LOYANG VIEW SECONDARY SCHOOL
Preliminary Examination 2021
Secondary Four Express

CANDIDATE NAME	MARKING SCHEME
CLASS	
CLASS INDEX NUMBER	0 0

ADDITIONAL MATHEMATICS

4049/02

Paper 2

1 SEP 2021

2 hours 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

For examiner's use only	
Question number	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
Total	

READ THESE INSTRUCTIONS FIRST

Write your Class, index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angle in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

Setter: Mr Sim LE

This document consists of **22** printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

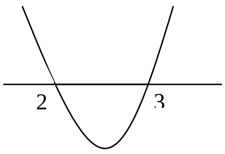
Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2} ab \sin C$$

1 **(i)** Show that the line $y = 3x - 2$ does not meet the curve $y = 3k^2x^2 + (k+3)x + 1$ for all real values of x. [4]

(ii) If the roots of the equation $x^2 + 2x(1-k) + 3k - 5 = 0$ are real, show that k cannot lie between 2 and 3. [4]

[Turn Over

Q	Solutions	Marks	Remarks
1 (i)	$3x-2=3k^2x^2+(3+k)x+1$ $3x-2=3k^2x^2+(3+k)x+1$ $3k^2x^2+kx+3=0$ b^2-4ac $=(k)^2-4(3k^2)(3)$ $=-35k^2$ $k^2>0$ for all real values of k $\Rightarrow -35k^2<0$ $\Rightarrow b^2-4ac<0 \Rightarrow$ no real roots Hence, the line does not meet the curve.	**M1** **M1** **M1** **A1**	√M1 incorrect
(ii)	$x^2+2x(1-k)+3k-5=0$ for real roots, $b^2-4ac\geq 0$ $(2-2k)^2-4(1)(3k-5)\geq 0$ $4-8k+4k^2-12k+20\geq 0$ $4k^2-20k+24\geq 0$ $4(k^2-5k+6)\geq 0$ $4(k-2)(k-3)\geq 0$ $k\leq 2$ or $k\geq 3$ for real roots  Hence, k cannot lie between 2 and 3.	**M1** **M1** **M1** **A1**	√M1 incorrect

2 **(a)** Solve the equation $x\sqrt{30} = x\sqrt{3} + \sqrt{6}$. [3]

(b) Solve the equation $\log_3 x - \log_3 \frac{1}{2} = \log_9 (5x - 1)$ [5]

Q	Solutions	Marks	Remarks
2 (i)	$x\sqrt{30} = x\sqrt{3} + \sqrt{6}$ $x\left(\sqrt{30} - \sqrt{3}\right) = \sqrt{6}$ $x = \dfrac{\sqrt{6}}{\sqrt{30} - \sqrt{3}}$ $x = \dfrac{\sqrt{6}}{\sqrt{30} - \sqrt{3}} \times \dfrac{\sqrt{30} + \sqrt{3}}{\sqrt{30} + \sqrt{3}}$ $x = \dfrac{\sqrt{180} + \sqrt{18}}{\left(\sqrt{30}\right)^2 - \left(\sqrt{3}\right)^2}$ $x = \dfrac{6\sqrt{5} + 3\sqrt{2}}{30 - 3} = \dfrac{6\sqrt{5} + 3\sqrt{2}}{27}$ $x = \dfrac{2\sqrt{5} + \sqrt{2}}{9}$	 **M1** **M1** **A1**	√M1 incorrect
(ii)	$\log_3 x - \log_3 \frac{1}{2} = \log_9 (5x - 1)$ $\log_3 \dfrac{x}{\left(\frac{1}{2}\right)} = \dfrac{\log_3 (5x-1)}{\log_3 9}$ $\log_3 2x = \dfrac{\log_3 (5x-1)}{\log_3 3^2}$ $\log_3 2x = \dfrac{\log_3 (5x-1)}{2}$ $\log_3 2x = \frac{1}{2} \log_3 (5x - 1)$ $\log_3 2x = \log_3 (5x-1)^{\frac{1}{2}}$ $2x = (5x-1)^{\frac{1}{2}}$ $(2x)^2 = 5x - 1$ $4x^2 - 5x + 1 = 0$	 **M1** **M1** **M1** **M1**	√M1 incorrect quotient law change of base law power law

