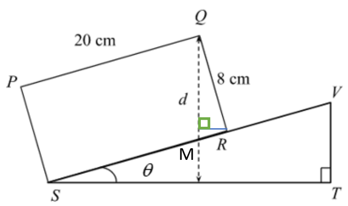
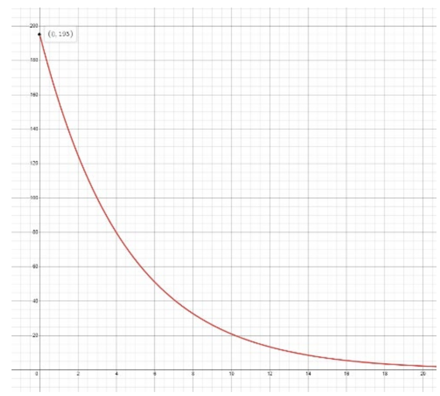
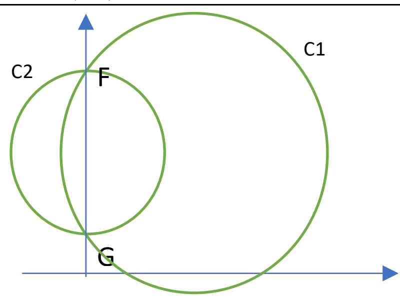
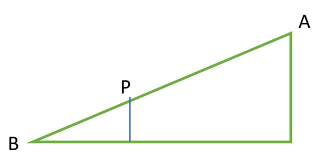
Marking Scheme BSSS AM4 4049 P1 2023

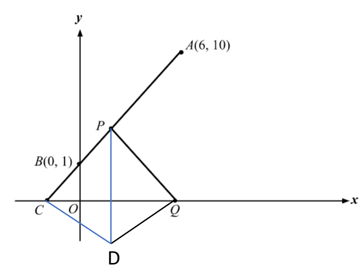
1	$\frac{3}{x}+\frac{2}{y}=\frac{1}{4}$ $\frac{3y+2x}{xy}=\frac{1}{4}$ $4(2x+3y)=xy$ ... eqn A $2x+3y+4=0$ $2x+3y=-4$ ... eqn B Sub eqn B into A, $4(-4)=xy$ $y=-\frac{16}{x}$ ... eqn C Sub eqn C into B, $2x+3\left(-\frac{16}{x}\right)=-4$ $2x^2-48=-4x$ $2x^2+4x-48=0$ $x^2+2x-24=0$ $(x+6)(x-4)=0$ $x=-6$ or $x=4$ $y=2\frac{2}{3}$ or $y=-4$ $\therefore$ intersections are $\left(-6,2\frac{2}{3}\right)$ and $(4,-4)$	M1 – form eq in terms of x/y M1 - factorization M1 – solve for x & y A1, A1
2a	$\frac{30+18\sqrt{2}}{2+\sqrt{2}}$ $=\frac{30+18\sqrt{2}}{2+\sqrt{2}}\times\frac{(2-\sqrt{2})}{(2-\sqrt{2})}$ $=\frac{60+36\sqrt{2}-30\sqrt{2}-36}{4-2}$ $=\frac{24+6\sqrt{2}}{2}$ $=12+3\sqrt{2}$	M1 A1
2b	Consider area of $\triangle$XYZ, $\frac{1}{2}(4+2\sqrt{2})(a+b\sqrt{2})\sin 135^o=15+9\sqrt{2}$ $(2+\sqrt{2})(a+b\sqrt{2})\frac{\sqrt{2}}{2}=15+9\sqrt{2}$ $(\sqrt{2}+1)(a+b\sqrt{2})=15+9\sqrt{2}$ $a+b\sqrt{2}=\frac{15+9\sqrt{2}}{\sqrt{2}+1}\times\frac{\sqrt{2}-1}{\sqrt{2}-1}$ $a+b\sqrt{2}=\frac{15\sqrt{2}+9\times 2-15-9\sqrt{2}}{2-1}$ $a+b\sqrt{2}=3+6\sqrt{2}$ $\therefore\ a=3,\ b=6$	M1 M1 A1

