TANJONG KATONG GIRLS' SCHOOL
PRELIMINARY EXAMINATION
SECONDARY FOUR EXPRESS

CANDIDATE NAME

CLASS	E	INDEX NUMBER	

ADDITIONAL MATHEMATICS 4049/01

PAPER 1

14 August 2023

2 hour 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 90.

For Examiner's use

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formula for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\Delta = \tfrac{1}{2}bc\sin A$$

1 A right circular cylinder has a volume of $\left(6+2\sqrt{3}\right)\pi$ cm^3 and a base radius of $\left(1+\sqrt{3}\right)$ cm. Find, **without using a calculator**, the height of the cylinder, in cm, in the form $\left(a+b\sqrt{3}\right)$, where a and b are integers. [3]

2 The curve $y = \dfrac{5}{x} + 3$ and the line $x - 2y - 3 = 0$ intersect at the points P and Q.
Find the coordinates of P and of Q. [3]

3 Express $3-3x-2x^2$ in the form $a(x+b)^2+c$ and hence state the coordinates of the turning point of the curve $y=3-3x-2x^2$. [4]

4 Integrate $\frac{8}{2x-1}+\frac{4}{x^3}+1$ with respect to x. [3]

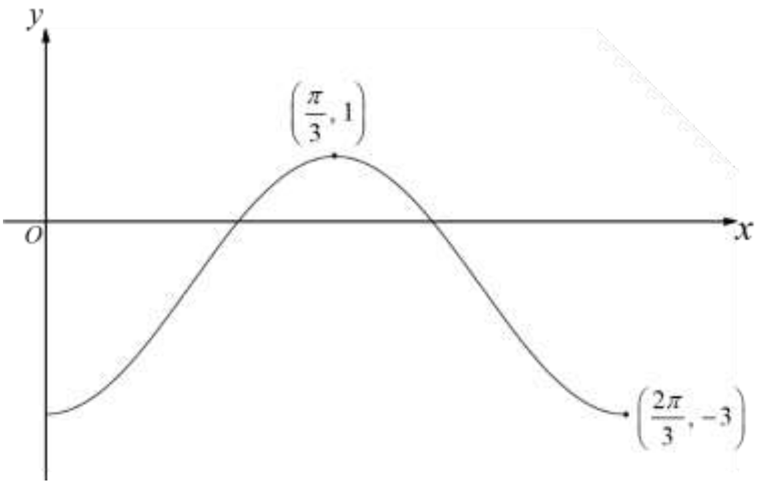
5 Find the value of A, B, C, and D for which $\dfrac{4x^3+7x^2-13x-2}{(x^2+3)(x-1)} = A + \dfrac{Bx+C}{x^2+3} + \dfrac{D}{x-1}$. [6]

6 A polynomial, P, is $x^3 - x^2 - x + k$, where k is a constant.

(a) Find the value of k given that P leaves a remainder of 3 when divided by $x - 2$. [2]

(b) In the case where $k = -2$, the quadratic expression $x^2 + ax + 1$ is a factor of P. Find the value of the constant a. [3]

7



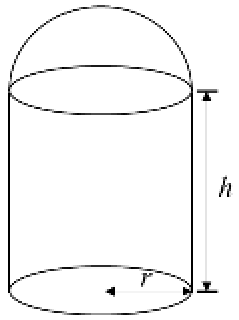
The diagram shows the curve $y = a\cos bx + c$ for $0 \le x \le \frac{2\pi}{3}$ radians, where a, b and c are integers. The curve has a maximum point at $\left(\frac{\pi}{3}, 1\right)$ and one of its minimum point at $\left(\frac{2\pi}{3}, -3\right)$.

(a) Find b. [2]

(b) Explain the effect of c on the curve, and show that $c = -1$. [2]

(c) State the amplitude, and write down the equation of the curve. [2]

8 [The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$, and its surface area is $4\pi r^2$.]

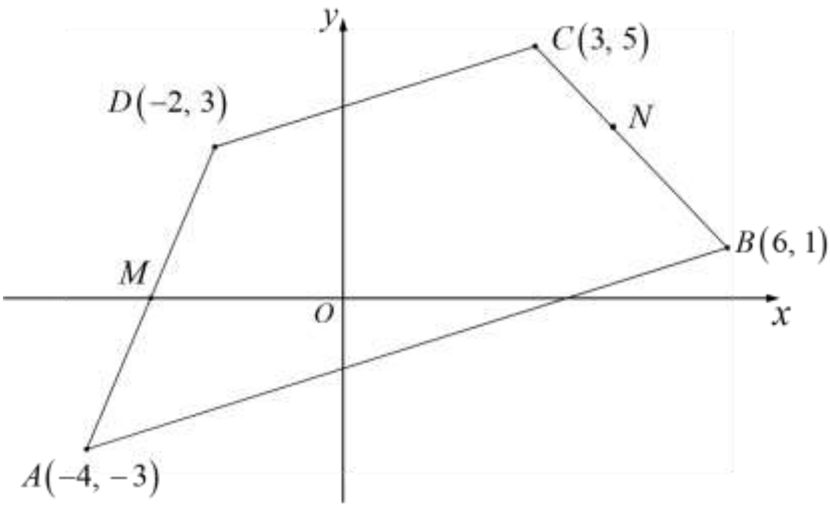


The diagram shows a bubble tea cup of capacity 360π cm^3. It consists of a cylindrical body of base radius r cm and height h cm, and a hemispheric cap. Let S cm^2 be the total surface area of the cup.