	$(4x-1)(x-1)=0$ $x=\frac{1}{4}$ or $x=1$	**A1**	

3 The first four terms, in ascending powers of *x*, of $(2+px)^n$ is $32-240x+80p^2x^2+qx^3$.

(i) Find the value of *p*, of *n* and of *q*. [5]

(ii) Using your answer in **(i)**, estimate, to 5 decimal places, the value of $(1.97)^5$. [3]

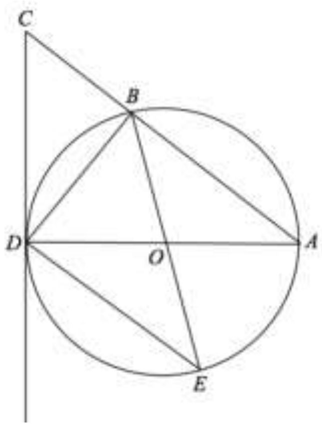
Q	Solutions	Marks	Remarks
3 (i)	$(2+px)^n=2^n+\binom{n}{1}2^{n-1}px+\binom{n}{2}2^{n-2}(px)^2+\binom{n}{3}2^{n-3}(px)^3+\ldots$ $=2^n+n2^{n-1}px+\frac{n(n-1)}{2!}2^{n-2}p^2x^2+\frac{n(n-1)(n-2)}{3!}2^{n-3}p^3x^3+\ldots$ Equating terms, $2^n=32\Rightarrow 2^n=2^5$ equating indices, $n=5$ $2^{n-1}np=-240$ $2^4\times5\times p=-240$ $p=-\frac{240}{80}=-3$ $\frac{n(n-1)(n-2)}{3!}2^{n-3}p^3=q$ $\frac{5(5-1)(5-2)}{6}2^2(-3)^3=q$ $q=-1080$	 **B1** **M1** **A1** **M1** **A1**	√M1 incorrect
(ii)	$(2-3x)^5=32-240x+720x^2-1080x^3+\ldots$ $2-3x=1.97$ $\Rightarrow x=\frac{2-1.97}{3}=0.01$ Substitute x = 0.01 into the expansion, $(2-3\times0.01)^5=32-240(0.01)+720(0.01)^2-1080(0.01)^3+\ldots$ $(1.97)^5=32-2.4+0.072-0.00108+\ldots$ $(1.97)^5=29.67092$	 **M1** **M1** **A1**	√M1 incorrect

4 It is given that $f'(x) = \sin 3x - \cos\frac{x}{2}$. Given that $f\left(\frac{\pi}{3}\right) = 0$, show that
$$12f''(x) + 3f(x) = 35\cos 3x + 2.$$
[6]

Commented [1]:

Q	Solutions	Marks	Remarks
4	$f'(x) = \sin 3x - \cos\frac{x}{2}$ $f(x) = \int \sin 3x - \cos\frac{x}{2}\,dx$		
	$f(x) = -\frac{\cos 3x}{3} - 2\sin\frac{x}{2} + C$	√M1	√M1 without "C"
	$0 = -\frac{\cos 3\left(\frac{\pi}{3}\right)}{3} - 2\sin\frac{\pi}{6} + C$ $0 = -\frac{\cos\pi}{3} - 2\sin\frac{\pi}{6} + C$ $0 = +\frac{1}{3} - 1 + C$		
	$C = \frac{2}{3}$	√M1	
	$f(x) = -\frac{\cos 3x}{3} - 2\sin\frac{x}{2} + \frac{2}{3}$	√A1	
	$f''(x) = 3\cos 3x + \frac{1}{2}\sin\frac{x}{2}$	√M1	
	$12f''(x) + 3f(x)$ $= 36\cos 3x + 6\sin\frac{x}{2} - \frac{3\cos 3x}{3} - 6\sin\frac{x}{2} + 2$	√M1	√M1 due to imperfect answers from above
	$= 35\cos 3x + 2$ (shown)	√AG1	

