MATHEMATICS

4052/02

Paper 2

15 August 2024

Candidates answer on the Question Paper.

**2 hours 15 minutes
(08:30 – 10:45)**

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total mark for this paper is 90.

This document consists of **20** printed pages.

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1 **(a)** In 2024, apples cost \$$x$ per kilogram and oranges cost \$$y$ per kilogram. The total cost of 5kg of apples and 10kg of oranges is \$40.

(i) Show that $8 - 2y = x$.

Answer

[2]

(ii) In 2023, the cost, per kilogram, of apples were 9% lower than in 2024 while the cost of oranges, per kilogram were 9% higher than in 2024.

Find an expression, in terms of x and y, for the total cost of 3kg of apples and 2kg of oranges in 2023. Give your answer in its simplest form.
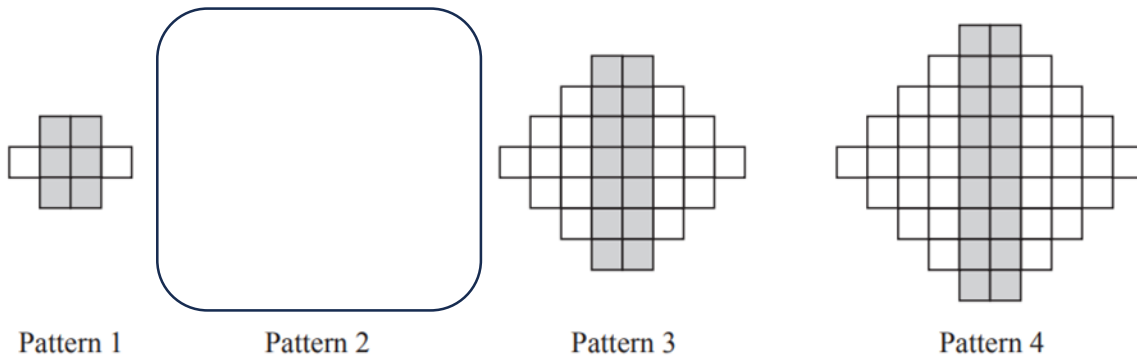
Answer ……………………....... [3]

[TURN OVER

(b) Write as a single fraction in its simplest form $\dfrac{x}{x^2 - x - 12} + \dfrac{3x}{4 - x}$.

Answer …………………............ [4]

2 The diagram shows a sequence made by using grey and white squares.

Pattern 1   Pattern 2   Pattern 3   Pattern 4

(a) Draw Pattern 2 in the space above.   [1]

(b) Complete the table for the next three pattern in the sequence.

Pattern Number	4	5	6
Number of grey squares	18		
Number of white squares	32		

[2]

(c) Find an expression, in terms of k, for the number of grey squares in Pattern k.
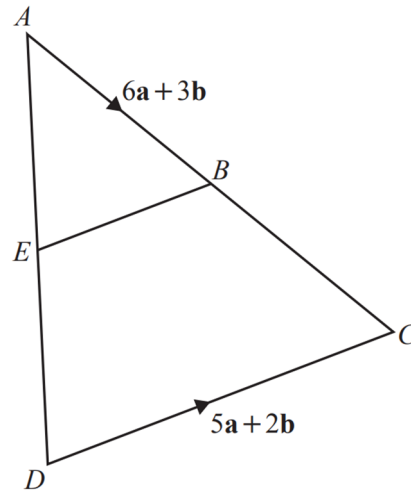
Answer   ………………............... [2]

(d) Determine if any pattern can be formed with 100 grey squares.
Show your working.
Answer

[2]

[TURN OVER

3

In triangle ACD, B is the midpoint of AC and E is the midpoint of AD.
$\overrightarrow{AB} = 6\mathbf{a} + 3\mathbf{b}$ and $\overrightarrow{DC} = 5\mathbf{a} + 2\mathbf{b}$.

(a) Express $\overrightarrow{AD}$, as simply as possible, in terms of $\mathbf{a}$ and $\mathbf{b}$.

Answer $\overrightarrow{AD} =$ ……………… [3]

(b) Show that triangles AEB and ADC are similar.

Answer

[2]

(c) *S* is a point on *AE* such $AS : SE = 2 : 3$.

It is given that $\overrightarrow{SC} = \dfrac{1}{k}\left(53\mathbf{a} + 26\mathbf{b}\right)$.

Find the value of *k*.

Answer   *k* =   ………………….   [2]

(d) Find the numerical value of $\dfrac{\text{area of triangle } AEB}{\text{area of } EBCD}$.

Answer   …………………………   [2]

(e) Use vectors to show that $\overrightarrow{EB}$ is parallel to $\overrightarrow{DC}$.

Answer

[2]

4   A tank is a closed cylinder of radius 5 m and height 8 m.
    It is made of negligible thickness and stands with its base horizontal.

(a)   The tank contains fuel to a depth of 3 m.

Find correct to the nearest cubic metre, the volume of the fuel in the tank.

Answer   …………………… m³   [1]

(b)   20 000 litres of fuel are added.

