Name:	Register No.:	Class:

CRESCENT GIRLS' SCHOOL
SECONDARY TWO
END OF YEAR EXAMINATION 2022

MATHEMATICS 4052

11 October 2022
2 hours

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use paper clips, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, submit the whole Question Paper.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

For Examiner's Use

Question	1	2	3	4	5	6	7	8	9
Marks									
Question	10	11	12	13	14	15	16	17	18
Marks									

Table of Penalties		**Qn. No.**		
Presentation	–1			
Significant Figures/ Units	–1		**Parent's/ Guardian's Signature**	**80**

This question paper consists of 20 printed pages.

Mathematical Formulae

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

Volume of a sphere = $\frac{4}{3}\pi r^3$

Statistics

Mean = $\frac{\sum fx}{\sum f}$

Answer **all** the questions on the question paper.

1 Expand and simplify $3p^2-(4-5p)(4p+3)$.

Answer …….. [2]

2 Factorise $35ad-21bd+10ac-6bc$ completely.

Answer …….. [2]

3 **(i)** Solve $\frac{-7x+3}{6} \geq -17$.

Answer ..... [2]

(ii) Hence, given that x is a prime number, state the greatest possible value of x.

Answer x = ... [1]

4 Tracy has \$50.
A shop sells a pack of eggs for \$$x$ each and a carton of milk for \$$y$ each.
If Tracy buys 2 packs of eggs and 5 cartons of milk, she will receive \$0.10 in change.
If she buys 5 packs of eggs and 2 cartons of milk, she will be short of \$6.50.
Find the price of each item.

Answer Price of one pack of eggs = \$……..

Price of one carton of milk = \$…….. [4]

5 **(i)** Factorise $3a^2 - 48b^2$ completely.

Answer ..... [1]

(ii) Hence, or otherwise, find the values of a and b if $3a^2 - 48b^2 = 51$

Answer a = ……..

b = …….. [2]

6 **(a)** Shannon has a picture measuring 70 cm by 100 cm. She puts the picture in a frame that gives it a border of uniform width x cm on all four sides.

(i) If the combined area of the picture and the border is 8800 cm^2, form an equation in terms of x, and show that it reduces to $x^2 + 85x - 450 = 0$. [2]

(ii) Hence, find the width of the border.

Answer …… cm [2]

(b) Explain why $5x^2 + 3 = 0$ has no real solutions.

Answer ……..

……

……

…… [1]

7 **(i)** Express $\frac{4x}{3x^2-2x-8}-\frac{7}{2-x}$ as a single fraction in its simplest form.

Answer ……..... [3]

(ii) Hence, or otherwise, solve $\frac{4x}{3x^2-2x-8}-\frac{7}{2-x}=0$.

Answer $x =$ ……..... [2]

(iii) For $y=\frac{4x}{3x^2-2x-8}-\frac{7}{2-x}$, explain why $x \neq 2$ and $x \neq -\frac{4}{3}$.

Answer ……...

……

……

…… [1]

8 It is given that T^2 is inversely proportional to r^3. When $T = 5$, $r = 2$.

(a) Find an equation connecting T and r.

Answer ……..…… [2]

(b) Find the values of T when $r = 4$.

Answer T = …… or …… [2]

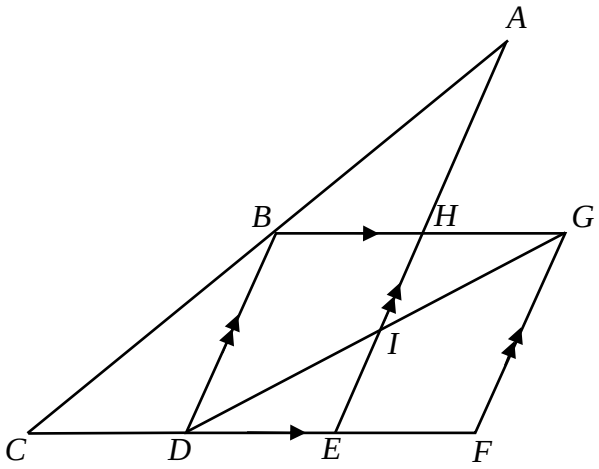
9 Two variables, p and q are connected by an equation.
The table below shows some values of p and q.

p	3	7	11	13	19
q	7	15	23	27	39

Explain why p is not directly proportional to q.