5 In the diagram, O is the centre of the circle $ABDE$ with diameters AD and BE. CD is a tangent to the circle at D.



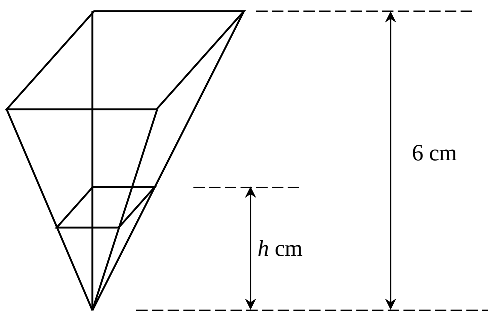
(i) Show that ΔCAD is similar to ΔBED. [2]

(ii) It can be proven that ΔDAB is congruent to ΔBED. Given that $\frac{CD}{BD} = \sqrt{2}$, show that $CD = AD$. [4]

Q	Solutions	Marks	Remarks
5 (i)	$\angle CAD = \angle BED$ (angles in the same segment) $\angle BDE = 90°$ (right angle in semicircle) $\angle CDA = 90°$ (tangent $\perp$ radius) Hence ΔCAD is similar to ΔBED (*AAA* similarity) $\angle BAD = \angle BED$ (angles in the same segment) $\angle DBA = \angle BDE = 90°$ (right angle in semicircle) $AD = DE$ (diameter of circle) Hence, ΔDAB is congruent to ΔBED (*ASA*)	 **M1** **A1**	√M1 incorrect *for practice after exam given in question*

(ii)	ΔCAD is similar to ΔBED ΔDAB is congruent to ΔBED $\Rightarrow \Delta CAD$ is similar to ΔDAB $\frac{CD}{DB} = \frac{CA}{DA} = \sqrt{2}$ $\frac{CA^2}{DA^2} = 2$ $\Rightarrow CA^2 = 2AD^2$ By Pythagoras' Theorem, $CD^2 + AD^2 = CA^2$ $CD^2 + AD^2 = 2AD^2$ $CD^2 = AD^2$ $\Rightarrow CD = AD$ (shown)	**M1** **M1** **M1** **A1**	√M1 incorrect

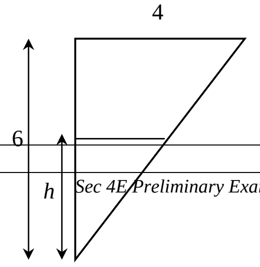
6 The diagram shows a container in the shape of an inverted pyramid. The top of the pyramid is a square base with an area of 64 cm^2. The height of the container is 6 cm.



(i) The container is filled with oil to depth of h cm. Show that the volume, V cm^3, of the empty space in the container is given by $V = \frac{16}{27}(216 - h^3)$. [3]

(ii) Oil is poured into the container and the depth of the oil increases at a constant rate of 0.4 cm/s. Find the rate of decrease of the volume of the empty space in the container when the depth of the oil is 3 cm. [4]

Q	Solutions	Marks	Remarks
6(i)	Side of square top = 8 cm Volume of Pyramid $V = \frac{1}{3}(\text{base area})(\text{height})$ $V = \frac{1}{3}b^2h$ By similar triangles, $\frac{h}{6} = \frac{b}{8}$ $b = \frac{4}{3}h$ **Alternative:** Similar figures; area and volume	**M1**	√M1 incorrect



	$V=\frac{1}{3}\left(\frac{4}{3}\right)^2 h^3$ $V=\frac{1}{3}\left(\frac{16}{9}\right)h^3$ $V=\frac{16}{27}h^3$ Volume of empty space $=\frac{16}{27}\left(6^3-h^3\right)$ $=\frac{16}{27}\left(216-h^3\right)$	**M1** **A1**	
(ii)	$\frac{\mathrm{d}V}{\mathrm{d}h}=-\frac{16}{9}h^2$ $\frac{\mathrm{d}h}{\mathrm{d}t}=0.4$ cm/s $\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}h}\times\frac{\mathrm{d}h}{\mathrm{d}t}$ $\frac{\mathrm{d}V}{\mathrm{d}t}=-\frac{16}{9}h^2\times\frac{2}{5}=-\frac{32}{45}h^2$ when $h=3$ cm, $\frac{\mathrm{d}V}{\mathrm{d}t}=-\frac{32}{45}h^2$ $\frac{\mathrm{d}V}{\mathrm{d}t}=-\frac{32}{45}(3)^2=-\frac{32}{5}$ $\frac{\mathrm{d}V}{\mathrm{d}t}=-6.4\ \text{cm}^3/\text{s}$ Rate of decrease of volume of the empty space in the container when the height is 3 cm is 6.4 cm^3/s.	**M1** **M1** **M1** **A1**	√M1 incorrect