Find the increase in depth of the fuel in the tank. Give your answer in cm.
(1 litre = 1000 cm³)

Answer   …………………… cm   [3]

(c)   The outer curved surface and the top of the tank are to be painted.
    The paint is sold in tins, each of which contains 5 litres.
    One litre of paint can cover 7 m².

Calculate the number of tins required.

Answer   …………………… tins   [3]

(d) The tank is completely emptied of all fuel and then partially filled with water.
It is now placed with its curved surface on a horizontal floor.
Diagram 1 shows the dimensions of the tank.
Diagram 2 shows a circular end of the fuel tank. O is the centre of the circle
and D is vertically below O. AB represents the level of the water surface and
angle $AOB = \dfrac{2}{3}\pi$.

Diagram 1   Diagram 2
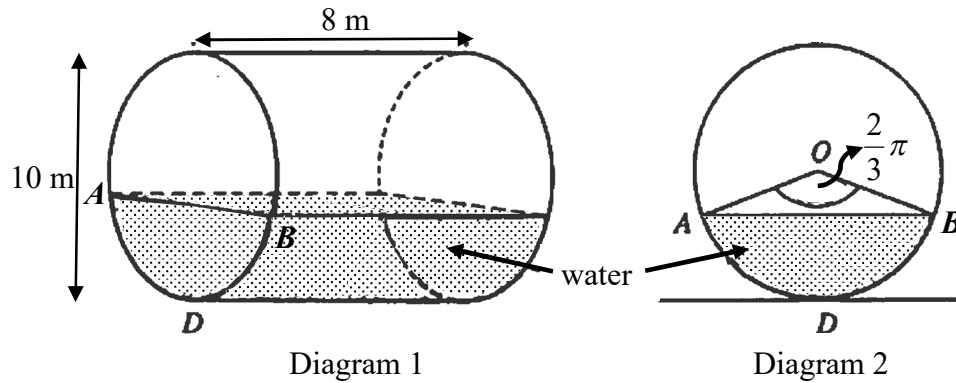
Calculate the area of the inside surface of the tank which is **not** in contact with
the water.

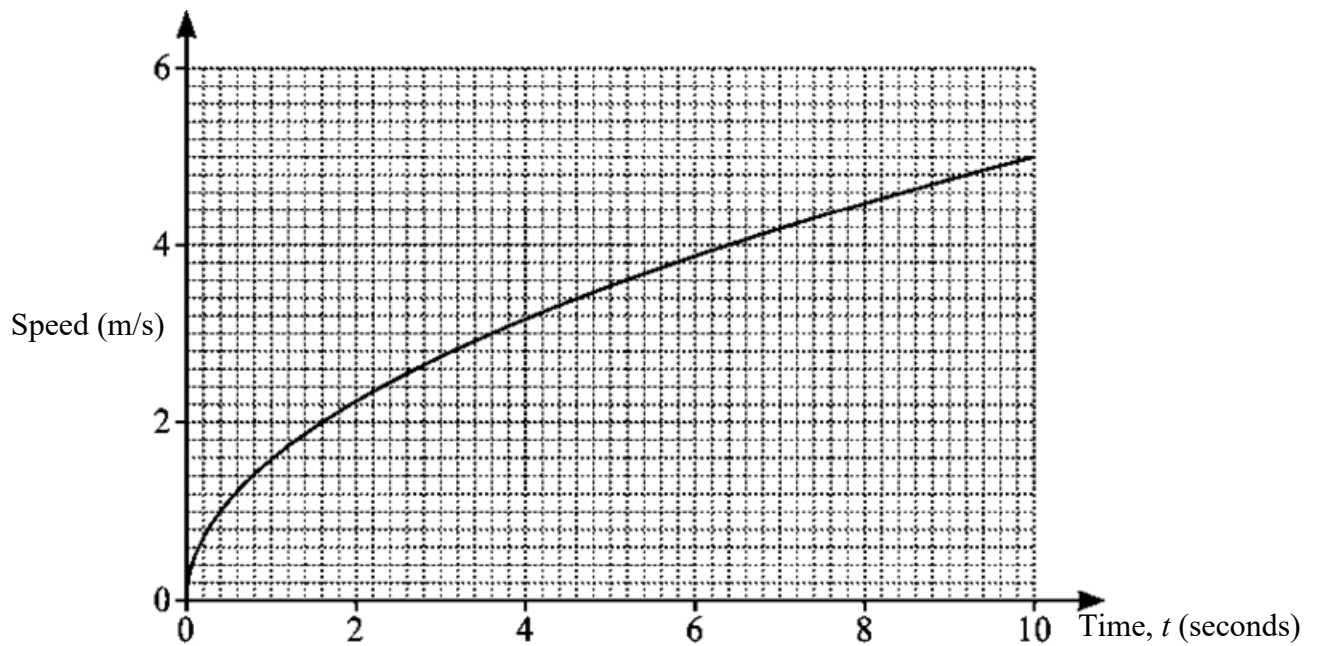*Answer*   …………………………   m²   [5]

[TURN OVER

5 (a) (i) Convert 14 km/h to m/s.

