2023 VJC H2 Math Promo Solutions

1 (a)

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}x}\tan^{-1}\sqrt{x^2-1} &= \frac{1}{1+\left(\sqrt{x^2-1}\right)^2}\left(\frac{1}{2}\right)\left(x^2-1\right)^{-\frac{1}{2}}(2x)\\ &= \frac{1}{1+x^2-1}\frac{x}{\sqrt{x^2-1}}\\ &= \frac{1}{x\sqrt{x^2-1}}\end{aligned}$$

(b)

$$y = x^{\cos x}$$

$$\ln y = \cos x \ln x$$

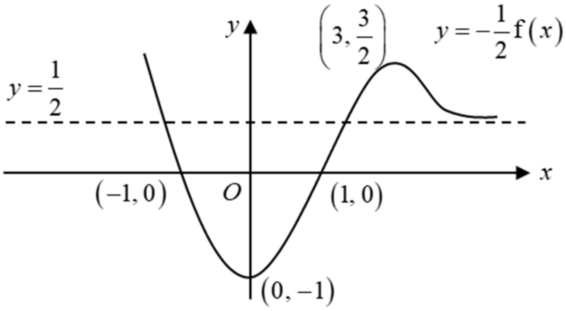
$$\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x} = \cos x\frac{1}{x} + \ln x(-\sin x)$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = x^{\cos x}\left(\frac{\cos x}{x} - \sin x \ln x\right)$$

2

(1) $y=\mathrm{f}(x) \xrightarrow{y \text{ replaced by } -y} y=-\mathrm{f}(x)$

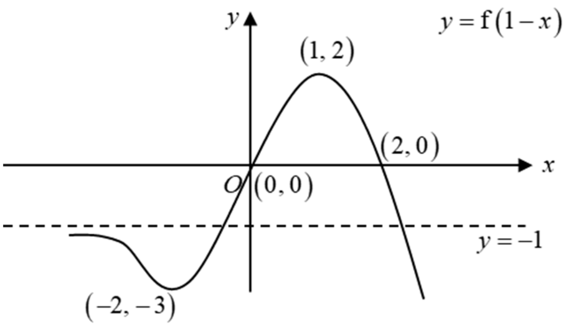
(2) $y=-\mathrm{f}(x) \xrightarrow{y \text{ replaced by } \frac{1}{2}y} y=-\frac{1}{2}\mathrm{f}(x)$



(b) $y=\mathrm{f}(1-x)$.

(1) $y=\mathrm{f}(x) \xrightarrow{x \text{ replaced by } x+1} y=\mathrm{f}(x+1)$

(2) $y=\mathrm{f}(x+1) \xrightarrow{x \text{ replaced by } -x} y=\mathrm{f}(1-x)$



3

$$\frac{ax-a+2}{(x-a)(1-x)} \le 1$$

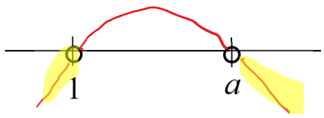
$$\frac{ax-a+2-(x-a)(1-x)}{(x-a)(1-x)} \le 0$$

$$\frac{ax-a+2-x+a+x^2-ax}{(x-a)(1-x)} \le 0$$

$$\frac{x^2-x+2}{(x-a)(1-x)} \le 0$$

$$\frac{\left(x-\frac{1}{2}\right)^2+\frac{7}{4}}{(x-a)(1-x)} \le 0$$

Since $\left(x-\frac{1}{2}\right)^2+\frac{7}{4} \ge \frac{7}{4} > 0$ for all $x \in \mathbb{R}$, $(x-a)(1-x) < 0$.



$x < 1$ or $x > a$

For $\frac{a|x|-a+2}{(|x|-a)(1-|x|)} \le 1$, replace x with $|x|$ in the result above to get the following.