7 (a) Show that the curve $y = \frac{1}{x\ln x}$ has no stationary point if $x > 1$. [3]

(b) The curve $y = \ln(x-k)^3$ crosses the x-axis at $x = 3$. Find the value of k and the equation of the normal to the curve at this point. [5]

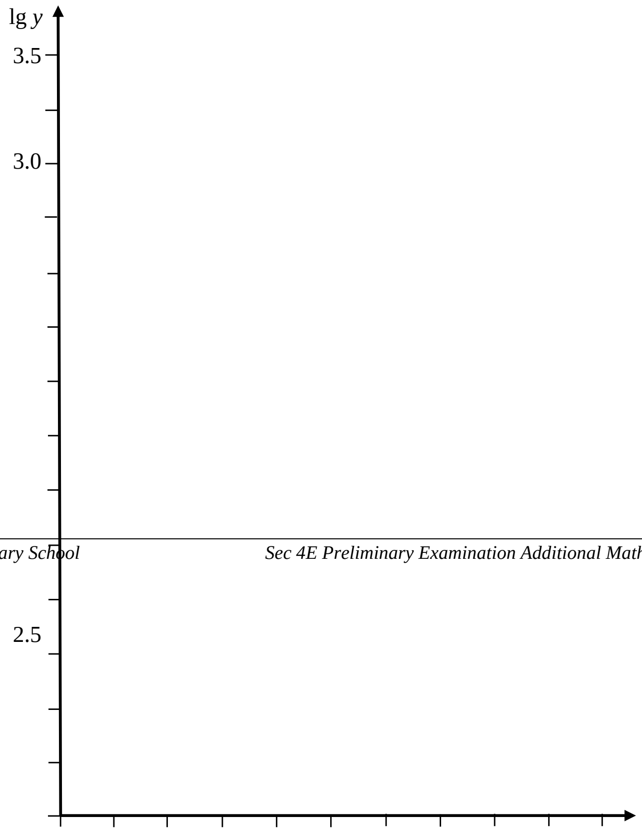
7 (a)	$y = \frac{1}{x\ln x}$ $\frac{dy}{dx} = \frac{0 - \ln x - x\frac{1}{x}}{(x\ln x)^2}$ $\frac{dy}{dx} = \frac{-\ln x - 1}{(x\ln x)^2}$ *or* $-\frac{\ln x + 1}{(x\ln x)^2}$ *or* $-\frac{1+\ln x}{(x\ln x)^2}$ If $x > 1$, $1 + \ln x > 0$ $x\ln x > 0$ $\therefore \frac{dy}{dx} \neq 0$ Hence, there is no stationary point. (shown)	√M1 √M1 √A1	quotient rule applied correctly √M1 for incorrect sign concluding that first derivative not equal to 0 with explanation that $x \ln x > 0$.
(b)	$y = \ln(x-k)^3$ at $x = 3$, $0 = \ln(3-k)^3$ $e^0 = (3-k)^3$ $1 = (3-k)^3$ $\Rightarrow 3 - k = 1$ $k = 2$	√M1 √A1 √M1	Substituting the value of x correct differentiation