Answer …………………………. m/s [1]

(ii) Haley rode a bike at a speed of 14 km/h for 1 hour 20 minutes from Kensington Town to Brood City.
After a 30 minutes rest at Brood City, she continued her 25 km journey to Bristol Plains at a speed of 10 km/h.

Calculate Haley's average speed, in km/h, for the entire journey.

Answer ……………………….. km/h [4]

(b) The speed-time graph shows the first 10s for the motion of a car.

(i) **Using the graph above**, estimate the acceleration of the car at $t = 2$s.

Answer ………………………… m/s² [2]

(ii) State if you agree or disagree with the statement. Explain your choice.

Statement	Agree / Disagree	Explanation
The speed of the car is increasing at a decreasing rate.		

[2]

[TURN OVER

6 (a) A scientist researched on the effectiveness of a new drug to combat a disease.
The probabilities of a patient's condition upon receiving the drug are as follows:

Patient's condition	Probability
No effect	q
Improving but still ill	$2p$
Getting worst	$5p - \dfrac{4q}{5}$

There are three times as many patients whose condition was not affected by the drug as compared to those whose condition was getting worst.

Calculate the value of p and of q.

Answer p = …………………...

q = …………………... [4]

(b) Baloxavir marboxil (trade name Xofluza®) is the latest influenza (flu) antiviral drug.
The drug is immediately stopped once the patient is cured.
At most, each patient can only receive two doses of the drug.
The probabilities of a patient's condition after receiving each dose is as follows:

Patient's condition	Probability
Fully cured	$\dfrac{4}{7}$
Improving but still ill	$\dfrac{5}{14}$
Getting worst	$\dfrac{1}{14}$

(i) Draw a tree diagram showing the possible outcomes.

Answer

[3]

(ii) Calculate the probability that a patient remains ill despite 2 doses of Xofluza®.

Answer ………………………….. [3]

[TURN OVER

7 The manufacturing cost, y, of a watch when x watches are produced, is related by the equation $y = 20 + \dfrac{1200}{x}$.
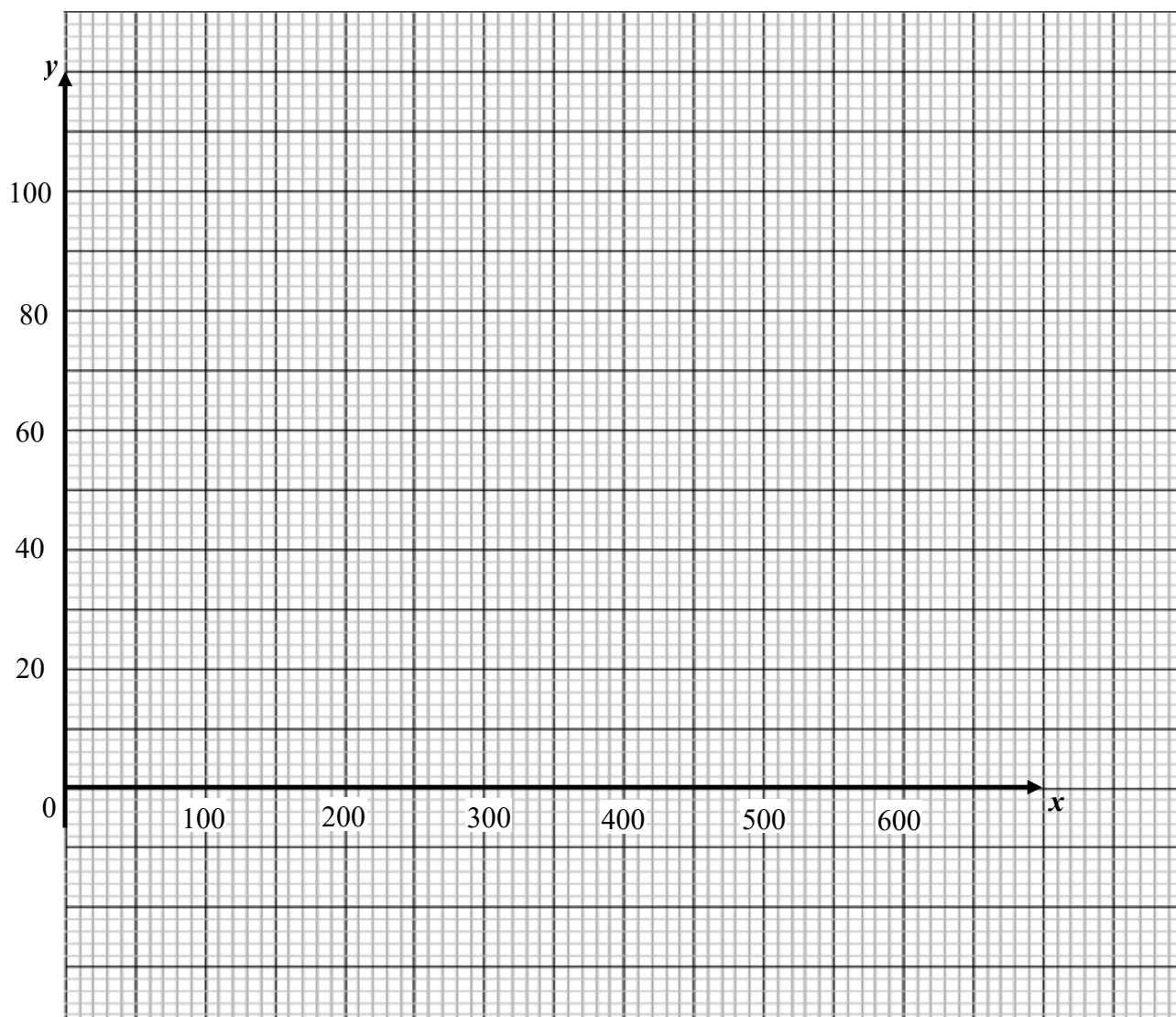
Some values of x and y are given in the table below.

