PEICAI SECONDARY SCHOOL
SECONDARY 3 EXPRESS
END-OF-YEAR EXAMINATION 2023

CANDIDATE NAME

CLASS | REGISTER NUMBER

MATHEMATICS **4052/02**

Paper 2 **3 October 2023**

2 hours 15 minutes

Candidates answer on Question Paper

READ THESE INSTRUCTIONS FIRST

Write your register number, class and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

	ANnotations	**AC**curacy
Marks Deducted	1	1

For Examiner's Use

This document consists of **19** printed pages and **1** blank page.

Setter: Mr Lim Jit Chong

[Turn over

Mathematical Formulae

Compound Interest

$$\text{Total Amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

Volume of a sphere = $\frac{4}{3}\pi r^3$

Area of triangle $ABC = \frac{1}{2}ab\sin C$

Arc length = $r\theta$, where θ is in radians

Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Answer **all** questions

1 **(a)** It is given that $m(3x+y)+3y=k$.

(i) Find the value of k when $x=5$, $y=-7$ and $m=2$.

Answer $k=$ 1 B1 [1]

(ii) Express y in terms of k, m and x.

$3mx+my+3y=K$ M1

$my+3y=K-3mx$

$y(m+3)=K-3mx$ M1

Answer $y=\frac{K-3mx}{m+3}$ A1 [3]

(b) Solve $(2x-7)^2=25$

$2x-7=\pm\sqrt{25}$ M1

Answer $k=$ 6 or 1 A1 [2]

(c) Simplify

(i) $\frac{6ac^2}{25x^3}\div\frac{2ac}{5x}$

$\frac{6ac^2}{25x^3}\times\frac{5x}{2ac}$ M1

Answer $\frac{3c}{5x^2}$ A1 [2]

(ii) $\frac{2}{x^2-9}-\frac{3x}{x-3}$

$\frac{2}{(x-3)(x+3)}-\frac{3x}{x-3}$ M1

$\frac{2}{(x-3)(x+3)}-\frac{3x(x+3)}{(x-3)(x+3)}$ M1

Answer $\frac{-3x^2-9x+2}{(x-3)(x+3)}$ A1 [3]

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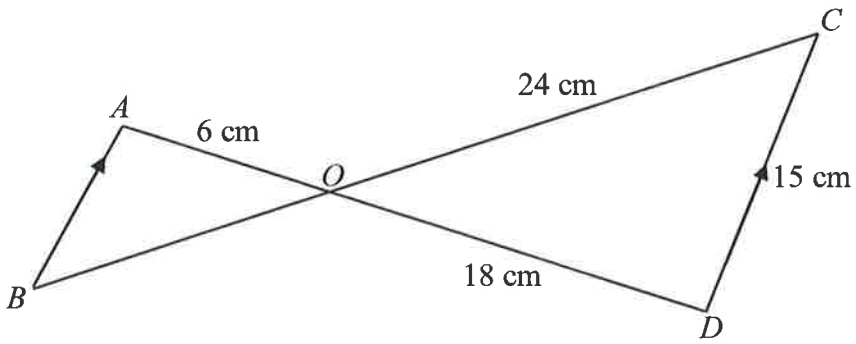
2 **(a)** The straight lines AD and BC intersect at O. AB is parallel to DC. $AO = 6$ cm, $OD = 18$ cm, $CD = 15$ cm and $OC = 24$ cm.



(i) Show that ΔAOB is similar to ΔDOC.

Answer:

$\angle AOB = \angle DOC$ (vert opp $\angle$s) } either 1 M1

$\angle BAO = \angle CDO$ (alt $\angle$s, $AB // DC$)

$\therefore \Delta AOB$ is similar to ΔDOC (AA similarity test) A1

[2]

(ii) Find the length of AB.