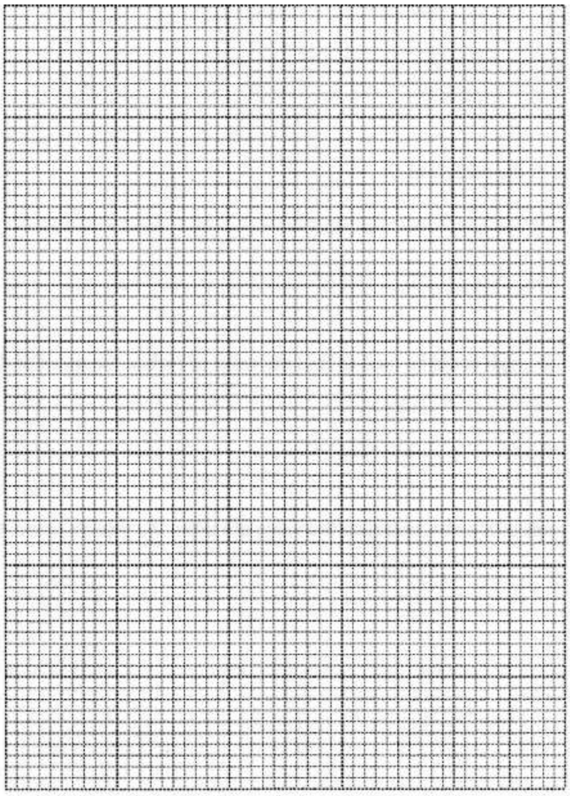
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1\times 3(x-2)^2}{(x-2)^3}=\frac{3}{x-2}$ at $x = 3$, $y = 0$, $\frac{\mathrm{d}y}{\mathrm{d}x}=3$, gradient of normal = $-\frac{1}{3}$ $\frac{y-0}{x-3}=-\frac{1}{3}$ $3y=-x+3$ *or* $y=-\frac{x}{3}+1$ *or* $3y+x-3=0$ (o.e.)	√M1 √A1	Forming equation for line with correct gradient of –1/3.

8 The table shows experimental values of x and y. It is known that the true values of x and y are connected by the equation $y = ax^n$, where a and n are constants.

x	2	3	5	7	8
y	17	47	170	390	540

(i) On the grid below plot lg y against lg x and draw a straight line graph. [2]



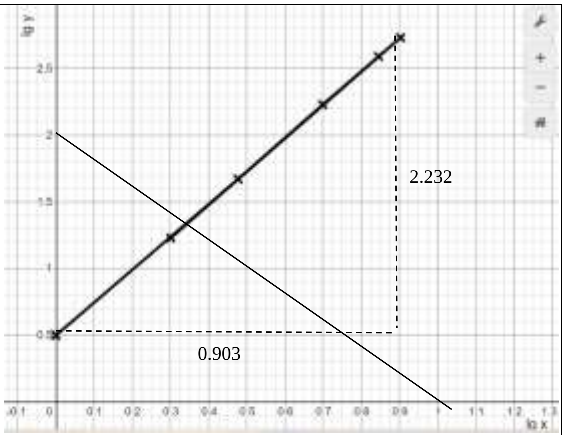


(ii) Using **(i)** above, find the value of a and of n. [4]

(iii) By drawing a suitable straight line on the same axes, solve the equation $ax^{n+2} = 100$. [3]

Q	Solutions	Marks	Remarks
8(i)			√M1 incorrect

x	2	3	5	7	8
y	17	47	170	390	540
lg x	0.301	0.477	0.699	0.845	0.903
lg y	1.230	1.672	2.230	2.591	2.732

	 gradient = $\frac{2.232}{0.903} = 2.472$ $\Rightarrow n = 2.47$ (3sf) y-intercept = 0.5 (accept 0.45 – 0. 55) $\lg a = 0.5$ $a = 10^{0.5} = 3.162$ $a = 3.16$ (3sf)	**B1** **B1** **M1** **A1** **M1** **A1**	all points plotted correct straight line drawn line cuts lg y axis
(ii)	$ax^{n+2} = 100$ $ax^n x^2 = 100$ $ax^n = \frac{100}{x^2} = 100x^{-2}$ $y = 100x^{-2}$ $\lg y = \lg 100x^{-2}$ $\lg y = \lg 100 - 2\lg x$ $\lg y = 2 - 2\lg x$ line drawn on graph, lg x = 0.335 $\Rightarrow x = 10^{0.335} = 2.187 = 2.19$ (3sf)	 **M1** **M1** **A1**	√M1 incorrect product law from graph only

9 **(i)** Given that $y = \ln(\sec x)$, show that $\frac{dy}{dx} = \tan x$. [2]

(ii) Differentiate $x \tan x$ with respect to x. [1]

(iii) Using the results from part **(i)** and part **(ii)**, show that

[6]