x	15	30	60	150	300	400	600
y	100	60	40	28	24	q	22

(a) Find the value of q.

Answer q = ………………….. [1]

(b) On the grid below, draw the graph of $y = 20 + \dfrac{1200}{x}$ for $0 < x \le 600$. [3]

Use your graph to find

 (i) the manufacturing cost when 80 watches are produced,

Answer $ ………………………… [1]

(ii) the minimum number of watches to be produced for the manufacturing cost to be at most $25.
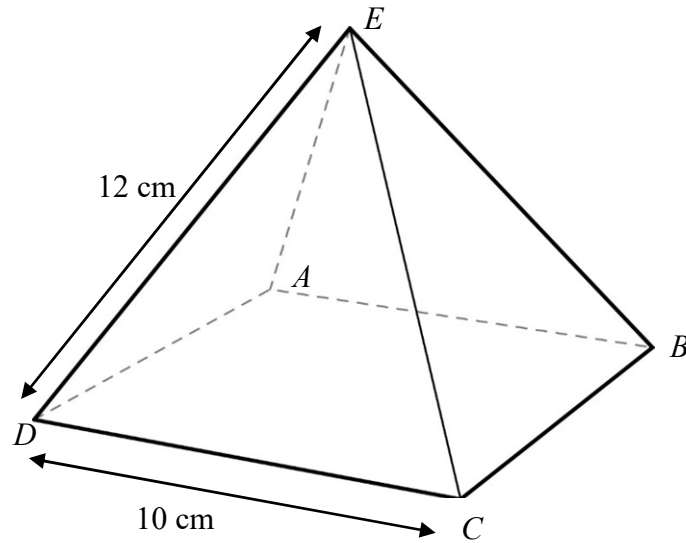
Answer …………………... watches [1]

(c) In order to sell x watches, the selling price of a watch needs to be $\left(90 - \dfrac{21x}{100}\right)$.

By inserting a suitable line, determine the range of number of watches that needs to be sold to ensure a profit.

Answer ……………………………… [3]

8 *ABCDE* is a right square-based pyramid. *DE* = 12 cm and *DC* = 10 cm.

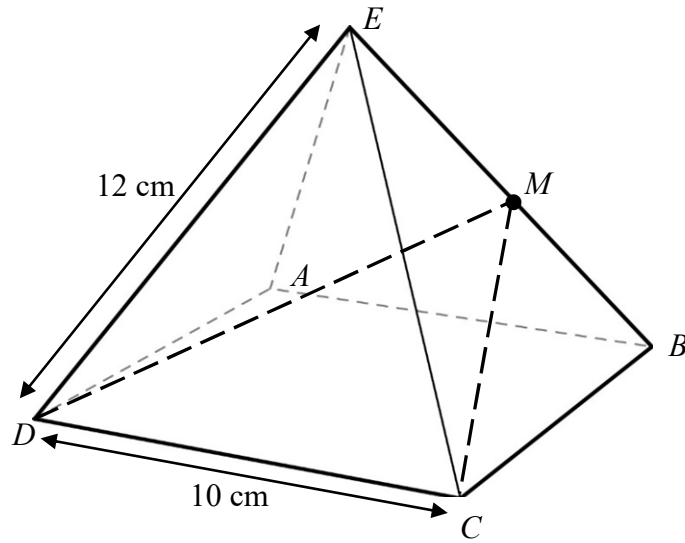**(a)** Show that the vertical height of the pyramid is 9.695 cm.

Answer

[4]

(b) Angle *DEC* = 49.2°.

Calculate angle *EDC*.

Answer   …………………………….....   [2]

(c) *M* is the midpoint on *EB* such that angle *DMC* = 55.7°.
Angle *DEB* = 72.2° .

Calculate the area of triangle *DMC* giving your answer correct to the nearest whole number.