$|x| < 1$ or $|x| > a$

$-1 < x < 1$ or $x < -a$ or $x > a$

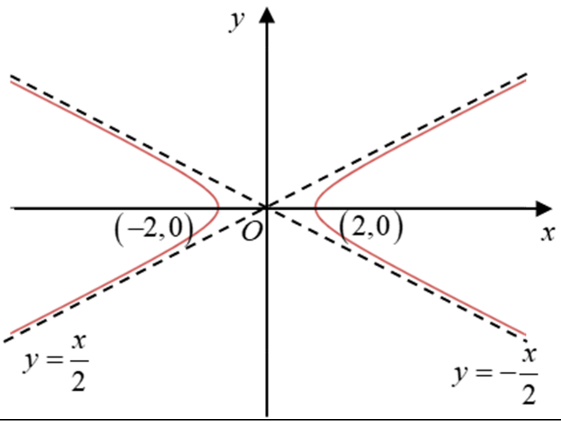
$\{x \in \mathbb{R} :\ x < -a \text{ or } -1 < x < 1 \text{ or } x > a\}$

4

$$\frac{\mathrm{d}}{\mathrm{d}x}\mathrm{e}^{\tan x}=\sec^2 x\ \mathrm{e}^{\tan x}$$

$$\begin{aligned}
&\int \sec^4 x\,\mathrm{e}^{\tan x}\,\mathrm{d}x\\
&=\int\left(\sec^2 x\right)\sec^2 x\,\mathrm{e}^{\tan x}\,\mathrm{d}x\\
&=\sec^2 x\cdot\mathrm{e}^{\tan x}-\int \mathrm{e}^{\tan x}\cdot 2\sec x\sec x\tan x\,\mathrm{d}x\\
&=\sec^2 x\cdot\mathrm{e}^{\tan x}-2\int \mathrm{e}^{\tan x}\cdot\sec^2 x\cdot\tan x\,\mathrm{d}x\\
&=\sec^2 x\cdot\mathrm{e}^{\tan x}-2\left[\left(\mathrm{e}^{\tan x}\right)\cdot\tan x-\int \mathrm{e}^{\tan x}\cdot\sec^2 x\,\mathrm{d}x\right]\\
&=\mathrm{e}^{\tan x}\sec^2 x-2\mathrm{e}^{\tan x}\tan x+2\mathrm{e}^{\tan x}+C\\
&=\mathrm{e}^{\tan x}\left(\sec^2 x-2\tan x+2\right)+C
\end{aligned}$$

5



$$x^2 - 4y^2 = 4 \xrightarrow{\text{Replace } y \text{ with } \frac{y}{2}} x^2 - y^2 = 4$$

$$x^2 - y^2 = 4 \xrightarrow{\text{Replace } x \text{ with } x-2} (x-2)^2 - y^2 = 4$$

(1) Scale the graph of C by a factor of 2, parallel to the y-axis, followed by
(2) translating the resultant graph by 2 units in the positive x-direction.

6

$$\int \frac{3-2x}{\sqrt{5+4x-x^2}}\,\mathrm{d}x = \int \frac{4-2x-1}{\sqrt{5+4x-x^2}}\,\mathrm{d}x$$

$$= \int \frac{4-2x}{\sqrt{5+4x-x^2}} - \frac{1}{\sqrt{9-(x-2)^2}}\ \mathrm{d}x$$

$$= 2\sqrt{5+4x-x^2} - \sin^{-1}\left(\frac{x-2}{3}\right) + C$$

$$\int_{\sqrt{2}}^{4} |x-3|\ \mathrm{d}x = \int_{\sqrt{2}}^{3} -(x-3)\ \mathrm{d}x + \int_{3}^{4} x-3\ \mathrm{d}x$$

$$= -\left[\frac{x^2}{2}-3x\right]_{\sqrt{2}}^{3} + \left[\frac{x^2}{2}-3x\right]_{3}^{4}$$

$$= -\left(\frac{9}{2}-9-1+3\sqrt{2}\right) + \left(8-12-\frac{9}{2}+9\right)$$

$$= 6-3\sqrt{2}$$

$$\int_{\frac{\sqrt{2}}{2}}^{\frac{\sqrt{3}}{2}} \frac{1+x^2}{\sqrt{1-x^2}}\ \mathrm{d}x \qquad\qquad x=\sin u \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}u}=\cos u$$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{1+\sin^2 u}{\sqrt{1-\sin^2 u}} \times \cos u\ \mathrm{d}u$$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{1+\sin^2 u}{|\cos u|} \times \cos u\ \mathrm{d}u$$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{1+\sin^2 u}{\cos u} \times \cos u\ \mathrm{d}u \qquad \left(\because \frac{\pi}{4} < u < \frac{\pi}{3} \Rightarrow \cos u > 0\right)$$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} 1+\frac{1-\cos 2u}{2}\ \mathrm{d}u = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{3-\cos 2u}{2}\ \mathrm{d}u$$

$$= \left[\frac{3}{2}u - \frac{\sin 2u}{4}\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$$

$$= \frac{3}{2}\left(\frac{\pi}{3}-\frac{\pi}{4}\right) - \frac{1}{4}\left(\sin\frac{2\pi}{3} - \sin\frac{\pi}{2}\right)$$

$$= \frac{\pi}{8} - \frac{\sqrt{3}}{8} + \frac{1}{4}$$

7

Since all the coefficients are real, $1-\sqrt{2}\,\mathrm{i}$ is also a root.