(a) Show that $S = \pi\left(\frac{5}{3}r^2 + \frac{720}{r}\right)$. [4]

(b) Given that r can vary, find the stationary value of S and determine its nature. [5]

9



The diagram shows a quadrilateral *ABCD* with vertices $A(-4, -3)$, $B(6, 1)$, $C(3, 5)$ and $D(-2, 3)$. *M* which lies on the *x*-axis is the midpoint of the side *AD*.

(a) Explain why *ABCD* is a trapezium. [3]

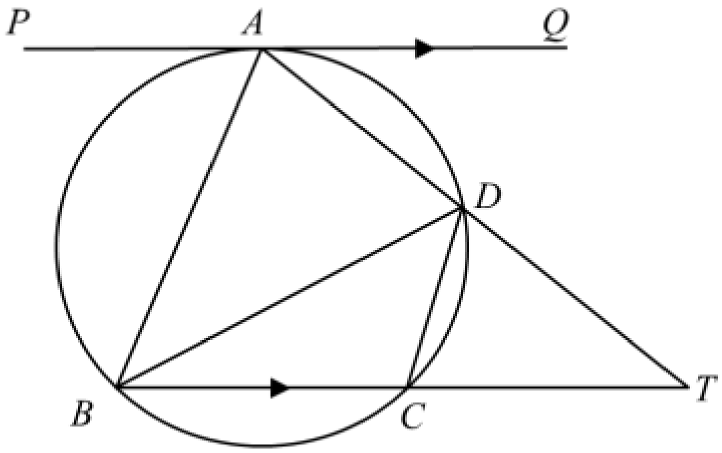
(b) Find the coordinates of *M*. [2]

(c) A point N is on the side BC. Given that the area of the quadrilateral $MNCD$ is $\frac{65}{4}$ units^2, find the coordinates of N. [5]

10 **(a)** Prove the identity $\dfrac{\tan A - \cot A}{\tan A + \cot A} = 1 - 2\cos^2 A$. [4]

(b) Hence solve the equation $\dfrac{\tan 2\theta - \cot 2\theta}{\tan 2\theta + \cot 2\theta} = \dfrac{1}{2}$ for $0 < \theta < \pi$, giving your answers in terms of π. [4]

11



In the diagram, *PAQ* is the tangent to the circle at *A*. The line *BCT* is parallel to *PAQ* and *ADT* is a straight line.

(a) Prove that angle $ADB =$ angle CDT. [4]

(b) Prove that triangles *TCD* and *BAD* are similar. [4]

12 **(a)** Show that the solution of the equation $3^{2x+4} \times 5^{x} = 3^{3x} \times 25^{x}$ is $x = \dfrac{\lg 81}{\lg 15}$. [4]

(b) Express the equation $\log_2 x + \log_4 (x+6) = 2$ as a cubic equation in *x*. [4]

13 The volume of liquid in a container, V m^3, is given by $V = 0.05\left[(3x+2)^3 - 8\right]$, where x is the height of the liquid in metres.

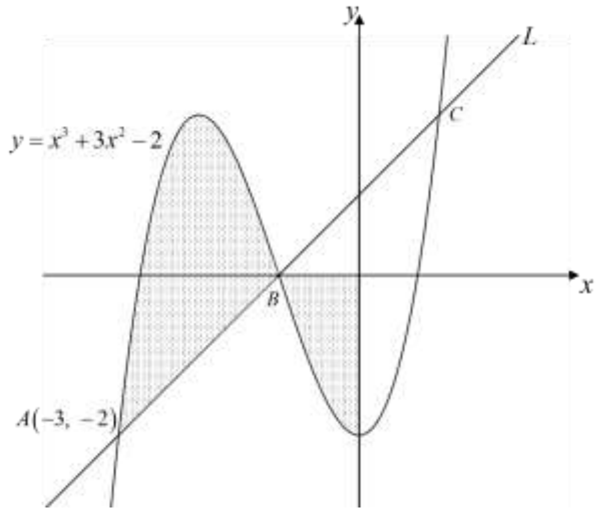
(a) Find an expression for $\frac{\mathrm{d}V}{\mathrm{d}x}$. [2]

The liquid enters the container at a constant rate of 0.081 m^3/s.

(b) Find the value of x when $V = 0.95$. [2]

(c) Hence find $\frac{\mathrm{d}x}{\mathrm{d}t}$ when $V = 0.95$, and explain the significance of your answer. [4]

14



The diagram shows part of the curve $y = x^3 + 3x^2 - 2$. The straight line L cuts the curve at $A(-3, -2)$, the x-axis at B, and intersects the curve again at C.

(a) The gradient of the tangent to the curve at $B = -3$, use it to find the coordinates of B. [3]

(b) Hence find the area of the shaded region. [6]

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Qn	Answers
1	$3-\sqrt{3}$
2	The coordinates of P and Q are $(-1, -2)$ and $\left(10, \frac{7}{2}\right)$.
3	$-2\left(x+\frac{3}{4}\right)^2+\frac{33}{8}$ maximum point at $\left(-\frac{3}{4}, \frac{33}{8}\right)$.
4	$4\ln(2x-1)-\frac{2}{x^2}+x+c$
5	$A=4,\ B=12,\ C=-13,\ D=-1$
6	$k=1$ $a=1$
7	$b=3$ $c=-1$ Amplitude $=2$ $y=-2\cos 3x-1$
8	$r=6$
9	$(-3,\ 0)$ $\left(\frac{9}{2}, 3\right)$
10	$\theta=\frac{\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6}$
12	$x^3+6x^2-16=0$
13	$\frac{dV}{dx}=0.45(3x+2)^2$ $x=\frac{1}{3}$ $\frac{dx}{dt}=0.02$ m/s When the volume of the liquid in the tub is 0.95m^3, the height of liquid in the container is increasing at 0.02 m/s.
14	$y=x+1$ $\frac{21}{4}\text{units}^2$