$\frac{AB}{DC} = \frac{AO}{DO}$

$\frac{AB}{15} = \frac{6}{18}$ M1

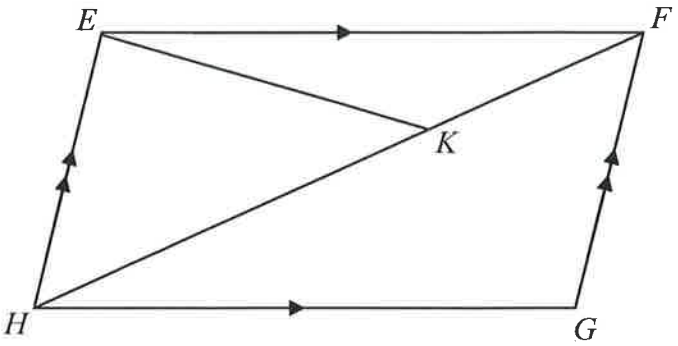
Answer 5 A1 cm [2]

(iii) Given that the area of ΔDOC is 126 cm^2, find the area of ΔAOB.

$\frac{\Delta AOB}{\Delta DOC} = \left(\frac{6}{18}\right)^2$ M1

Answer 14 A1 cm^2 [2]

(b) In the diagram below, $EFGH$ is a parallelogram.



(i) Show that ΔEHF is congruent to ΔGFH.

Answer:

$EH = GF$ (opp sides of //gram)
$EF = GH$ (opp sides of //gram)
$\angle HEF = \angle FGH$ (opp $\angle$s of //gram)

M1 for any 1

$\therefore \Delta EHF \equiv \Delta GFH$ (SAS congruence test) A1 [2]

(ii) Given that $HK : KF = 2:1$, find $\dfrac{\text{Area of } \Delta EKF}{\text{Area of } \Delta EHK}$.

Answer $\frac{1}{2}$ B1 ..... [1]

(iii) Find $\dfrac{\text{Area of } \Delta EKF}{\text{Area of } EFGH}$.

Answer $\frac{1}{6}$ B1 ..... [1]

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3 On his holiday trip, Mr Lim drove 230km from town A to town B at an average speed of x km/h.

(a) Write an expression in terms of x for the time that he had taken to drive from town A to town B.

$\frac{230}{x}$ B1

Answer ..... hours [1]

(b) On his return trip from town B to town A, Mr Lim increased his average speed by 10 km/h. Write an expression in terms of x for the time that he had taken for his return trip.

$\frac{230}{x+10}$ B1

Answer ..... hours [1]

(c) Mr Lim took 45 minutes less for his return journey.
Form an equation and show that it reduces to $3x^2 + 30x - 9200 = 0$.

Answer:

$\frac{230}{x} - \frac{230}{x+10} = \frac{3}{4}$ M1

$920(x+10) - 920x = 3x(x+10)$ A1

[2]

(d) Solve the equation $3x^2 + 30x - 9200 = 0$, correct to 3 significant figures.

$$x = \frac{-30 \pm \sqrt{(30)^2 - 4(3)(-9200)}}{2(3)} \quad \text{M1}$$

$$= \frac{-30 \pm \sqrt{111300}}{6} \quad \text{M1}$$

$$= 50.603 \text{ or } -60.603$$

Answer $x =$ 50.6 or −60.6 A1 [3]

(e) Hence, find the time that Mr Lim reached town A for his return trip given that he left town B at 09 00.