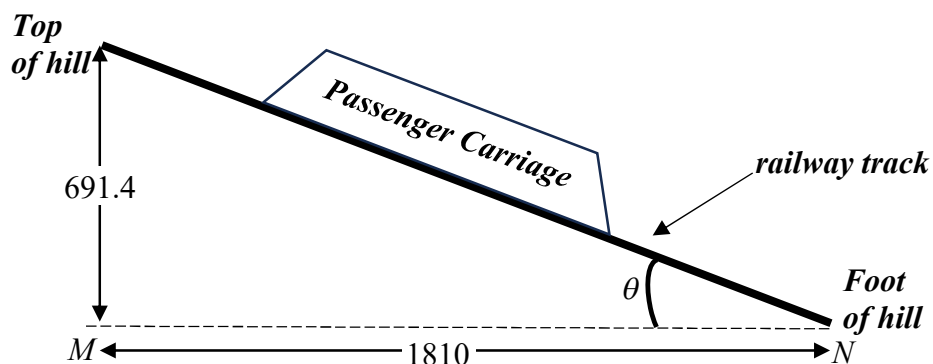*Answer* …………………... cm^2 [6]

9 The Penang Hill funicular railway which climbs the Penang Hill was built in 1923.

Supporting structures in the form of arches.

The diagram below shows a schematic drawing of the railway.

During the journey from the start (foot of the hill) to the end point (top of the hill), the passenger carriage makes a vertical gain of 691.4 m.
The horizontal distance between *M* and *N* is 1810 m.

(a) Calculate the angle of elevation, θ, of the Penang Hill funicular railway.

Answer θ = …………………… [2]

**(b)** Calculate the length of the railway track.

Answer ……………………... m [2]

(c) The railway is built on supporting structures in the form of arches which can withstand an overall maximum load of 140 tonnes.
The information table shows some technical parameters of the Penang Hill funicular railway.
(1 tonne = 1000kg)

	Railway track	Passenger Carriage
Material used	1084 Hot rolled steel	5052 H-32 welded Aluminium
Density of material	8.03 g/cm^3	2.68 g/cm^3
Specifications	Shape — Asymmetric I-Beam	Mass of Carriage — 14 500 kg
	Cross-sectional area (in mm^2) — *Refer to drawing*	

Drawing and measurements of the Asymmetric I-Beam
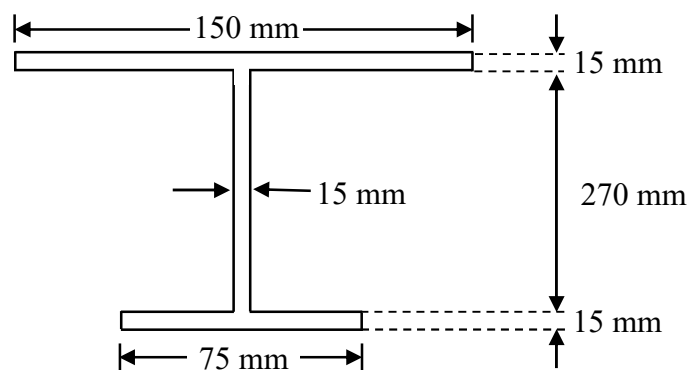
Some further details of the Penang Hill funicular railway are as follows:
- Operating hours are from 0900 till 1900.
- Ticket for the 2-way ride (up and down the hill) costs $45 per person.
- Rides up and down the hill are scheduled every hour, with each ride taking 30 minutes non-stop.
- Only 1 passenger carriage is used for the rides up and down the hill.
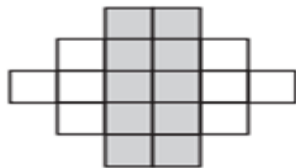
[Question 9 is continued on the next page.]

[TURN OVER

The government of Penang claims that the Penang Hill funicular railway receives a daily estimated total of at least $80 000 from the sale of tickets for the rides.

Assuming the average weight of a passenger is 75 kg, justify with calculations, if the claim made by the government of Penang is true.

[7]

No	Solution	
1a i	$5x+10y=40$ $\qquad x+2y=8$ $\qquad 8-2y=x(shown)$	
aii	$\quad$ Apple cost in 2023=$0.91x$ Orange cost in 2023 $=1.09y$ $\quad$ Total cost in 2023 $=3(0.91x)+2(1.09y)$ $\qquad\qquad\qquad =2.73x+2.18y$	
iii	$\dfrac{x}{x^2-x-12}+\dfrac{3x}{4-x}$ $=\dfrac{x}{(x-4)(x+3)}+\dfrac{3x}{4-x}$ $=\dfrac{x}{(x-4)(x+3)}-\dfrac{3x}{x-4}$ $=\dfrac{x-3x(x+3)}{(x-4)(x+3)}$ $=\dfrac{-3x^2-8x}{(x-4)(x+3)}$	
2i	Pattern 2	

ii	Pattern Number	4	5	6		
	Number of grey squares	18	**22**	**26**		
	Number of white squares	32	**50**	**72**		