Answer [2]

10 In the figure below, *BDFG* is a parallelogram, *AE* // *BD* // *GF*, *ABC* and *CDEF* are straight lines. The length of *CD* = *DE* = *EF* = 6 cm.



(a) **(i)** Name the quadrilateral that is congruent to quadrilateral *BHED*.

Answer Quadrilateral ……..... [1]

(ii) Name the quadrilateral that is congruent to quadrilateral *BHID*.

Answer Quadrilateral ……..... [1]

(b) Name the triangle that is similar to triangle *BAH* and triangle *CBD*.

Answer Triangle ……..... [1]

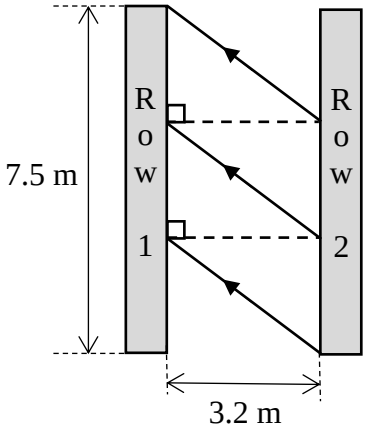
(c) Given that *GF* = 10 cm, find the length of *AE*.

Answer ……..... cm [2]

11 Flag buntings are to be hung as decoration between two rows of stalls at a carnival. Figure 1 shows a picture of the flag buntings. Figure 2 shows the top view of the stalls with the flag buntings. The 3 strings of flag buntings are equally spaced out along the row of stalls and are parallel to one another.

Figure 1

Figure 2



(a) It is given that the two rows of stalls are 3.2 metres apart and the length of each row is 7.5 m. Assuming the flag buntings are put up taut without any slack, calculate the total length of flag buntings required for decoration.

Answer ……..…… m [2]

(b) Alice has to hang the flag buntings from the top edge of the support pillars of the two rows of stalls. She uses a ladder of length 2.9 metres to complete this task. When the ladder leans against the top edge of the support pillars, it forms an angle of 58° with the flat ground. Find the height of the flag buntings above the ground.

Answer ……..…… m [2]

12 In a lucky draw, there is one for each of the top three prizes, five 'Prize A', eight 'Prize B' and x number of 'Prize C' to be won.

Given that the probability to win Prize C is $\frac{3}{7}$, find the total number of prizes to be won in this lucky draw.

Answer ……. [3]

13 The number of men and women watching a street performance are $3x$ and 36 respectively. Halfway through the performance, $x-14$ women join in while 9 men leave. The probability to select a women volunteer during the performance becomes $\frac{8}{17}$. Find the total number of people watching the performance at first.

Answer …….. [3]

14 The heights of some students were measured and recorded in the table below.

Height (x/cm)	**Number of students**
$140 < x \leq 145$	1
$145 < x \leq 150$	p
$150 < x \leq 155$	15
$155 < x \leq 160$	12
$160 < x \leq 165$	6
$165 < x \leq 170$	2

(a) Write down the largest value of p given that the modal class is $150 < x \leq 155$.

Answer p = …….**.....** [1]

(b) Write down the largest value of p given that the median class is $155 < x \leq 160$.

Answer p = ……**.....** [2]

(c) Find the value of p given that the estimated mean height of the students is 155.5 cm.

Answer p = ……**.....** [2]

15 A frustum is a solid that remains when a smaller cone is cut off from a larger cone. Figure 1 shows a frustum not drawn to scale.

Figure 2 shows a drinking glass in the shape of a frustum. The height of the frustum is 16 cm. The radii of the base and the opening of the glass are 3.5 cm and 4 cm respectively.

