H

ANGLO-CHINESE JUNIOR COLLEGE
JC1 PROMOTIONAL EXAMINATION

Higher 1

/50

CANDIDATE NAME

TUTORIAL/ FORM CLASS

INDEX NUMBER

MATHEMATICS 8865/01

Paper 1

29 September 2023
1 hr 30 min

Candidates answer on the Question Paper.
Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your index number, class and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the Question Paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

Question	Marks
1	/4
2	/4
3	/7
4	/7
5	/7
6	/8
7	/13

This document consists of **14** printed pages.

1 Use the substitution $u = \ln x$ to solve the inequality

$$(\ln x)^2 - \ln\left(\mathrm{e}x^2\right) > 7,$$

leaving your answer in terms of e. [4]

2 Show that the expression $5x^2 - 5x + 2$ is positive for all real values of x. Hence or otherwise, find the set of values of k for which the equation

$$4x^2 + (12k - 8)x - k(k + 2) = 0$$

has two distinct roots. [4]

3 **(a)** Differentiate $\ln\left(\dfrac{\mathrm{e}^{-2x}}{\sqrt{2+5x}}\right)$, leaving your answer in the form $A + \dfrac{B}{4 + Cx}$. [3]

(b) Given that $\displaystyle\int_{\frac{2}{3}}^{k} \frac{6}{\sqrt{2+3x}}\,\mathrm{d}x = 4$ where $k > 2$. Find the value of k. [4]

4 The curve C has equation $y = \ln(x - 1) + 2$.

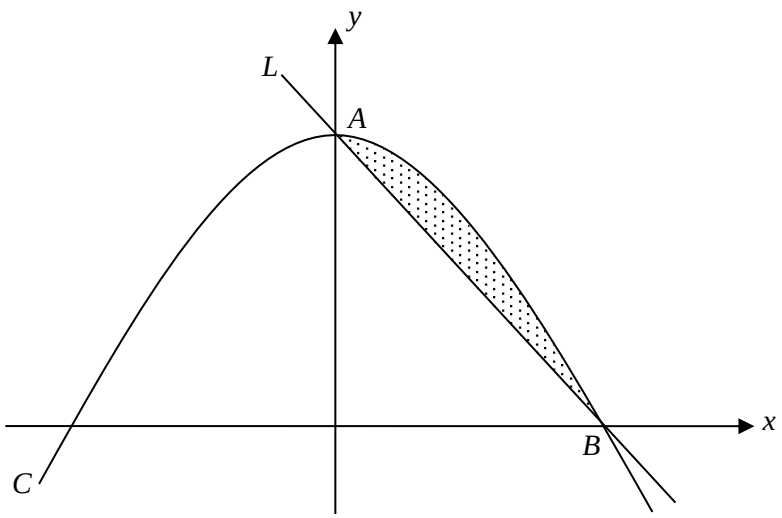
(i) Sketch the graph of C, stating the exact coordinates of any point(s) of intersection with the axes and the equation of the asymptote. [3]

(ii) By using an appropriate line and the graph of C, solve the equation

$$\ln(x - 1) + 7 = 2x.$$ [3]

(iii) Find the numerical value of the area of the region bounded by C, the x-axis, and the lines $x = 1.5$ and $x = 4$. [1]

5



The curve C has the equation $y = k^2 - x^2$ and the line L has equation $y = -kx + k^2$, where k is a positive constant. Curve C and line L intersect at the points A and B as shown.

(i) Find, in terms of k, the coordinates of A and B. [3]

(ii) Find the area of the shaded region enclosed by C and L, leaving your answer in terms of k. [4]

6 The curve C has the equation $y = \dfrac{x^2 + 12}{\sqrt{x}}$, where $x > 0$.

(i) Use differentiation to show that the curve C has a stationary point at $\left(2, \dfrac{16}{\sqrt{2}}\right)$.

Determine the nature of this stationary point. [5]

(ii) Find the equation of the tangent to C at the point where $x = 1$, giving your answer in the form $y = mx + c$, where m and c are constants. [3]

7 The total sales, S thousand dollars, of a jewellery store is modelled by

$$S = 2500 - \frac{5000}{2 + 0.4t},$$

where t is the time in years since the year 2010.

(i) Sketch the graph of S against t for $t \geq 0$, stating the coordinates of any point(s) of intersection with the axes and the equation of any asymptotes. [2]

(ii) By using differentiation, find $\frac{\mathrm{d}S}{\mathrm{d}t}$ when $t = 2$. Explain what this value represents in the context of the question. [3]

(iii) Explain what will happen to the sales in the long run. [1]

(iv) Find algebraically the value of $\int_2^4 2500 - \frac{5000}{2+0.4t}\,\mathrm{d}t$, giving your answer to the nearest integer. Explain what this value represents in the context of the question. [4]

7 [continued]

The jewellery store is also interested in modelling its advertising costs, C thousand dollars for each year. At time t years for $t \geq 0$, it is modelled by

$$C = t^3 - 12t^2 + (k+36)t,$$

where k is a positive constant.

(v) Find the set of values of k for which C is an increasing function of t. [3]

Summary of Areas for Improvement			
Knowledge (K)	**Careless Mistakes (C)**	**Read/Interpret Qn wrongly (R)**	**Presentation (P)**