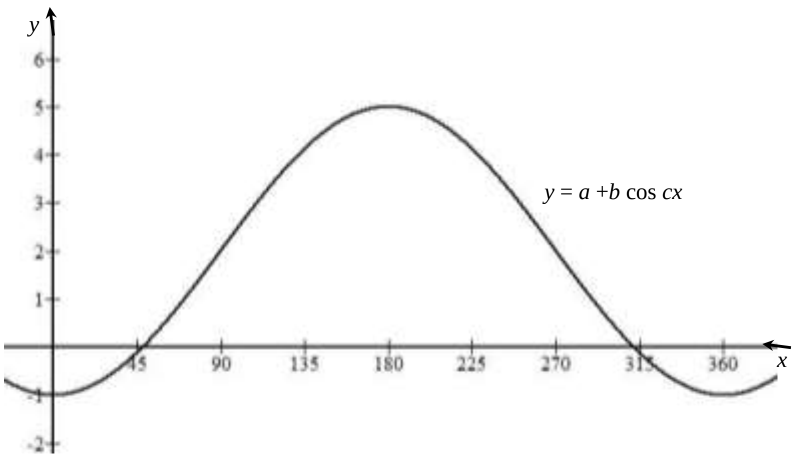
$$\int_0^{\frac{\pi}{4}} 2x\sec^2 x \, dx = \frac{\pi}{2} - \ln 2$$

.

Q	Solutions	Marks	Remarks
9 (i)	$y = \ln(\sec x) = \ln\left(\frac{1}{\cos x}\right)$ $y = \ln 1 - \ln\cos x$ $y = -\ln\cos x$ $\frac{dy}{dx} = -\frac{1}{\cos x}(-\sin x) = \tan x$ (shown) Alternative:	 M1 A1	√M1 incorrect
(ii)	$y = x\tan x$ $\frac{dy}{dx} = \tan x + x\sec^2 x$	B1	√M1 incorrect

(iii)	$\int_0^{\frac{\pi}{4}} x\sec^2 x + \tan x \, dx = [x\tan x]_0^{\frac{\pi}{4}}$ $\int_0^{\frac{\pi}{4}} x\sec^2 x \, dx = [x\tan x]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} \tan x \, dx$ $\int_0^{\frac{\pi}{4}} x\sec^2 x \, dx = \left(\frac{\pi}{4}\tan\frac{\pi}{4} - 0\right) - [\ln\sec x]_0^{\frac{\pi}{4}}$ $\int_0^{\frac{\pi}{4}} x\sec^2 x \, dx = \frac{\pi}{4} - \left[\ln\sec\frac{\pi}{4} - \ln\sec 0\right]$ $\int_0^{\frac{\pi}{4}} x\sec^2 x \, dx = \frac{\pi}{4} - \left[\ln\frac{1}{\cos\frac{\pi}{4}} - \ln\frac{1}{\cos 0}\right]$ $\int_0^{\frac{\pi}{4}} x\sec^2 x \, dx = \frac{\pi}{4} - \left[\ln\sqrt{2} - 0\right]$ $\int_0^{\frac{\pi}{4}} x\sec^2 x \, dx = \frac{\pi}{4} - \ln\sqrt{2}$ $\int_0^{\frac{\pi}{4}} x\sec^2 x \, dx = \frac{\pi}{4} - \ln 2^{\frac{1}{2}}$ $\int_0^{\frac{\pi}{4}} x\sec^2 x \, dx = \frac{\pi}{4} - \frac{1}{2}\ln 2$ Hence, $\int_0^{\frac{\pi}{4}} 2x\sec^2 x \, dx = \frac{\pi}{2} - \ln 2$ (shown)	**M1** **M1** **M1** **M1** **M1** **A1**	

10 The diagram shows the graph of $y = a + b\cos cx$ for $0° \le x \le 360°$.



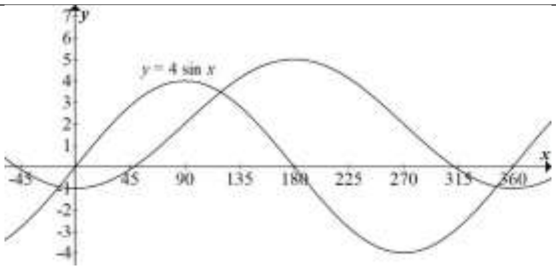
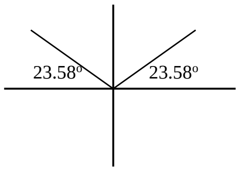
(i) Find the value of *a*, of *b* and of *c*. [3]