3a	$f(x)=\frac{(5-x^2)}{(x^2+3)},\ x>0.$ $f'(x)=\frac{(x^2+3)(-2x)-(5-x^2)(2x)}{(x^2+3)^2}$ $=\frac{-2x^3-6x-10x+2x^3}{(x^2+3)^2}$ $=\frac{-16x}{(x^2+3)^2}$ Since $(x^2+3)^2>0$, for $x>0$, $\frac{-16x}{(x^2+3)^2}<0$ Therefore, f is a decreasing function.	M1 – Quotient Rule M1 A1
3b	when $x=4$, $f'(x)=\frac{-16x}{(x^2+3)^2}$ $=\frac{-64}{(19)^2}$ $\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}$ $0.2=\frac{-64}{(19)^2}\times\frac{dx}{dt}$ $\frac{dx}{dt}=0.2\times\frac{(19)^2}{-64}$ $=1.1281$ $=1.13$ (to 3 s.f.)	M1 M1 A1
4a	At intersections, $5(24-hx)+x^2=20$ $x^2-5hx+100=0$ Discriminant $=(-5h)^2-400$ $=25h^2-400$ $=25(h^2-16)$ $=25(h-4)(h+4)$ For no intersections, $25(h-4)(h+4)<0$ $-4<h<4$	M1 M1 A1
4b	x^2-4x+7 $=(x-2)^2-4+7$ $=(x-2)^2+3$ Minimun point is $(2,3)$ $\therefore\quad x^2-4x+7\geq 3.$ Alternatively, Since $(x-2)^2\geq 0$ $(x-2)^2+3\geq 3$ $\therefore\quad x^2-4x+7\geq 3.$	B1 – complete sq only B1

5a	$\sin(A+B)\sin(A-B)$ $=(\sin A\cos B+\cos A\sin B)(\sin A\cos B-\cos A\sin B)$ $=\sin^2 A\cos^2 B-\cos^2 A\sin^2 B$ $=(1-\cos^2 A)\cos^2 B-\cos^2 A(1-\cos^2 B)$ $=\cos^2 B-\cos^2 A\cos^2 B-\cos^2 A+\cos^2 A\cos^2 B$ $=\cos^2 B-\cos^2 A$	M1 M1 A1
5b	Let $A=45^\circ$ and $B=30^\circ$, $\sin 75^\circ \sin 15^\circ$ $=\sin(45^\circ+30^\circ)\sin(45^\circ-30^\circ)$ $=\cos^2 30^\circ-\cos^2 45^\circ$ $=\left(\frac{\sqrt{3}}{2}\right)^2-\left(\frac{\sqrt{2}}{2}\right)^2$ $=\frac{3}{4}-\frac{2}{4}$ $=\frac{1}{4}$	M1 A1
6a	 $\angle RQM=\angle RSM=\theta$ $d=8\cos\theta+20\sin\theta$	M1 A1
6b	$R\sin\alpha=8$ $\quad R\cos\alpha=20$ $R^2=8^2+20^2$ $\quad \alpha=\tan^{-1}\left(\frac{8}{20}\right)$ $=\sqrt{464}$ $\quad =21.54^o$	B1 – r B1– α
6c	Max $d=21.54$ $=21.5$ (to 3 sig. fig.) Corresponding $\theta=90^o-21.54^o$ $=68.45^o$ $=68.5^o$ (to 1 dec. pl.)	B1 – maximum d B1– corresponding θ
7a	$y=\frac{3x+2}{\sqrt{5x-2}}$ $\frac{dy}{dx}=\frac{\sqrt{5x-2}(3)-\left[\frac{1}{2}(5x-2)^{-\frac{1}{2}}(5)\right](3x+2)}{5x-2}$ $=\frac{1}{5x-2}\times\frac{1}{\sqrt{5x-2}}\times\frac{1}{2}\times[(5x-2)6-5(3x+2)]$ $=\frac{30x-12-15x-10}{2\sqrt{(5x-2)^3}}$ $=\frac{15x-22}{2\sqrt{(5x-2)^3}}$	M1 Qoutient Rule A1
7b	<u>b = 1</u> ∵ <u>there is only 1 cycle in for</u> $0^\circ\le\theta\le 360^\circ$. At $(0,-3)$, $-3=a\sin(0-30^\circ)-1$ $-2=a\left(-\frac{1}{2}\right)$ $a=4$	B1 M1 A1
8ai	when $t=0$, $m=195(0.8)^0$ $m=195$	B1