$$\begin{aligned}\text{Quadratic factor} &= \left[x-\left(1-\sqrt{2}\,\mathrm{i}\right)\right]\left[x-\left(1+\sqrt{2}\,\mathrm{i}\right)\right]\\ &= (x-1)^2-2(-1)\\ &= x^2-2x+3\end{aligned}$$

Let $3x^3+px^2+qx+3=\left(x^2-2x+3\right)(ax+b)$.

By inspection or by comparing coefficient of x^3 and constant term, $a=3$ and $b=1$.

$3x^3+px^2+qx+3=\left(x^2-2x+3\right)(3x+1)$

By comparing coefficients,

$p=1-6=-5$

$q=-2+9=7$

$3x^3+px^2+qx+3=\left(x^2-2x+3\right)(3x+1)=0$

Other than $1+\sqrt{2}\,\mathrm{i}$, the other roots are $1-\sqrt{2}\,\mathrm{i}$ and $-\dfrac{1}{3}$.

Alternative

$3\left(1+\sqrt{2}\mathrm{i}\right)^3+p\left(1+\sqrt{2}\mathrm{i}\right)^2+q\left(1+\sqrt{2}\mathrm{i}\right)+3=0$

$3\left(1+3\sqrt{2}\mathrm{i}-6-2\sqrt{2}\mathrm{i}\right)+p\left(1+2\sqrt{2}\mathrm{i}-2\right)+q\left(1+\sqrt{2}\mathrm{i}\right)+3=0$

$(-12-p+q)+(3+2p+q)\sqrt{2}\mathrm{i}=0$

Comparing real and imaginary parts,

$$\begin{cases}-12-p+q=0\\ 3+2p+q=0\end{cases}\Rightarrow\begin{cases}-p+q=12\\ 2p+q=-3\end{cases}\Rightarrow p=-5,\ q=7.$$

Since all the coefficients are real, $1-\sqrt{2}\,\mathrm{i}$ is also a root.

$$\begin{aligned}\text{Quadratic factor} &= \left[x-\left(1-\sqrt{2}\,\mathrm{i}\right)\right]\left[x-\left(1+\sqrt{2}\,\mathrm{i}\right)\right]\\ &= (x-1)^2-2(-1)\\ &= x^2-2x+3\end{aligned}$$

$3x^3-5x^2+7x+3=0$

$\left(x^2-2x+3\right)(3x+1)=0$

$x=1\pm 2\mathrm{i}$ or $x=-\dfrac{1}{3}$

Other than $1+\sqrt{2}\,\mathrm{i}$, the other roots are $1-\sqrt{2}\,\mathrm{i}$ and $-\dfrac{1}{3}$.

$w + z^* = 2 - \mathrm{i} \quad \Rightarrow \quad z^* = 2 - \mathrm{i} - w$

$$w(2 - \mathrm{i} - w) = 3 - \mathrm{i}$$
$$w^2 - (2 - \mathrm{i})w + (3 - \mathrm{i}) = 0$$
$$w = \frac{2 - \mathrm{i} \pm \sqrt{(2 - \mathrm{i})^2 - 4(3 - \mathrm{i})}}{2}$$
$$= \frac{2 - \mathrm{i} \pm \sqrt{4 - 4\mathrm{i} - 1 - 12 + 4\mathrm{i}}}{2}$$
$$= \frac{2 - \mathrm{i} \pm \sqrt{-9}}{2}$$
$$= \frac{2 - \mathrm{i} \pm 3\mathrm{i}}{2}$$
$w = 1 - 2\mathrm{i}$ or $w = 1 + \mathrm{i}$

When $w = 1 - 2\mathrm{i}$:
$z^* = 2 - \mathrm{i} - (1 - 2\mathrm{i}) = 1 + \mathrm{i}$
$z = 1 - \mathrm{i}$
(reject since it's given that $|w| < |z|$)

When $w = 1 + \mathrm{i}$:
$z^* = 2 - \mathrm{i} - (1 + \mathrm{i}) = 1 - 2\mathrm{i}$
$z = 1 + 2\mathrm{i}$

Hence, $w = 1 + \mathrm{i}$ and $z = 1 + 2\mathrm{i}$.