(ii) Show that $4\tan x+1=2(\sec x-1)$ can be reduced to the form $4\sin x=a+b\cos cx$. [2]

(iii) On the same axes, sketch the graph of $y=4\sin x$ and determine the number of solutions to the equation $4\tan x+1=2(\sec x-1)$ for $0^\circ\le x\le 360^\circ$. [2]

(iv) Solve, algebraically, $4\tan x+1=2(\sec x-1)$, for $0^\circ\le x\le 360^\circ$. [3]

Q	Solutions	Marks	Remarks
10 (i)	$a=2$ $b=-3$ $c=1$	**B1** **B1** **B1**	√M1 incorrect
(ii)	$4\tan x+1=2(\sec x-1)$ $\frac{4\sin x}{\cos x}+1=\frac{2}{\cos x}-2$ $4\sin x+\cos x=2-2\cos x$ $4\sin x=2-3\cos x$ (shown)	**M1** **A1**	√M1 their *a*, *b* , *c*
(iii)		**M1**	accept 

	 The solutions will be the intersection of the two curves. Number of solutions = 2	**A1**	
(iv)	$4\tan x + 1 = 2(\sec x - 1)$ $4\sin x = 2 - 3\cos x$ $4\sin x + 3\cos x = 2$ Let $4\sin x\ +\ 3\cos x\ = R(\sin x + \alpha)$ $R\cos x = 4$ $R\sin x = 3$ $\Rightarrow R = 5 \quad \tan\alpha = \frac{3}{4} \Rightarrow \alpha = 36.87°$ $\therefore 4\sin x + 3\cos x = 5\sin(x + 36.9°)$ (1 dp) $4\sin x + 3\cos x = 5\sin(x + 36.87°) = 2$ $\sin(x + 36.87°) = 0.4$ basic angle, $\alpha = 23.58°$  $x + 36.87 = 156.42°, \quad 383.58°$ $x = 119.55°, 346.71°$ $x = 119.6°, 346.7°$ (1 dp)	**M1** **M1** **A1**	

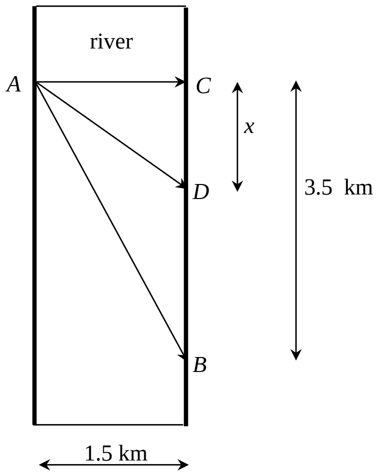
11 A segment of a competition that you are participating in requires you to cross a river from point *A* to point *B* as shown in the diagram. You are to launch a boat from point *A* on a bank of the straight river, 1.5 km wide. The destination is the point *B*, 3.5 km downstream on the opposite bank. To reach your destination, you have three options:

Option 1 row the boat horizontally across the river to point *C* and then run to *B*

Option 2 row the boat directly to *B*

Option 3 row to some point *D* between *C* and *B* and then run to *B*

It is known that you can row at a speed of 6 km/h and run at a speed of 8 km/h.



(i) If the distance between *CD* is *x* km, show that the total time, *T*, in hours, taken to reach the destination by both rowing and running is given by the equation

$$T = \frac{\sqrt{9+4x^2}}{12} + \frac{7-2x}{16}.$$ [2]

(ii) By stating and substituting an appropriate value of *x* into the expression *T* found in **(i)** find the time taken if

(a) option 1 is chosen,

(b) option 2 is chosen.