iii	$4k+2$	

iv		
	$4k + 2 = 100$ $4k = 98$ $k = 24.5(\text{ non-integer})$ Hence, it is not possible to have 100 grey squares.	
3a	$\overrightarrow{AB} = 6\mathbf{a} + 3\mathbf{b}$ $\overrightarrow{AC} = 2(6\mathbf{a} + 3\mathbf{b})$ $\quad = 12\mathbf{a} + 6\mathbf{b}$ $\overrightarrow{AD} + \overrightarrow{DC} = \overrightarrow{AC}$ $\overrightarrow{AD} = \overrightarrow{AC} - \overrightarrow{DC}$ $\quad = (12\mathbf{a} + 6\mathbf{b}) - (5\mathbf{a} + 2\mathbf{b})$ $\quad = 7\mathbf{a} + 4\mathbf{b}$	
b	$\angle DAC = \angle EAB$ (common angle) $\dfrac{AE}{ED} = \dfrac{AB}{BC} = \dfrac{1}{2}$ Hence, by SAS similarity test, ΔAEB is similar to ΔADC.	

c	$\overrightarrow{AS} + \overrightarrow{SC} = \overrightarrow{AC}$	
	$\overrightarrow{SC} = \overrightarrow{AC} - \overrightarrow{AS}$	
	$\qquad = \overrightarrow{AC} - \dfrac{2}{5}\overrightarrow{AE}$	
	$\qquad = \overrightarrow{AC} - \dfrac{2}{5}\left(\dfrac{1}{2}\overrightarrow{AD}\right)$	
	$\qquad = \overrightarrow{AC} - \dfrac{1}{5}\overrightarrow{AD}$	
	$\qquad = 2(6\mathbf{a} + 3\mathbf{b}) - \dfrac{1}{5}\left(7\mathbf{a} + 4\mathbf{b}\right)$	
	$\qquad = \dfrac{53}{5}\mathbf{a} + \dfrac{26}{5}\mathbf{b}$	
	$\qquad = \dfrac{1}{5}\left(53\mathbf{a} + 26\mathbf{b}\right)$	
	$\Rightarrow k = \dfrac{1}{5}$	
d	$\dfrac{\text{area of } \Delta AEB}{\text{area of } \Delta ADC} = \left(\dfrac{1}{2}\right)^2$	
	$\qquad\qquad\qquad = \dfrac{1}{4}$	
	$\dfrac{\text{area of } \Delta AEB}{\text{area of } EBCD} = \dfrac{1}{4-1}$	
	$\qquad\qquad\qquad = \dfrac{1}{3}$	
e	$\overrightarrow{AD} = 2\overrightarrow{AE}$	
	$\overrightarrow{AE} = \dfrac{1}{2}\left(7\mathbf{a} + 4\mathbf{b}\right)$	
	$\overrightarrow{AB} = \overrightarrow{AE} + \overrightarrow{EB}$	
	$6a + 3b = \dfrac{1}{2}\left(7\mathbf{a} + 4\mathbf{b}\right) + \overrightarrow{EB}$	
	$\overrightarrow{EB} = \dfrac{5}{2}\mathbf{a} + \mathbf{b}$	
	$\overrightarrow{DC} = 5\mathbf{a} + 2\mathbf{b}$	
	$\qquad = 2\left(\dfrac{5}{2}\mathbf{a} + \mathbf{b}\right)$	
	$\qquad = 2\overrightarrow{EB}$	
	Since $\overrightarrow{DC} = k\overrightarrow{EB}$, where $k=2$ and do not have a common point, they are parallel.	

4a	$V = \pi(5)^2(3)$ $= 75\pi$ or $235.619449 m^3$ $= 236 \text{ m}^3 (3sf)$
b	$20000l = 20\ 000\ 000 cm^3 = 20 m^3$ $V_{new} = 20 + 235.619449$ $\qquad = 255.619449 m^3$ $\qquad \Rightarrow \pi(5)^2 h_{new} = 255.619449$ $h_{new} = 3.2546479 m$ $increase = 3.2546479 - 3$ $\qquad = 0.2546479 m$ $\qquad = 25.5 cm (3sf)$ OR $20000(1000) = \pi(500)^2(h)$ $h = \dfrac{20000000}{500^2 \pi}$ $\quad = \underline{\mathbf{25.5\ cm}} \text{ (to 3 sf)}$
c	Area to be painted $= \pi(5)^2 + 2\pi(5)(8)$ $\qquad = 105\pi m^2$ $5l = 0.005 m^3$ $\qquad 0.001 m^3 \text{ rep } 7 m^2$ $\qquad 0.005 m^3 \text{ rep } 35 m^2$ No. of tins needed $= \dfrac{105\pi}{35}$ $\qquad = 9.42477961$ $\qquad = 10 \text{ tins (nearest integer)}$