8

$a \quad +b \quad +c = \ln 3$

$2.25a + 1.5b + c = \ln 5.5$

$4a \quad +2b \quad +c = \ln 9$

By GC, $a = -0.227$, $b = 1.78$ and $c = -0.455$ (to 3 s.f.).

$$\mathrm{f}(x) = \ln\left(1+2x^2\right) = 2x^2 - \frac{1}{2}\left(2x^2\right)^2 + \frac{1}{3}\left(2x^2\right)^3 + \ldots = 2x^2 - 2x^4 + \frac{8}{3}x^6 + \ldots$$

The expansion is valid when $-1 < 2x^2 \le 1$.

$2x^2 \le 1 \qquad \left(\because 2x^2 \ge 0 \text{ for all } x \in \mathbb{R}\right)$

$2x^2 - 1 \le 0$

$\left(\sqrt{2}x - 1\right)\left(\sqrt{2}x + 1\right) \le 0$

$\left\{x \in \mathbb{R} : -\frac{1}{\sqrt{2}} \le x \le \frac{1}{\sqrt{2}}\right\}$

$\ln\left(1+2x^2\right) \approx 2x^2 - 2x^4 + \frac{8}{3}x^6$

Substitute $x^2 = \frac{1}{6}$:

$\ln\left[1 + 2\left(\frac{1}{6}\right)\right] \approx 2\left(\frac{1}{6}\right) - 2\left(\frac{1}{6}\right)^2 + \frac{8}{3}\left(\frac{1}{6}\right)^3$

$\ln\left(\frac{4}{3}\right) \approx \frac{47}{162}$

$\therefore m = 47$

$$\ln\left(\frac{1+2x^2}{1+2x+x^2}\right) = \ln\left(1+2x^2\right) - \ln\left(1+x\right)^2 = \ln\left(1+2x^2\right) - 2\ln\left(1+x\right)$$

$$= \left(2x^2 - 2x^4 + \ldots\right) - 2\left(x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots\right) = -2x + 3x^2 - \frac{2}{3}x^3 - \frac{3}{2}x^4 + \ldots$$

__Alternative__

$$\ln\left(\frac{1+2x^2}{1+2x+x^2}\right) = \ln\left(1+2x^2\right) - \ln\left(1+2x+x^2\right)$$

$$= \left(2x^2 - 2x^4 + \ldots\right) - \left(\left(2x+x^2\right) - \frac{\left(2x+x^2\right)^2}{2} + \frac{\left(2x+x^2\right)^3}{3} - \frac{(2x)^4}{4} + \ldots\right)$$

$$= 2x^2 - 2x^4 - 2x - x^2 + \frac{4x^2 + 4x^3 + x^4}{2} - \frac{8x^3 + 3\left(4x^4\right)}{3} + 4x^4 + \ldots = -2x + 3x^2 - \frac{2}{3}x^3 - \frac{3}{2}x^4 + \ldots$$

9

$\alpha = -\sqrt{3} + \mathrm{i} = 2\mathrm{e}^{\frac{5\pi}{6}\mathrm{i}}$ and $\beta = \cos\frac{\pi}{3} + \mathrm{i}\sin\frac{\pi}{3} = \mathrm{e}^{\frac{\pi}{3}\mathrm{i}}$

$$\left|\frac{\alpha^5}{\beta^*}\right| = \frac{|\alpha|^5}{|\beta^*|} = \frac{2^5}{1} = 32$$

$$5\arg(\alpha) - \arg(\beta^*) = 5\left(\frac{5\pi}{6}\right) - \left(-\frac{\pi}{3}\right) = \frac{27\pi}{6} = \frac{9\pi}{2}$$

$$\arg\left(\frac{\alpha^5}{\beta^*}\right) = \frac{\pi}{2}$$

$$\frac{\alpha^5}{\beta^*} = 32\left(\cos\frac{\pi}{2} + \mathrm{i}\sin\frac{\pi}{2}\right)$$