[2]

(iii) Show that if option 3 is used, to reach the destination, point B, as soon as possible, $x=\frac{9\sqrt{7}}{14}$.
Hence, determine the total time taken, T hours, if this option is used.
Given : $\frac{\mathrm{d}^2T}{\mathrm{d}x^2}=\frac{3}{\left(9+4x^2\right)^{\frac{3}{2}}}$. [5]

(iv) Hence, which option would you choose in order to reach point B from A as quickly as possible? Justify your answer. [1]

(v) State one assumption that you made in the calculations above. [1]

Q	Solutions	Marks	Remarks
11 (i)	By Pythagoras' Theorem, $AD^2=\left(\frac{3}{2}\right)^2+x^2$ $AD=\sqrt{\frac{9+4x^2}{4}}$ time taken , T = time taken for rowing + time taken for running time taken, T, in hours $T=\frac{\sqrt{\frac{9+4x^2}{4}}}{6}+\frac{\frac{7}{2}-x}{8}$ $T=\frac{\sqrt{9+4x^2}}{12}+\frac{7-2x}{16}$ (shown)	M1 A1	√M1 incorrect
(ii)	**(a)** option 1 is when he rows to C, $x=0$ $T=\frac{3}{12}+\frac{7}{16}=\frac{11}{16}=0.6875$ hours **(b)** option 2 is when he rows all the way to B, $x=3.5$ $T=\frac{\sqrt{58}}{12}=0.6346=0.635$ hours (3 sf)	B1 B1	√M1 incorrect

(iii)	$T=\dfrac{\sqrt{9+4x^2}}{12}+\dfrac{7-2x}{16}$ $\dfrac{dT}{dx}=\dfrac{\frac{1}{2}(9+4x^2)^{-\frac{1}{2}}(8x)}{12}-\dfrac{2}{16}$ $\dfrac{dT}{dx}=\dfrac{x}{3\sqrt{(9+4x^2)}}-\dfrac{1}{8}$ to reach as soon as possible, $\dfrac{dT}{dx}=0$ $\dfrac{x}{3\sqrt{(9+4x^2)}}=\dfrac{1}{8}$ $\Rightarrow x=\dfrac{3}{8}\sqrt{(9+4x^2)}$ $64x^2=81+36x^2$ $28x^2=81\Rightarrow x=\sqrt{\dfrac{81}{28}}=\dfrac{9\sqrt{7}}{14}$ km (shown) $\dfrac{dT}{dx}=\dfrac{x}{3\sqrt{(9+4x^2)}}-\dfrac{1}{8}$ $\dfrac{d^2T}{dx^2}=\dfrac{1(9+4x^2)^{\frac{1}{2}}-x\left(\frac{1}{2}\right)(9+4x^2)^{-\frac{1}{2}}(8x)}{3(9+4x^2)}$ $\dfrac{d^2T}{dx^2}=\dfrac{(9+4x^2)^{-\frac{1}{2}}\left[(9+4x^2)-4x^2\right]}{3(9+4x^2)}$ $\dfrac{d^2T}{dx^2}=\dfrac{3}{(9+4x^2)^{\frac{3}{2}}}$ If $x=\dfrac{9\sqrt{7}}{14}$ km, $\dfrac{d^2T}{dx^2}=\dfrac{3}{(9+4x^2)^{\frac{3}{2}}}>0\Rightarrow$ minimum T	M1 M1 A1 B1	Students need not do this part, it is given in question. Just need to state it is minimum as second derivation greater than 0. only this part needed just need to state. No need to differentiate again.

	$= \dfrac{\sqrt{9+4\left(\dfrac{9\sqrt{7}}{14}\right)^2}}{12} + \dfrac{7-2\left(\dfrac{9\sqrt{7}}{14}\right)}{16}$ $T = \dfrac{\sqrt{\dfrac{144}{7}}}{12} + \dfrac{7-\left(\dfrac{9\sqrt{7}}{7}\right)}{16}$ $T = 0.3780 + 0.2249$ $T = 0.6029$ $T = 0.603$ hours (3sf)	**A1**	
(iv)	Option 1 takes 0.6875 hours Option 2 takes 0.635 hours Option 3 takes 0.603 hours Therefore option 3 is the best option to from point *A* to point *B* in the shortest possible time.	**B1**	
(v)	Assumptions suggested answers: - no wind to affect course of rowing - no detours for any reason - the courses are all in a straight line - constant rowing / running speeds - the river banks on both sides are parallel (width of river is consistent) time in getting on and off boat are negligible	**B1**	any one

-END-

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