag. Very few students got full marks.

Many attempted to solve the hexic equation by hand rather than using GC. This was fruitless.

Some incorrectly took the square root of all the terms, which led to incorrect answers.

Many attempted to use the GC to solve the hexic equation, but still got incorrect answers.

Very few attempted to check that the answer was a minimum point. Of those that did, most did not substitute in values, instead just stating that the sign of the derivative (or second derivative).

Some students incorrectly assumed that $\frac{dA}{dx}=25x^6+16x^4-256$.

Some students incorrectly assumed that 3.10 was the minimum x value.

Some students failed to correct substitute their values, and consequently got incorrect answers.</td>
</tr>
</table>

(iv)	For sufficiently small θ , we have $\sin\theta \approx \theta$ $V = \dfrac{k\sin\theta}{5+5\sin\theta}$ $= \dfrac{k}{5}\left(\dfrac{\sin\theta}{1+\sin\theta}\right)$ $\approx \dfrac{k}{5}\left(\dfrac{\theta}{1+\theta}\right)$ $= \dfrac{k}{5}\theta(1+\theta)^{-1}$ $= \dfrac{k}{5}\theta(1-\theta+...)$ $\approx \dfrac{k}{5}\theta - \dfrac{k}{5}\theta^2$	Poorly done. Most students were able to substitute $\sin\theta \approx \theta$, although some swapped variable to x without explanation. Not all were able to see the need to apply the binomial series to the resulting expression.

12	Solution [12] Differential Equations	
(a)	$\frac{\mathrm{d}x}{\mathrm{d}t} \propto x$ $\frac{\mathrm{d}x}{\mathrm{d}t} = -k_1 x$ $\int \frac{1}{x}\mathrm{d}x = \int -k_1 \mathrm{d}t$ $\ln x = -k_1 t + c_1$ $x = A\mathrm{e}^{-k_1 t}$ When $t = 0$, $x = 1$, thus $A = 1$ $x = \mathrm{e}^{-k_1 t}$	The common error is missing the negative sign at the beginning of the DE; otherwise most students recognize the method of separable.
(ii)	$\frac{\mathrm{d}y_{\text{produced}}}{\mathrm{d}t} = -\frac{\mathrm{d}x}{\mathrm{d}t} = k_1 x$ $\frac{\mathrm{d}y_{\text{decay}}}{\mathrm{d}t} \propto y$ $\frac{\mathrm{d}y_{\text{decay}}}{\mathrm{d}t} = -k_2 y$ $\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\left(y_{\text{produced}}\right) - \frac{\mathrm{d}}{\mathrm{d}t}\left(y_{\text{decay}}\right)$ $\frac{\mathrm{d}y}{\mathrm{d}t} = k_1 x - k_2 y$ $= k_1 \mathrm{e}^{-k_1 t} - k_2 y$ $\frac{\mathrm{d}y}{\mathrm{d}t} + k_2 y = k_1 \mathrm{e}^{-k_1 t}$	This part is not so well-done as most students have difficulty explaining the DE; they tried to manipulate the terms around instead of explaining; some of them claimed $\frac{\mathrm{d}y}{\mathrm{d}t} = -k_2 y$ which is incorrect.
(iii)	$\frac{\mathrm{d}}{\mathrm{d}t}\left(\mathrm{e}^{k_2 t} y\right) = \mathrm{e}^{k_2 t}\frac{\mathrm{d}y}{\mathrm{d}t} + k_2 \mathrm{e}^{k_2 t} y$	Quite ok except missing the $\frac{\mathrm{d}y}{\mathrm{d}t}$
(iv)	$\frac{\mathrm{d}y}{\mathrm{d}t} + k_2 y = k_1 \mathrm{e}^{-k_1 t}$ $\mathrm{e}^{k_2 t}\frac{\mathrm{d}y}{\mathrm{d}t} + k_2 \mathrm{e}^{k_2 t} y = k_1 \mathrm{e}^{(k_2 - k_1)t}$ $\frac{\mathrm{d}}{\mathrm{d}t}\left(\mathrm{e}^{k_2 t} y\right) = k_1 \mathrm{e}^{(k_2 - k_1)t}$	Most students can follow the instruction by multiplying the term $\mathrm{e}^{k_2 t}$; the better ones managed to recognize the link with (iii); the good ones

	Integrating with respect to t , $e^{k_2 t}\ y = \int k_1\, e^{(k_2 - k_1)t}\ dt$ $e^{k_2 t}\ y = \dfrac{k_1}{k_2 - k_1} e^{(k_2 - k_1)t} + c_2$ $y = \dfrac{k_1}{k_2 - k_1} e^{-k_1 t} + c_2\, e^{-k_2 t}$ Since there is initially no element B present, when t = 0, y = 0. Therefore, $y = \dfrac{k_1}{k_2 - k_1} e^{-k_1 t} - \dfrac{k_1}{k_2 - k_1} e^{-k_2 t}$ $= \dfrac{k_1}{k_2 - k_1}\left(e^{-k_1 t} - e^{-k_2 t}\right)$	(only a few) went on to integrate but failed to either include a new constant C or if they did, they did not solve for the particular solution.