Alternative

$\alpha = -\sqrt{3} + \mathrm{i} = 2\mathrm{e}^{\frac{5\pi}{6}\mathrm{i}}$ and $\beta = \cos\frac{\pi}{3} + \mathrm{i}\sin\frac{\pi}{3} = \mathrm{e}^{\frac{\pi}{3}\mathrm{i}}$

$$\begin{aligned}\frac{\alpha^5}{\beta^*} &= \frac{\left(2\mathrm{e}^{\frac{5\pi}{6}\mathrm{i}}\right)^5}{\mathrm{e}^{-\frac{\pi}{3}\mathrm{i}}} \\ &= \frac{32\mathrm{e}^{\frac{25\pi}{6}\mathrm{i}}}{\mathrm{e}^{-\frac{\pi}{3}\mathrm{i}}} \\ &= \frac{32\mathrm{e}^{\frac{\pi}{6}\mathrm{i}}}{\mathrm{e}^{-\frac{\pi}{3}\mathrm{i}}} \qquad \left(\because \mathrm{e}^{\frac{25\pi}{6}\mathrm{i}} = \mathrm{e}^{\frac{25\pi}{6}\mathrm{i} - 4\pi\mathrm{i}}\right) \\ &= 32\mathrm{e}^{\frac{\pi}{2}\mathrm{i}} \\ &= 32\left(\cos\frac{\pi}{2} + \mathrm{i}\sin\frac{\pi}{2}\right)\end{aligned}$$

9b	$z = \cos\theta + \mathrm{i}\sin\theta = \mathrm{e}^{\mathrm{i}\theta} \quad \Rightarrow \quad z^* = \mathrm{e}^{-\mathrm{i}\theta}$ $\frac{1}{1+z^2} = \frac{1}{1+\mathrm{e}^{2\theta\mathrm{i}}}$ $= \frac{1}{\mathrm{e}^{\mathrm{i}\theta}\left(\mathrm{e}^{-\mathrm{i}\theta}+\mathrm{e}^{\mathrm{i}\theta}\right)}$ $= \frac{\mathrm{e}^{-\mathrm{i}\theta}}{\left(\mathrm{e}^{-\mathrm{i}\theta}+\mathrm{e}^{\mathrm{i}\theta}\right)}$ $= \frac{z^*}{2\cos\theta} \qquad \left(\because z^*+z = 2\,\mathrm{Re}(z) = 2\cos\theta\right)$ $\left\lvert\frac{1}{1+z^2}\right\rvert = \frac{\lvert z^*\rvert}{\lvert 2\cos\theta\rvert} = \frac{1}{2\cos\theta} \qquad \left(-\frac{\pi}{2} < \theta < \frac{\pi}{2} \Rightarrow 2\cos\theta > 0\right)$ $\arg\left(\frac{1}{1+z^2}\right) = \arg\left(\frac{z^*}{2\cos\theta}\right) = \arg(z^*) = -\arg(z) = -\theta$ *<u>Alternative</u>* $\frac{1}{1+z^2} = \frac{1}{1+(\cos\theta+\mathrm{i}\sin\theta)^2}$ $= \frac{1}{1+\cos^2\theta+2\mathrm{i}\sin\theta\cos\theta-\sin^2\theta}$ $= \frac{1}{2\cos^2\theta+2\mathrm{i}\sin\theta\cos\theta} \qquad \left(\because 1-\sin^2\theta = \cos^2\theta\right)$ $= \frac{1}{2\cos\theta(\cos\theta+\mathrm{i}\sin\theta)}$ $= \frac{\cos\theta-\mathrm{i}\sin\theta}{2\cos\theta(\cos\theta+\mathrm{i}\sin\theta)(\cos\theta-\mathrm{i}\sin\theta)}$ $= \frac{\cos\theta-\mathrm{i}\sin\theta}{2\cos\theta\left(\cos^2\theta+\sin^2\theta\right)}$ $= \frac{z^*}{2\cos\theta} \qquad \left(\because \cos^2\theta+\sin^2\theta = 1\right)$ $\left\lvert\frac{1}{1+z^2}\right\rvert = \frac{\lvert z^*\rvert}{\lvert 2\cos\theta\rvert} = \frac{1}{2\cos\theta} \qquad \left(-\frac{\pi}{2} < \theta < \frac{\pi}{2} \Rightarrow 2\cos\theta > 0\right)$ $\arg\left(\frac{1}{1+z^2}\right) = \arg\left(\frac{z^*}{2\cos\theta}\right) = \arg(z^*) = -\arg(z) = -\theta$

10

Since C does not cut the x-axis, $\Rightarrow y \neq 0$

$$\Rightarrow \frac{3x^2+ax+3}{x+1} \neq 0$$

$\Rightarrow 3x^2+ax+3 \neq 0$

i.e. $3x^2+ax+3=0$ has no real solutions.