Figure 1

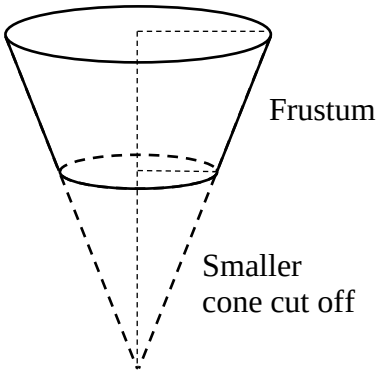
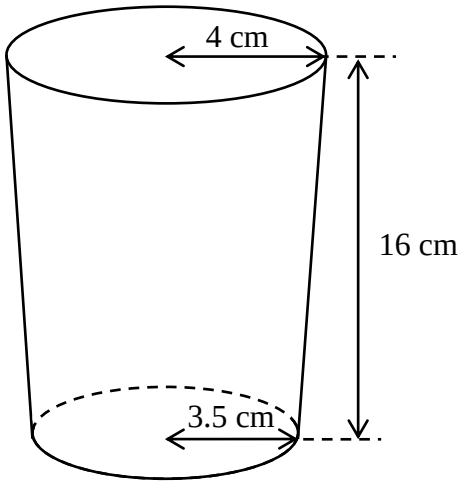


Figure 2



(a) Using similar triangles, show that the volume of the drinking glass is 707.9 cm^3 correct to 4 significant figures. [4]

Answer

(b) **(i)** Polly pours water into the glass. The surface of the water is 4 cm below the top of the glass. Polly thinks that the glass is filled to 75% of its total capacity. Without calculating the volume, explain why she is wrong.

Answer ……

……

…… [1]

(ii) Polly then pours a full glass of water into a hemispherical bowl.
The water filled to the brim of the bowl.
Using your answer in part (a), calculate the radius of the hemispherical bowl.

Answer …… cm [3]

(iii) Hence, find the internal surface area of the hemispherical bowl, assuming that the thickness of the bowl is negligible.

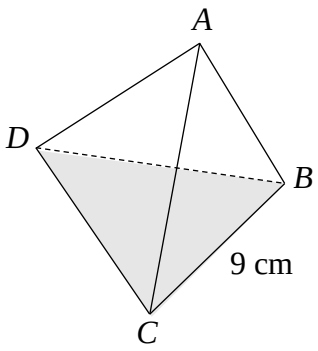
Answer …… cm^2 [2]

16 Diagram I shows a picture of traditional rice dumplings wrapped in leaves.

Diagram II shows the rice dumpling modelled as a tetrahedron with four equilateral triangles. The apex of the tetrahedron is at point *A* and *BCD* is its triangular base. The length of each side of the tetrahedron is 9 cm.

Diagram I

Diagram II



(a) Show that the height of each equilateral triangle is 7.7942 cm. [2]

Answer

(b) Find the total surface area of the tetrahedron *ABCD*.

Answer ……..…… ..... cm^2 [2]

(c) Caleb wants to make a tetrahedron with four equilateral triangles of side 9 cm using a rectangular piece of paper.

To minimise wastage, he figures out the net of a tetrahedron which requires him the least amount of paper.

Suggest the length and breadth of the rectangular piece of paper that would allow him to make this tetrahedron. Justify your suggested dimensions with calculations and explanations. [3]

Answer

17 A gardener has 110 m of fencing to make a rectangular enclosure.

The length of the enclosure is x m.

(a) By expressing the breadth of the enclosure in terms of x, show that the area y m^2 is given by $y = x(55 - x)$. [2]

Answer

(b) Some values of x and y are given in the table below.

x (m)	0	10	20	30	40	50	55
y (m^2)	0	450	700	750	600	p	0

Find the value of p.