8aii	when $t=6$, $m=195(0.8)^6$ $m=51.118$ $m=51.1$ (to 3 sig. fig.)	B1
8aiii	when $m=\frac{1}{4}(195)$, $\frac{1}{4}(195)=195(0.8)^t$ $\frac{1}{4}=(0.8)^t$ $t\ln 0.8=\ln\frac{1}{4}$ $t=6.2125$ $t=6.21$ (to 3 sig. fig.)	M1 M1 A1
8b	Since $0\le 0.8^t\le 1$, $\quad 195(0.8)^t\le 195$.	B1
8c		G1- Correct Shape G1 - Labels
9a	$\int_0^4\frac{3}{2x+1}dx$ $=3\int_0^4\frac{1}{2x+1}dx$ $=3\left[\frac{1}{2}\ln(2x+1)\right]_0^4$ $=\frac{3}{2}[\ln 9-\ln 1]$ $=3\ln 3$	M1 A1
9bi	$\int_0^5 2f(x)\,dx-\int_5^3 f(x)\,dx$ $=2\left(\int_3^5 f(x)\,dx+\int_0^3 f(x)\,dx\right)-\left(-\int_3^5 f(x)\,dx\right)$ $=2\times 6+3$ $=15$	M1, M1 A1
9bii	$\int_3^5\left[f(x)-\frac{k}{x^2}\right]dx$ $=\int_3^5 f(x)\,dx-\int_3^5\frac{k}{x^2}\,dx$ $=3-k\left[-\frac{1}{x}\right]_3^5$ $=3-k\left[-\frac{1}{5}+\frac{1}{3}\right]$ $=3-\frac{2}{15}k$ $\int_3^5\left[f(x)-\frac{k}{x^2}\right]dx=0$ $\Rightarrow 3-\frac{2}{15}k=0$ $\Rightarrow k=\frac{45}{2}$	M1 M1 A1

10a	$x^2+y^2+8x-12y+27=0$ $\Rightarrow (x+4)^2-16+(y-6)^2-36+27=0$ $\Rightarrow (x+4)^2+(y-6)^2=5^2$ Centre is $(-4,6)$ Radius = 5	M, A1, A1
10b	 Consider C_1, at y-axis, $x=0$, $y^2-12y+27=0$ $(y-9)(y-3)=0$ $y=3$ or $y=9$ Consider C_2, Centre $=\left(0,\frac{3+9}{2}\right)$ $=(0,6)$ Radius $=\frac{(9-3)}{2}$ $=3$ C_2 is $x^2+(y-6)^2=3^2$	M1 M1 M1 A1
10c	Distance from $(-3,3)$ to $C_1=\sqrt{(-3+4)^2+(3-6)^2}$ $=\sqrt{10}<5$, radius C_1 Distance from $(-3,3)$ to $C_2=\sqrt{(-3-0)^2+(3-6)^2}$ $=\sqrt{18}>3$, radius C_2 $\therefore(-3,3)$ lies in C_1 but not in C_2.	M1 A1
11ai	 Horizontal distance from A to B $=6-0=6$ Horizontal distance from P to B $=\frac{6}{3}=2$ Vertical distance from A to B $=10-1=9$ Vertical distance from P to B $=\frac{9}{3}=3$ $\therefore$ P is $(2,4)$ Alternatively, $\overline{AP}=2\overline{PB}$ $p-a=2(b-p)$ $p-a=2b-2p$ $3p=2b+a$ $3\begin{pmatrix}x\\y\end{pmatrix}=2\begin{pmatrix}0\\1\end{pmatrix}+\begin{pmatrix}6\\10\end{pmatrix}$ $3\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}6\\12\end{pmatrix}$ $\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}2\\4\end{pmatrix}$	B1