d	$$\text{reflex } \angle AOB = 2\pi - \frac{2}{3}\pi$$ $$= \frac{4\pi}{3}$$ $$\text{area of major sector AOB} = \frac{240^\circ}{360^\circ} \times \pi \times (5)^2$$ $$= \frac{50}{3}\pi\, m^2$$ $$\text{area of } \Delta AOB = \frac{1}{2}(5)^2 \sin\frac{2}{3}\pi$$ $$= \frac{25\sqrt{3}}{4}\, m^2$$ $$\text{major arc AB} = 2\pi(5) \times \frac{4\pi/3}{2\pi}$$ $$= \frac{20}{3}\pi\, m^2$$ $$\text{Dry curved area} = \frac{20}{3}\pi \times 8$$ $$= \frac{160}{3}\pi\, m^2$$ $$\text{Total area not in contact} = \frac{160}{3}\pi + 2\left(\frac{25\sqrt{3}}{4} + \frac{50}{3}\pi\right)$$ $$= \frac{160}{3}\pi + \frac{25\sqrt{3}}{2} + \frac{100}{3}\pi$$ $$= \frac{260}{3}\pi + \frac{25\sqrt{3}}{2}$$ $$= 293.9198\, m^2$$ $$= 294 \text{ m}^2 \,(3sf)$$	
5 ai	14000m in 60×60 s 14000m in 3600s $\dfrac{14000}{3600}$ m in 1s 3.8889m in 1s $\quad \therefore 14km/h = 3.89 m/s$ (accept $3\frac{8}{9}$)	

aii	
Distance between Kensington and Brood $= 14 \times \dfrac{4}{3}$ $= 18\dfrac{2}{3} km$ Time taken from Brood to Bristol $= \dfrac{25}{10} hr$ Total time $= \dfrac{4}{3} + \dfrac{25}{10} + \dfrac{1}{2}$ $= \dfrac{13}{3} hrs$ Average speed $= \dfrac{25 + 18\dfrac{2}{3}}{13 \big/ 3}$ $= 10.07692 km/h$ $= 10.1$ km/h (3sf)	

bi	
$0.5 - 0.6$	

ii				
	Statement	**Agree / Disagree**	**Explanation**	
	The velocity of the car is increasing at a decreasing rate.	**Agree**	The gradient of the tangent is decreasing.	

6a

$$q + 2p + 5p - \frac{4q}{5} = 1$$

$$7p + \frac{q}{5} = 1 ---(1)$$

Let the total number of people be P

$$qP = 3(5p - \frac{4q}{5})(P)$$

$$q = 15p - \frac{12q}{5}$$

$$\frac{17}{5}q = 15p$$

$$p = \frac{17}{75}q ---(2)$$

sub (2) into (1):

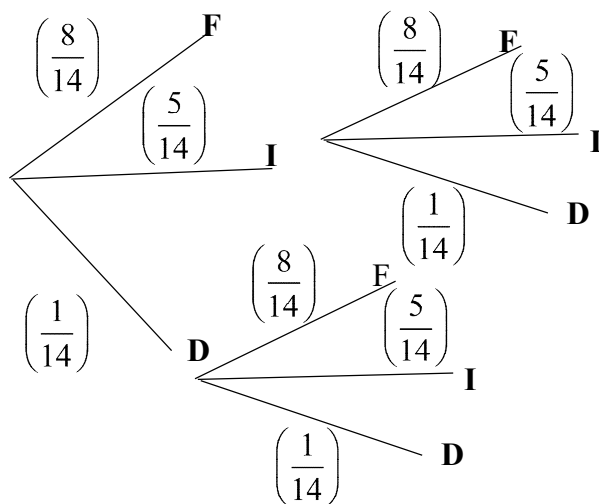$$7\left(\frac{17}{75}q\right) + \frac{q}{5} = 1$$

$$\frac{134}{75}q = 1$$

$$q = \frac{75}{134}$$

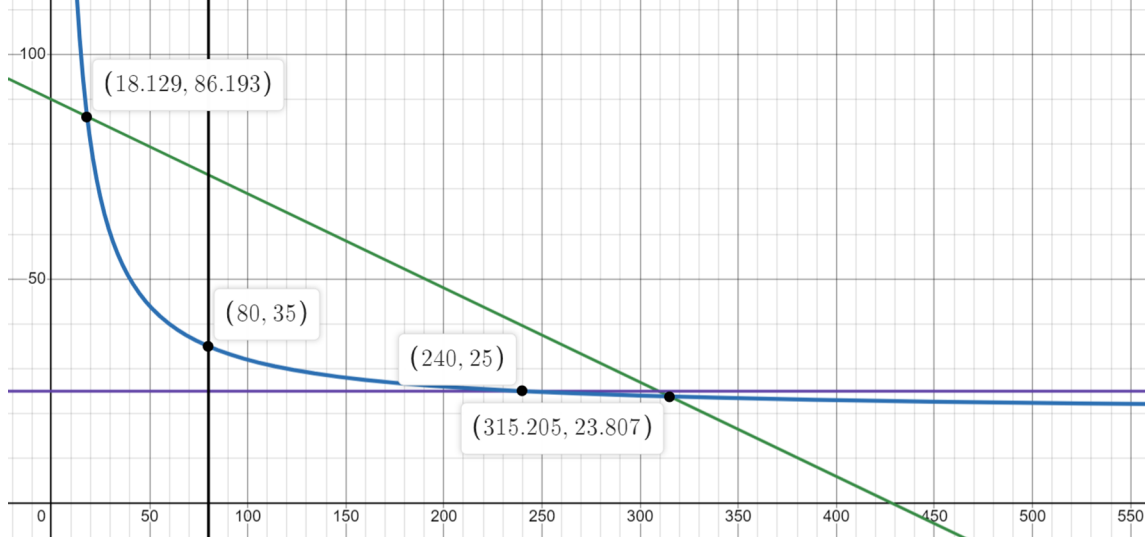$$\Rightarrow p = \frac{17}{75}\left(\frac{75}{134}\right)$$

$$= \frac{17}{134}$$

bi

B1 – for showing a tree diagram (award even if partially drawn) ;
B1 – all values in Tier 1, B1 – all values in Tier 2

