Mark Scheme for 2024 S4E5N Add Math Prelim **Paper 1**

1a	$2x^2+4x-1$			
	$2[(x+1)^2-1]-1$			
	$2(x+1)^2-3$	B2,1		-1 for each error
	$-x^2+x-8$			
	$-(x^2-x)-8$			
	$-\left[\left(x-\frac{1}{2}\right)^2-\frac{1}{4}\right]-8$			
	$-\left(x-\frac{1}{2}\right)^2-\frac{31}{4}$	B2,1		-1 for each error Accept $-\left(x-0.5\right)^2-7.75$
1b	Minimum point $(-1,-3)$, Maximum point $(0.5,-7.75)$	B1√		Only need the *y*-values
	max of $y=-x^2+x-8<$ min of $y=2x^2+4x-1$ → curves will not intersect	B1		Correct argument. Diagram may be used as long as all necessary working is shown
			[6]	
2a	Let $u=e^{2x}\rightarrow\ 9u+14=8u^{-1}$			
	$9u^2+14u-8=0$	M1		Form quadratic equation using substitution
	$(9u-4)(u+2)=0$ $\rightarrow u=\frac{4}{9}$ or $u=-2$ (impossible)	M1		Solving the quadratic and finding *u*
	$e^{2x}=\frac{4}{9}\rightarrow e^x=\frac{2}{3}$	A1		Do not accept 0.666…
			[3]	
3	$V=\pi r^2h$			
	$(26\sqrt{3}-20\sqrt{5})\pi=\pi(\sqrt{5}-\sqrt{3})^2h$	M1		Use of $V=\pi r^2h$ formula
	$(\sqrt{5}-\sqrt{3})^2=8-2\sqrt{15}$	B1		For correctly squaring $\sqrt{5}-\sqrt{3}$
	$h=\frac{(26\sqrt{3}-20\sqrt{5})}{8-2\sqrt{15}}$ or $h=\frac{(13\sqrt{3}-10\sqrt{5})}{4-\sqrt{15}}$			
	$\frac{(26\sqrt{3}-20\sqrt{5})}{8-2\sqrt{15}}\times\frac{8+2\sqrt{15}}{8+2\sqrt{15}}$	M1		For rationalising denominator
	$\frac{208\sqrt{3}+52\sqrt{45}-160\sqrt{5}-40\sqrt{75}}{64-4(15)}$	DM1		Depend on previous method. Correct expansion of numerator
	$\frac{8\sqrt{3}-4\sqrt{5}}{4}$			
	$2\sqrt{3}-\sqrt{5}=\sqrt{12}-\sqrt{5}$	A1 A1		A1 unsimplified answer A1 correct simplified
			[6]	
4	$f'(x)=-3\cos x-2\sin 2x\quad(+c)$	B2,1		-1 for each error
	$9=-3\cos(\pi)-2\sin(2\pi)+c$	M1		Use gradient of 9 with $x=\pi$
	$9=3-0+c\rightarrow\ c=6$			
	$f(x)=-3\sin x+\cos 2x+6x\quad(+d)$	B2,1√		-1 for each error. √ from $f'(x)$
	$8=-3\sin(\frac{\pi}{6})+\cos 2(\frac{\pi}{6})+6(\frac{\pi}{6})+d$			