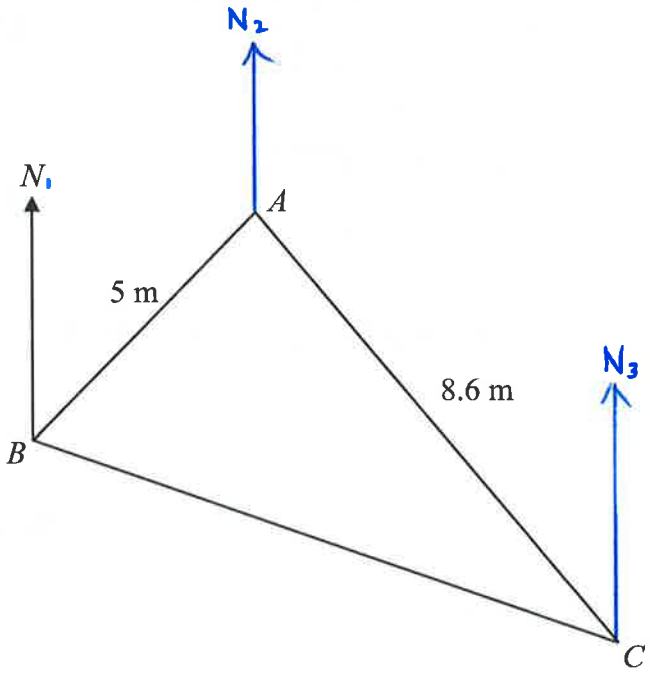
$$\frac{230}{50.603 + 10} = 3.7952$$

M1

$= 3\text{h } 48 \text{ min}$

Answer 1248 A1 [2]

4 The diagram, not drawn to scale, shows a field ABC.
$AB = 5$ m and $AC = 8.6$ m.
The bearing of A from B is 49° and the bearing of C from A is 141°.



(a) Show that $\angle BAC = 88°$.
State your reasons clearly.

Answer:

$\angle N_2AB = 180° - 49°$ (int Ls) M1 −1 if no/wrong reasons

$= 131°$

$\angle BAC = 360° - 131° - 141°$ (Ls at a pt) A1

$= 88°$ [2]

(b) Find the length of BC.

$BC^2 = 5^2 + 8.6^2 - 2(5)(8.6)\cos 88$ M1

$BC = 9.7958$

$= 9.80$ A1

Answer ..... m [2]

(c) Calculate $\angle ACB$.

$$\frac{\sin \angle ACB}{5} = \frac{\sin 88}{9.7958} \quad \text{M1}$$

$$\angle ACB = 30.671$$

$$= 30.7 \quad \text{A1}$$

Answer ..... ° [2]

(d) Find the shortest distance from A to BC.

$$\frac{1}{2} \times 9.7958 \times h = \frac{1}{2}(5)(8.6)\sin 88 \quad \text{M1}$$

$$h = 4.3870$$

$$= 4.39 \quad \text{A1}$$

Answer ..... m [2]

(e) A kite is hovering 6 m vertically above A. Mr Lim is walking along BC. Find the greatest angle of elevation of the kite from Mr Lim.

$$\tan \theta = \frac{6}{4.3870} \quad \text{M1}$$

$$\theta = 53.8 \quad \text{A1}$$

Answer ..... ° [2]

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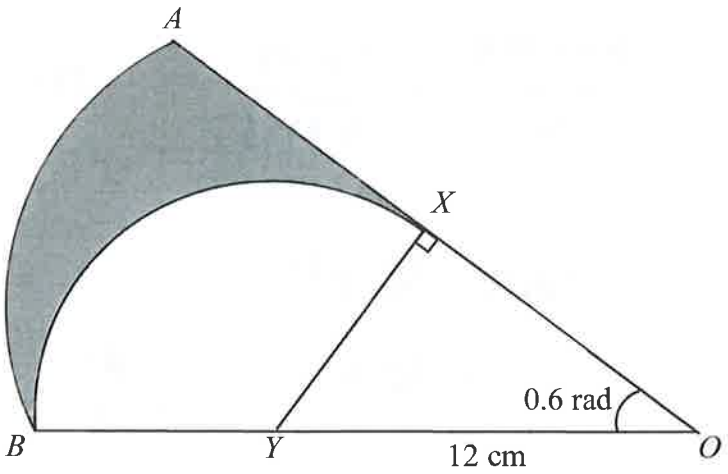
5 In the diagram below, OAB and XYB are sectors of two circles with centres O and Y respectively. $\angle YOX = 0.6$ radians, $OY = 12$ cm and XY is perpendicular to OX.



(a) Show that the length of $XY = 6.78$ cm.