7

ii	P (ill after 2 doses) $$= \left(\frac{5}{14}\right)^2 + \left(\frac{1}{14}\right)^2 + \left(\frac{5}{14} \times \frac{1}{14}\right) + \left(\frac{1}{14} \times \frac{5}{14}\right)$$ $$= \frac{9}{49}$$
7a	23
b	**B1 – showing all plotting points** **B1 – labels of axis and equation of curve** **B1 – smooth curve passing through all points**
ci	$28 \le y \le 42$
ii	$232.5 \le x \le 247.5$
d	$15(+5) < x \le 320(\pm 5)$
8a	diagonal length of base $= \sqrt{10^2 + 10^2}$ $$= \sqrt{200}\,cm$$ *hence,* $$h^2 + \left(\frac{\sqrt{200}}{2}\right)^2 = 12^2$$ $$h^2 + 50 = 144$$ $$h^2 = 94$$ $$h = \sqrt{94}\,cm$$ $$= 9.69535971\,cm$$ $$= 9.695 \text{ cm (shown)}$$

b	$\angle EDC = \dfrac{180^\circ - 49.2^\circ}{2}$ (base$\angle$s of isos.Δ,$ED = EC$) $= 65.4^\circ$ (1dp)
c	By Cosine Rule, $DM^2 = 12^2 + 6^2 - 2(12)(6)\cos 72.208409^\circ$ $DM = \sqrt{136}$ $= 11.6619038\,cm$ $\angle CBM = \dfrac{180^\circ - 49.24864^\circ}{2}$ (base angles of isos.Δ, $CM = BM$) $= 65.37568^\circ$ $CM^2 = 6^2 + 10^2 - 2(6)(10)\cos 65.37568^\circ$ $= 85.999996863$ $CM = 9.27361833\,cm$ $Area = \dfrac{1}{2}(\sqrt{136})(9.2736183)\sin 55.7^\circ$ $= 44.67045755416\,cm^2$ $= 45\ cm^2$ (nearest whole number) ***---Alternative Solution to 8c---*** By Cosine rule, $DM^2 = 12^2 + 6^2 - 2(12)(6)\cos 72.208409^\circ$ $DM = \sqrt{136}$ $= 11.6619038\,cm$ $\dfrac{\sqrt{136}}{\sin \angle DCM} = \dfrac{10}{\sin 55.7^\circ}$ $\sin \angle DCM = \dfrac{\sqrt{136}\sin 55.7^\circ}{10}$ $\angle DCM = 74.44807810^\circ$ $\angle MDC = 180^\circ - 55.7^\circ - 74.44807810^\circ$ (sum of angles in Δ) $= 49.8519219^\circ$ $Area = \dfrac{1}{2}(10)(\sqrt{136})\sin 49.8519219^\circ$ $= 44.570667097\,cm^2$ $= 45\ cm^2$ (nearest whole number)	

9a	$\tan\theta = \dfrac{691.4}{1810}$	
	$\theta = \tan^{-1}\left(\dfrac{691.4}{1810}\right)$	
	$\quad = 20.9063044^{\circ}$	
	$\quad = 20.9^{\circ}$	
b	$l = \sqrt{(691.4)^2 + (1810)^2}$	
	$\quad = 1937.5587763m$	
	$\quad = 1940$ m (3sf)	

c		
	Maximum load for the supporting arches $= 140 \times 1000$	
	$= 140000 kg$	
	Cross-sectional area of track $=(150\times15)+(75\times15)+(270\times15)$	
	$= 7425 mm^2$	
	Mass of track $= \dfrac{7425}{100} \times 1937.585712 \times 100 \times 8.03$	
	$= 115524.1885 kg$	
	Maximum allowable weight of passengers $= 140000 - 115524.1885 - 14500$	
	$= 9975.8115 kg$	
	No.of passengers carriage can carry on each ride $= \dfrac{9975.8115}{75}$	
	$=133.01082$	
	$=133$	
	Maximum no. of single-rides in one day $= \dfrac{10\times60}{30}$	
	$=20$	
	Maximum no. of two-way-rides in one day $= 10$	
	Maximum no. of passengers taking the rides each day $= 133\times10$	
	$= 1330$	
	Maximum money received from ticket sales each day $= 1330\times45$	
	$= \$59850\left(<\$80000\right)$	
	Hence, **the claim is false** as the maximum revenue from ticket sales is lesser than the stated $80 000	