	$8=-\frac{3}{2}+\frac{1}{2}+\pi+c \rightarrow\ c=9-\pi$			
	$\mathrm{f}(x)=-3\sin x+\cos 2x+6x+9-\pi$	A1		Use $\mathrm{f}(x)$ of 8 with $x=\frac{\pi}{3}$, c.a.o.
			[6]	
5ai	$-\dfrac{\pi}{3}$	B1		
5aii	Principal value for $\cos^{-1}x$ lies between 0 and π inclusive	B1		Accept $0\le\cos^{-1}x\le\pi$ and $0\le\theta\le\pi$
5b	$2\sin(A+B)=1-2\sin(A-B)$			
	$2(\sin A\cos B+\cos A\sin B)$ $=1-2(\sin A\cos B-\cos A\sin B)$	B1		Use of addition formula
	$4\sin A\cos B=1$			
	$4(\sin A)(\frac{1}{3})=1 \rightarrow \sin A=\frac{3}{4}$	M1		For $\sin A=k$
	$\sqrt{4^2-3^2}$	M1		For finding adjacent
	$\tan A=\dfrac{3}{\sqrt{7}}$	A1		c.a.o.
			[6]	
6a	$x-$coord of $C=4.5$	B1		
	$\frac{1}{2}\times(6-3)\times h=6 \rightarrow h=4$			
	$y-$coord of $C=3-4=-1$	M1		For $3-$height or use of shoelace
	$m_{AC}=\dfrac{3-(-1)}{3-4.5}=-\dfrac{8}{3}$	M1		For finding gradient of *AC*
	$m_{DE}=\dfrac{k-7}{10-7.5}=-\dfrac{8}{3}$	M1		For $m_{DE}=m_{AC}$
	$k=\dfrac{1}{3}$	A1		
6b	$\dfrac{1}{2}\begin{vmatrix}4.5 & 10 & 7.5 & 4.5\\ -1 & \frac{1}{3} & 7 & -1\end{vmatrix}$			
	$\dfrac{1}{2}\left\|\frac{1}{3}(4.5)+10(7)-1(7.5)-(-10)-\frac{1}{3}(7.5)-7(4.5)\right\|$	M1		Correct use of shoelace method
	20 units^2	A1		
			[7]	
7a	$\frac{\mathrm{d}y}{\mathrm{d}x}=\sec^2 x$	B1		
	$y=1\rightarrow x=\frac{\pi}{4}$	M1		For finding *x* value
	$\frac{\mathrm{d}y}{\mathrm{d}t}=\frac{\mathrm{d}y}{\mathrm{d}x}\times\frac{\mathrm{d}x}{\mathrm{d}t}$			
	$-0.12=\sec^2(\frac{\pi}{4})\times\frac{\mathrm{d}x}{\mathrm{d}t}$	M1		Chain rule used correctly. Ignore $-$ in $\frac{\mathrm{d}y}{\mathrm{d}t}$
	$\frac{\mathrm{d}x}{\mathrm{d}t}=-0.06$ units/second	A1		a.e.f.
7b	$k=(120)(2)=240$	B1		
	$PA=k \rightarrow A=kP^{-1}$	M1		Make *A* the subject
	$\frac{\mathrm{d}A}{\mathrm{d}P}=-kP^{-2}$	A1		
	$\frac{\mathrm{d}A}{\mathrm{d}P}=-240(120)^{-2}=-\dfrac{1}{60}$	A1		Accept $-0.0166...$
			[8]	
8a	$6x+3y=48$	B1		

	Perpendicular height $=\sqrt{x^2-\left(\frac{1}{2}x\right)^2}$ or Area of triangle $=\frac{1}{2}(x)(x)\sin 60°$	M1		For finding perpendicular height or area of equilateral triangle
	$V=\frac{1}{2}\left(\frac{\sqrt{3}}{2}x\right)(x)(y)$ or $\left(\frac{\sqrt{3}}{4}x^2\right)y$	M1		For cross section area $\times$ height
	$V=\left(\frac{\sqrt{3}}{4}x^2\right)(16-2x)=\frac{\sqrt{3}x^2(8-x)}{2}$	A1		Answer was given – so all working must be correct
8b	$V=4\sqrt{3}x^2-\frac{1}{2}\sqrt{3}x^3$			
	$\frac{\mathrm{d}V}{\mathrm{d}x}=8\sqrt{3}x-\frac{3}{2}\sqrt{3}x^2$	B1		
	$8\sqrt{3}x-\frac{3}{2}\sqrt{3}x^2=0$	M1		Sets $\frac{\mathrm{d}V}{\mathrm{d}x}$ to 0 and solves
	$\sqrt{3}x(8-\frac{3}{2}x)=0$			
	$x=0$ (rejected) or $x=\frac{16}{3}$	A1		a.e.f.
	$V=65.7$	A1		Accept 65.68 …
			[8]	
9ai	$(x+1)(x^2-x+1)$	B1		
	$(x-1)(x^2+x+1)$	B1		
9aii	$6^6-1=(6^3)^2-1^2$			
	$(6^3+1)(6^3-1)$	M1		For $a^2-b^2=(a+b)(a-b)$
	$(6+1)(6^2-6+1)(6-1)(6^2+6+1)$			
	$(7)(31)(5)(43)$	A1		SR1 for $(6^2)^3-1^3=(6^2-1)((6^2)^2+(6^2)+1)$ $=(35)(1333)$ but A0 thereafter
9bi	$2(-\frac{1}{2})^3-3(-\frac{1}{2})^2-11(-\frac{1}{2})+6$	M1		Use of remainder theorem or long division
	10.5	A1		
9bii	$2x^3-3x^2-11x+6=(Ax+B)(2x^2+ax+3)$ or $\begin{array}{r l} & \quad 2x^2 \quad +ax \quad +3 \\ Ax+B & \overline{)2x^3 \quad -3x^2 \quad -11x \quad +6} \end{array}$	M1		Sets up $P(x)=(Ax+B)(\text{factor})$ or long division with factor as dividend
	$2x^3-3x^2-11x+6=(x+2)(2x^2+ax+3)$	B1		For $(Ax+B)=(x+2)$
	$-3x^2=ax^2+4x^2$ or $-11x=3x+2ax$			
	$a=-7$	A1		
	$(x+2)(2x-1)(x-3)$	A1		
			[10]	
10ai	$T_{r+1}=\binom{12}{r}x^{12-r}\left(\frac{1}{2x^3}\right)^r$	B1		i.s.w.
10aii	$x^{12-r}\times(\frac{1}{x^3})^r=x^{12-4r}$			
	power of $x=12-4r$	B1		Accept x^{12-4r}