Answer p =…….. [1]

(c) Using a scale of 2 cm to represent 10 m on the x-axis, and a scale of 2 cm to represent 100 m^2 on the y-axis, draw the graph of $y = x(55 - x)$ for $0 \le x \le 55$ on the grid opposite. [3]

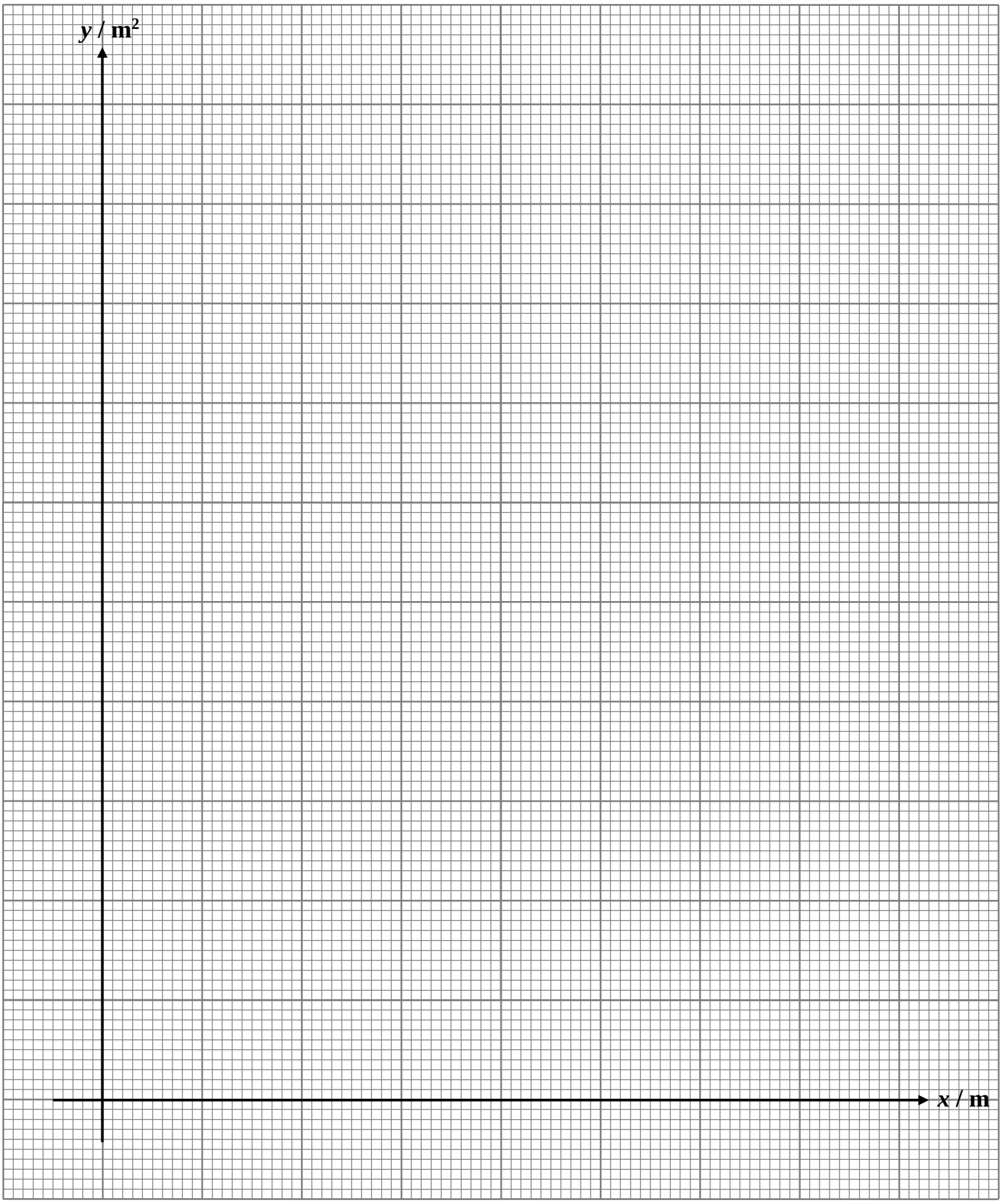
(d) **(i)** Use your graph in **(b)** to find the maximum value of y.