Discriminant < 0

$a^2-4(3)(3)<0$

$(a-6)(a+6)<0$

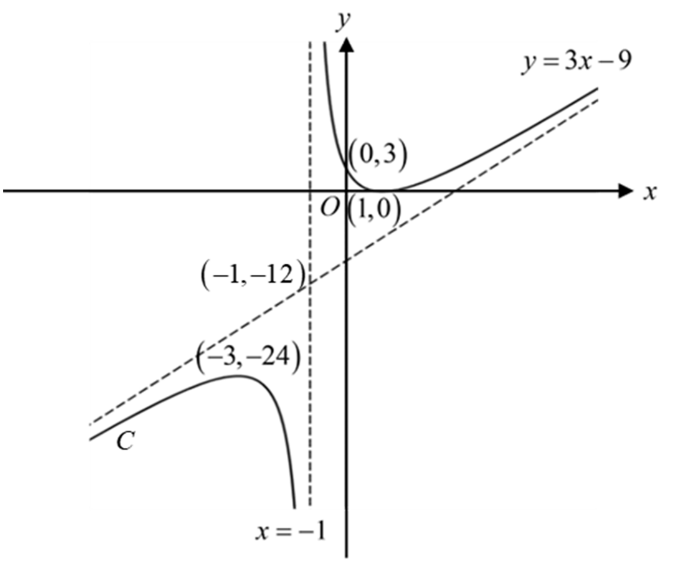
$-6<a<6$

$\{a \in \mathbb{R} : -6<a<6\}$

$$y=\frac{3x^2-6x+3}{x+1}=3x-9+\frac{12}{x+1}$$

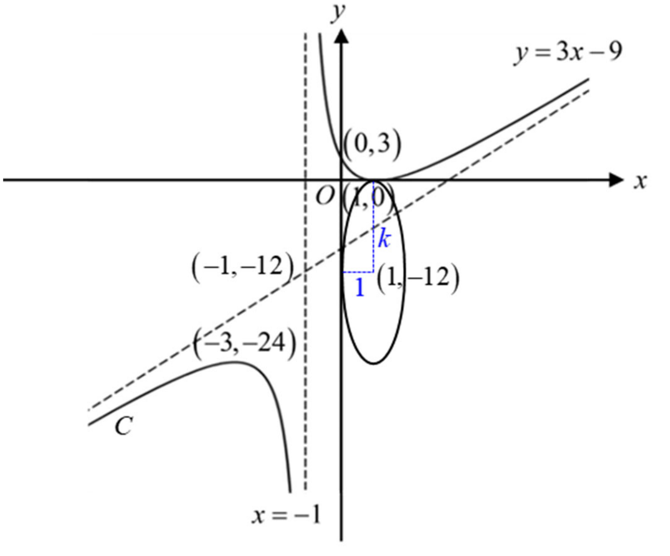
Equations of asymptotes: $y=3x-9, \quad x=-1$

Intercepts: $(0,3), \quad (1,0)$



$$k^2(x-1)^2+\left(\frac{3x^2-6x+3}{x+1}+12\right)^2=k^2$$

Sketch $(x-1)^2+\frac{(y+12)^2}{k^2}=1$ in the same diagram as C.



From the sketch, $k > 12$.

11

$$\left(\frac{1}{2}\right)\left(\frac{4}{3}\pi r^3\right)+\pi r^2 h=108\pi$$

$$\frac{2}{3}r^3+r^2h=108$$

$$h=\frac{108-\frac{2}{3}r^3}{r^2}=\frac{108}{r^2}-\frac{2}{3}r$$

$$A=\frac{1}{2}\left(4\pi r^2\right)+2\pi rh+\pi r^2$$

$$=3\pi r^2+2\pi r\left(\frac{108}{r^2}-\frac{2}{3}r\right)$$

$$=3\pi r^2+\frac{216\pi}{r}-\frac{4\pi}{3}r^2$$

$$=\frac{5\pi}{3}r^2+\frac{216\pi}{r}$$

$$=\left(\frac{5r^2}{3}+\frac{216}{r}\right)\pi$$

$$A=\left(\frac{5r^2}{3}+\frac{216}{r}\right)\pi$$

$$\frac{\mathrm{d}A}{\mathrm{d}r}=\left(\frac{10r}{3}-\frac{216}{r^2}\right)\pi$$