Answer:

$\sin(0.6) = \frac{XY}{12}$ B1

$XY = 12\sin 0.6$

$= 6.7757$

[1]

(b) Show that $\angle BYX = 2.17$ rad. $= 6.78$

Answer:

$\frac{\pi}{2} + 0.6 = 2.17$ OR $\pi - \frac{\pi}{2} - 0.6 = 0.97079$ B1

B1

$\pi - 0.97079 = 2.1708$

≈ 2.17

[1]

(c) **Hence**, find the perimeter of the shaded region ABX.

arc $BX = (6.78)(2.17)$ M1

$= 14.7126$

arc $AB = (6.78 + 12)(0.6)$ M1

$= 11.268$

$OX^2 + 6.78^2 = 12^2$ M1

$OX = 9.9011$

$AX = 18.78 - 9.9011$

$= 8.8789$

Perimeter $= 14.7126 + 11.268 + 8.8789 = 34.9$ A1

Answer ..... cm [4]

(d) Find the area of the shaded region ABX.

Sector $AOB = \frac{1}{2}(18.78)^2(0.6)$ M1

$= 105.80652$

Sector $BYX = \frac{1}{2}(6.78)^2(2.17)$ M1

$= 49.8757$

$\Delta OXY = \frac{1}{2} \times 9.9011 \times 6.78$ M1

$= 33.565$

Shaded area $= 105.80652 - 49.8757 - 33.565$

$= 22.4$ A1

Answer ..... cm [4]

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6 A triangle has vertices $A(-3, 4)$, $B(3, 4)$ and $C(0, 7)$

(a) Find the equation of the line AC.

$m = \frac{7-4}{0+3}$ M1

$= 1$

Answer $y = x + 7$ A1 [2]

(b) Find the distance of line BC.

$\sqrt{(7-4)^2 + (0-3)^2}$ M1

Answer 4.24 A1 [2]

(c) State the equation of line AB.

Answer $y = 4$ B1 [1]

(d) State the equation of the line of symmetry of triangle ABC.

Answer $x = 0$ B1 [1]

(e) Find the area of triangle *ABC*.

$\frac{1}{2} \times 6 \times 3$ M1

Answer 9 A1 [2]

(f) Find the equation of the line that passes through point B and is parallel to the line *AC*.

$y = x + c$ M1

$4 = 3 + c$

$c = 1$

Answer $y = x + 1$ A1 [2]

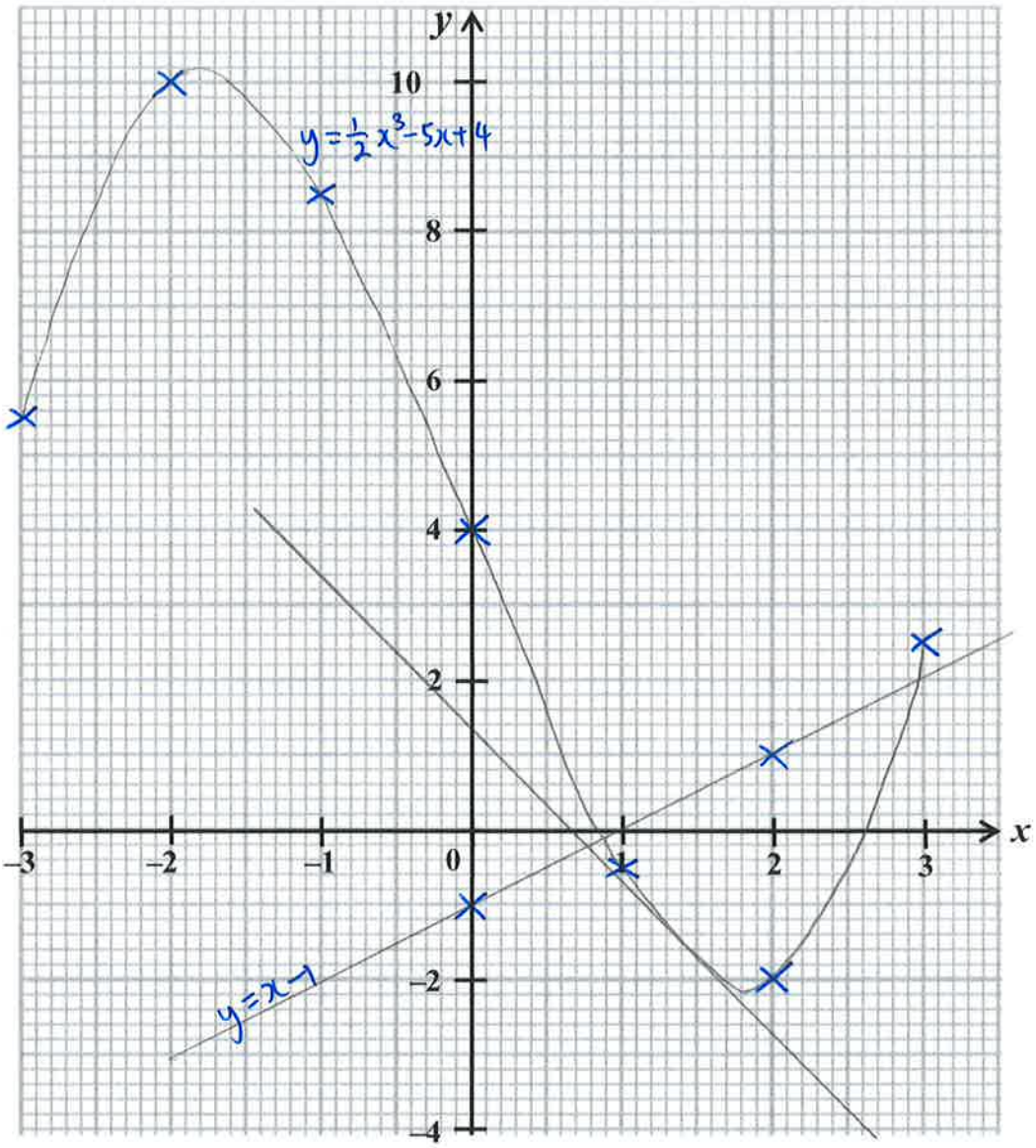
7 The table below is for $y = \frac{1}{2}x^3 - 5x + 4$.