Answer y = ……..…….. [1]

(ii) Hence, state the breadth and length of the rectangular enclosure that encompasses the greatest area.

Answer breadth = …….. m

length = …….. m [1]


y / m²
x / m

18 Two groups of students entered a Mathematics competition. Their scores are shown in the stem-and-leaf diagram.

Group A							Group B					
			7	1	0	5	2	5				
	8	7	6	5	5	6	4	5	6			
5	5	4	3	2	2	7	1	2	2	4	5	8
		4	2	2	1	8	2	3	3	5	7	
				7	3	9	0	2	5	5		

Key: 0 | 5 | 2 means a score of 50 in Group A and a score of 52 in Group B

(a) Find the median score for Group B.

Answer ……..... [1]

(b) Find the modal score for the Mathematics competition.

Answer ……..... [1]

(c) Students who scored more than 75 were awarded a distinction.

Use this information to compare the performance of the students in the two groups.

Answer …….

…….....

…….....

……..... [2]

- End of Paper -

Answer Key

1. $23p^2 - p - 12$

2. $(5a - 3b)(7d + 2c)$

3i. $x \leq 15$
3ii. $x = 13$

4. Price of one pack of eggs = \$8.70
Price of one carton of milk = \$6.50

5i. $3(a + 4b)(a - 4b)$
5ii. $a = 9, \quad b = 2$

6ai. $(70 + 2x)(100 + 2x) = 8800$

$4x^2 + 340x + 7000 = 8800$

$x^2 + 85x - 450 = 0$

6aii. 5 cm
6b. From $5x^2 + 3 = 0$, $5x^2 = -3$.
For all real values of x, $x^2 \geq 0$.
Hence $5x^2 + 3 = 0$ has no real solutions.

7i. $\dfrac{25x + 28}{(x - 2)(3x + 4)}$

7ii. $x = -1\dfrac{3}{25}$

7iii. If $x = 2$ or $x = -\dfrac{4}{3}$, the denominator would be equal to zero/the numerator would be divided by 0. The fraction would therefore be **undefined.**

8a. $T^2 = \dfrac{200}{r^3}$

8a. $T^2 = \dfrac{200}{r^3}$

8b. $T = \pm 1.77$ (to 3 s.f.)

9. $\dfrac{p_2}{p_1} = \dfrac{7}{3} = 2.33$

$\dfrac{q_2}{q_1} = \dfrac{15}{7} = 2.14$

Since $\dfrac{p_2}{p_1} \neq \dfrac{q_2}{q_1}$,

p is not directly proportional to q

9. $\frac{p_2}{p_1} = \frac{7}{3} = 2.33$

$\frac{q_2}{q_1} = \frac{15}{7} = 2.14$

Since $\frac{p_2}{p_1} \neq \frac{q_2}{q_1}$,

p is not directly proportional to q

10ai. Quadrilateral *HGFE*
10aii. Quadrilateral *FEIG*
10b. Triangle *CAE*
10c. *AE* = 20 cm

11a. 12.2 m (3 s.f.)
11b. 2.46 m (3 s.f.)

12. 28 prizes

13. 90 people

14a. $p = 14$
14b. $p = 3$
14c. $p = 4$

15a. Hint: Apply concept of similar triangles
15bi. She is wrong because the radius along the height of the frustum is not constant / not similar.