10aiii	$12-4r=0$ or $x^{12}+\binom{12}{1}x^{11}\left(\frac{1}{2x^3}\right)+\binom{12}{2}x^{10}\left(\frac{1}{2x^3}\right)^2+\ldots$	M1		Sets power to 0 or Correct expansion of first 3 terms
	$T_{3+1}=\binom{12}{3}x^{12-3}\left(\frac{1}{2x^3}\right)^3=\frac{55}{2}$	A1		a.e.f.
10b	$(1+ax)(1+\frac{x}{2})^n$			
	$(1+ax)(1+n(\frac{x}{2})+\frac{n(n-1)}{2}(\frac{x}{2})^2+\ldots)$	B2		B1 for each $\binom{n}{1}=n$ and $\binom{n}{2}=\frac{n(n-1)}{2}$
	$(1+ax)(1+\frac{n}{2}x+\frac{n(n-1)}{8}x^2+\ldots)$			
	$1+\frac{n}{2}x+\frac{n(n-1)}{8}x^2+ax+\frac{an}{2}x^2+\ldots$			
	$1+\frac{n}{2}x+ax+\frac{n(n-1)}{8}x^2+\frac{an}{2}x^2+\ldots$			
	$\frac{1}{2}n+a=1$	M1		Equate coeff. of x terms to 1
	$\frac{n(n-1)}{8}+\frac{an}{2}=-5$	M1		Equate coeff. of x^2 terms to -5
	$\frac{n(n-1)}{8}+\frac{4(1-\frac{1}{2}n)n}{8}=-5$			
	$n(n-1)+4n(1-\frac{1}{2}n)=-40$			
	$n^2-n+4n-2n^2=-40$			
	$n^2-3n-40=0$			$4a^2-2a-42=0$
	$n=8$ or $n=-5$ (rejected)	A1		$a=\frac{7}{2}$ (rejected) or $a=-3$
	$a=1-\frac{1}{2}(8)=-3$	A1		$n=8$
			[10]	
11	$\frac{dy}{dx}=2x-10$	B1		
	$2x-10=-4\rightarrow x=3$	M1		For finding *x*-coord of *P*
	$x=3\rightarrow y=3^2-10(3)+24=3$			
	$m_{PR}=\frac{1}{4}$	B1		
	$3=\frac{1}{4}(3)+c$ $c=\frac{9}{4}$ $y=\frac{1}{4}x+\frac{9}{4}$ $y=0\rightarrow x=-9$ or $\frac{3-0}{3-x}=\frac{1}{4}\rightarrow x=-9$	M1		For finding *x*-coord of *R*
	$\frac{1}{2}(9+3)(3)=18$ or $\int_{-9}^{3}\frac{1}{4}x+\frac{9}{4}\ dx$	M1		For finding area of triangle. Answer 18 may be implied
	$x^2-10x+24=0\rightarrow x=4$	B1		For finding *x*-coord of *Q*
	Integrate curve $\rightarrow\left[\frac{1}{3}x^3-5x^2+24x\right]$	M1 A1		Knowing to integrate. A1 might be implied
	Limits 3 to 4 → $\left[\frac{1}{3}(4)^3-5(4)^2+24(4)\right]-\left[\frac{1}{3}(3)^3-5(3)^2+24(3)\right]$ $=1\frac{1}{3}$	M1		Uses definite integral method on antiderivative
	$18+1\frac{1}{3}=19\frac{1}{3}$	A1		
			[10]	

12ai	Plot all points correctly (allow ± 1 mm)	M1		
	Straight line drawn through all points	A1		Dep. on method
12aii	Read off at $t=0$ (allow ± 0.005)	M1		
	$e^6 = 403 \to 403$ thousands	A1		Accept 403 000
12aiii	Read off at $\ln 200 = 5.30 \to 1995+8=2003$	M1 A1		c.a.o.
12b	$s = ut + \frac{1}{2}at^2$			
	$\frac{s}{t} = \frac{1}{2}at + u \to$ Plot graph of $\frac{s}{t}$ against t	M1 A1		Divide throughout by t
	$\frac{1}{2}a$ value obtained from gradient of graph	DB1		Dep. on method
	u value obtained from y-intercept of graph	DB1		Dep. on method
	Alternative Answer:			
	$\frac{s}{t^2} = \frac{u}{t} + \frac{1}{2}a \to$ Plot graph of $\frac{s}{t^2}$ against $\frac{1}{t}$	M1 A1		Divide throughout by t^2
	$\frac{1}{2}a$ value obtained from y-intercept of graph	DB1		Dep. on method
	u value obtained from gradient of graph	DB1		Dep. on method
			[10]