11aii	Gradient AB $=\frac{10-1}{6-0}=\frac{3}{2}$ Gradient PQ $=\tan(180^o-\angle PQC)$ $=-\tan(\angle PQC)$ $=-\tan(\angle PCQ)$ $=-$Gradient AB $=-\frac{3}{2}$ Equation PQ is $\frac{y-4}{x-2}=-\frac{3}{2}$ $2y-8=-3x+6$ $2y+3x=14$	M1 M1 – find gradient M1 – valid attempt to find equation (ecf) A1
11aiii	When $y=0$, $x=\frac{14}{3}$ $\therefore Q$ is $\left(\frac{14}{3},0\right)$.	B1
11b	 Note $x_D=x_P=2$ Area $=2\times\left(\frac{1}{2}\left(\frac{14}{3}-2\right)(4-y_D)\right)=104$ $(4-y_D)=\frac{104\times3}{8}$ $y_D=4-39$ $=-35$ $\therefore D$ is $(2,-35)$	M1 – find x_D M1 – find area A1
12	$2\log_7 p=3+\log_p 49$ $\frac{2\lg p}{\lg 7}=3+\frac{2\lg 7}{\lg p}$ $2(\lg p)^2-3\lg 7(\lg p)-2(\lg 7)^2=0$ $(2\lg p+\lg 7)(\lg p-2\lg 7)=0$ $\lg p=-\frac{1}{2}\lg 7$ or $\lg p=2\lg 7$ $p=\frac{1}{\sqrt{7}}$ or $p=49$	M1 – Change of Base M1 – Quadratic Eqn M1- Solve for lg p A1 - Solve for p
12bi	$\log_3 x=a$ $\qquad$ $\log_9 y=b$ $x=3^a$ $\qquad$ $y=3^{2b}$ $\log_x 9y=\frac{\log_9 9+\log_9 y}{\log_9 x}$ $=\frac{1+b}{\left(\frac{\log_3 x}{\log_3 9}\right)}$ $=\frac{2+2b}{a}$	M1 – change of base M1 – Add/Subtract Law A1
12bii	$x^3y=(3^a)^3(3^{2b})$ $=3^{3a+2b}$	M1 – equivalent exponential A1

13a	$For\left(x-3\right)^{11}, \mathrm{T}_{n+1}=\binom{11}{n}x^{11-n}\left(-3\right)^{n}$	M1- find T_{n+1}
	When $n=2$, $\mathrm{T}_3=\binom{11}{2}x^{11-2}\left(-3\right)^2$	M1- find T_3
	$=\left(55\right)x^9\left(9\right)$	
	$=495x^9$	M1 - Correct T_3
	When $n=3$, $\mathrm{T}_4=\binom{11}{3}x^{11-3}\left(-3\right)^3$	M1- find T_4
	$=\left(165\right)x^8\left(-27\right)$	
	$=-4455x^8$	M1 – Correct T_4
	Considering coefficient of x^9, $495-4455k=1386$	M1 – Correct coefficient of x^9
	$k=\frac{1}{5}$	A1
	Alternatively,	
	$\left(x-3\right)^{11}=x^{11}+11x^{10}\left(-3\right)+\binom{11}{2}x^{11-2}\left(-3\right)^2+\binom{11}{3}x^{11-3}\left(-3\right)^3+\ldots$	M1
	$=x^{11}-33x^{10}+495x^9-4455x^8+\ldots$	M2 – 1 mark for 2 terms
	$\left(1+kx\right)\left(x-3\right)^{11}=\left(1+kx\right)\left(x^{11}-33x^{10}+495x^9-4455x^8+\ldots\right)$	M1
	terms with $x^9=-4455kx^9+495x^9$	M1
	$-4455k+495=1386$	M1
	$k=\frac{1}{5}$	A1

13b	$\left(a+b\right)^n=a^n+na^{n-1}b^1+\binom{n}{2}a^{n-2}b^2+\ldots$	
	$=a^n+na^{n-1}b^1+\frac{n(n-1)}{2}a^{n-2}b^2+\ldots$	M1
	$=p+q+r+\ldots$	
	$\frac{q^2}{pr}=\frac{\left(na^{n-1}b^1\right)^2}{a^n\left(\frac{n(n-1)}{2}a^{n-2}b^2\right)}$	M1
	$=\frac{n^2a^{2n-2}b^2}{\frac{n(n-1)}{2}a^{2n-2}b^2}$	
	$=\frac{2n}{(n-1)}$	A1