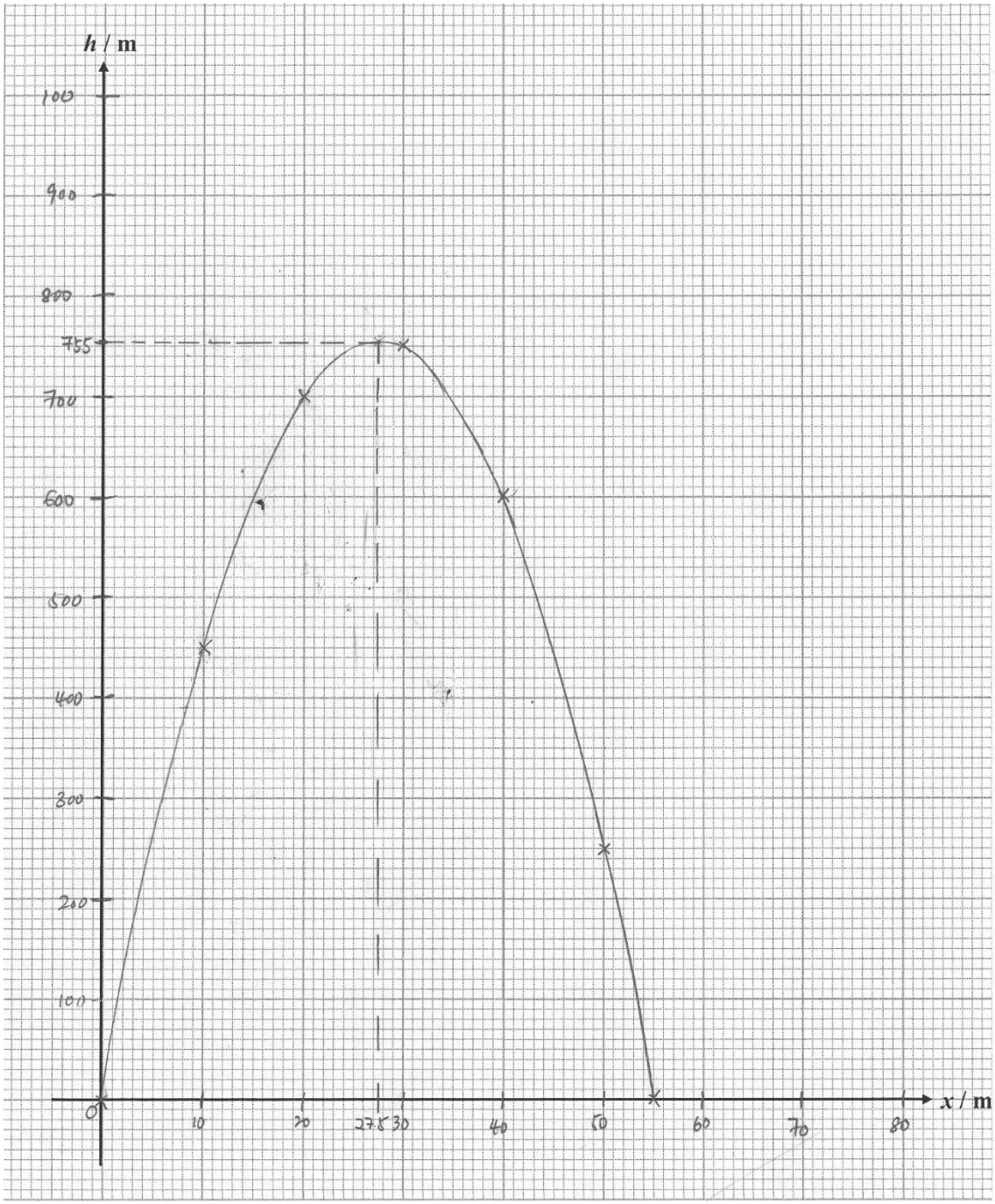
15bii. 6.97 cm (3 s.f.)
15biii. 305 cm^2 (3 s.f.)

16a. Hint: Apply concept of Pythagoras' Theorem
16b. 140 cm^2 (3 s.f.)
16c. Suggested length: 22.5 cm
Suggested breadth: 7.8 cm

17a. $y = x(55 - x)$
17b. $p = 250$
17di. $y = 755$ or $y = 760$
17dii. $x = 27.5$ (x is defined as length; Accept $x = 28$, $x = 29$)
Breadth = 27.5 m, Length = 27.5 m

18a. 76.5
18b. 72
18c. 6 students scored more than 75 in Group A while 10 students scored more than 75 in Group B. Hence, students in Group B performed better than students in Group A.

17c.


h / m
1000
900
800
755
700
600
500
400
300
200
100
0
10
20
27.5
30
40
50
60
70
80
x / m