For stationary value of A, $\frac{\mathrm{d}A}{\mathrm{d}r}=0$.

$$\frac{10r}{3}-\frac{216}{r^2}=0$$

$$10r^3=648$$

$$r^3=\frac{324}{5}$$

$$r=\sqrt[3]{\frac{324}{5}}$$

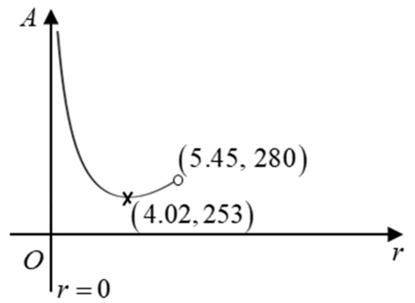
$$\frac{\mathrm{d}A}{\mathrm{d}r}=\left(\frac{10r}{3}-\frac{216}{r^2}\right)\pi$$

$$\frac{\mathrm{d}^2A}{\mathrm{d}r^2}=\left(\frac{10}{3}+\frac{432}{r^3}\right)\pi$$

When $r=\sqrt[3]{\frac{324}{5}}$, $\frac{\mathrm{d}^2A}{\mathrm{d}r^2}=10\pi=31.416>0$. [*OR evaluate by GC*]

Hence, A is minimum when $r=\sqrt[3]{\frac{324}{5}}$.

When $h=\frac{108-\frac{2}{3}r^3}{r^2}=0$, $r=\sqrt[3]{\frac{324}{2}}\approx 5.4514$.



After 14 seconds, amount of liquid leaked is 70 cm^3.
Total volume of hemisphere
$=\frac{2}{3}\pi(3)^3=18\pi=56.549<70$.
After 14 seconds, the remaining liquid lies within the cylinder.

Since the cylinder has a uniform cross-section, rate of decrease of height is $\frac{5}{\pi(3)^2}=\frac{5}{9\pi}\approx 0.177$ cm per second.

<u>*Alternative*</u>
After 14 seconds, amount of liquid leaked is 70 cm^3.
Total volume of hemisphere
$=\frac{2}{3}\pi(3)^3=18\pi=56.549<70$.
Let V cm^3 be the volume of the liquid in the cylinder, and l cm be the height of the liquid.
$V=\pi r^2 l=9\pi l\Rightarrow\frac{\mathrm{d}V}{\mathrm{d}l}=9\pi$
$\frac{\mathrm{d}l}{\mathrm{d}t}=\frac{\mathrm{d}l}{\mathrm{d}V}\times\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{1}{9\pi}\times(-5)=-\frac{5}{9\pi}\approx -0.177$
Height of liquid decreases at 0.177 cm per second.

12

$$A_n = \left(\frac{1}{2^n}\right)^2 = \frac{1}{2^{2n}} = \frac{1}{4^n}$$

Total area of new squares added in stage $n = \frac{1}{4^n} \times 4 \times 3^{n-1} = \left(\frac{3}{4}\right)^{n-1}$

Total area in stage $n = 1 + \left(\frac{3}{4}\right)^0 + \left(\frac{3}{4}\right)^1 + \left(\frac{3}{4}\right)^2 + \ldots + \left(\frac{3}{4}\right)^{n-1}$

$$= 1 + \frac{\left(\frac{3}{4}\right)^0 \left[1 - \left(\frac{3}{4}\right)^n\right]}{1 - \frac{3}{4}}$$

$$= 1 + 4\left[1 - \left(\frac{3}{4}\right)^n\right]$$

$$= 5 - 4\left(\frac{3}{4}\right)^n$$

As $n \to \infty$, $\left(\frac{3}{4}\right)^n \to 0$

$5 - 4\left(\frac{3}{4}\right)^n \to 5$

Hence, area of the square fractal converges.

Area of square fractal is 5.

$5 - 4\left(\frac{3}{4}\right)^n > 0.9(5)$

$\left(\frac{3}{4}\right)^n < \frac{1}{8}$

$n > \frac{\ln\frac{1}{8}}{\ln\frac{3}{4}} = 7.23$

Hence, $m = 8$.

Total number of squares $= 1 + \sum_{n=1}^{8}\left(4 \times 3^{n-1}\right) = 13121$