x	–3	–2	–1	0	1	2	3
y	5.5	a	8.5	4	–0.5	–2	2.5

(a) Calculate the value of a.

Answer a = 10 [1]

(b) Draw the graph of $y = \frac{1}{2}x^3 - 5x + 4$ for $-4 \le x \le 3$. [2]



(c) By drawing a tangent, find the gradient of the curve at $x = 1.5$.

tangent : M1

Answer -1·625 A1 [2]

(d) Using your graph, find

(i) the value of y when $x = 0.5$,

Answer $y =$ 1·6 ± 0·05 B1 [1]

(ii) the solutions of $\frac{1}{2}x^3 - 5x + 4 = -1$.

Answer $x =$ 1·2 ± 0·1 B1 or 2·35 ± 0·1 B1 [2]

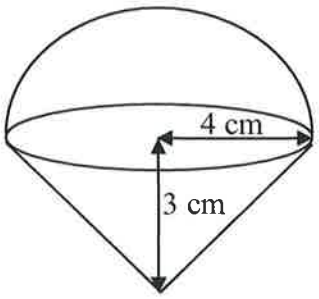
(iii) the solutions of $\frac{1}{2}x^3 - 6x + 2 = -3$.

$y = x - 1$ M1

graph drawn M1

Answer $x =$ 0·9 ± 0·1 or 2·95 ± 0·1 [3] A1

8 The diagram shows a solid that consists of a hemispherical top and a right conical bottom. The radius of each of the two parts is 4 cm. The perpendicular height of the cone is 3 cm. The solid is made of pure gold.



(a) Find the volume of the solid.
Leave your answer in terms of π.

Cone $= \frac{1}{3} \times \pi (4)^2 \times 3$ (M1) $= 16\pi$

hemisphere $= \frac{1}{2} \times \frac{4}{3} \pi (4)^3$ (M1) $= \frac{128}{3}\pi$

Vol $= 16\pi + \frac{128}{3}\pi$

$= \frac{176}{3}\pi$ / $58\frac{2}{3}\pi$ (A1)

Answer ..... cm^3 [3]

(b) Find the total surface area of the solid.

Slant height $= \sqrt{4^2 + 3^2}$ (M1)

$= 5$

Cone $= \pi (4)(5)$ (M1)

$= 20\pi$ / 62.832

hemisphere $= \frac{1}{2} \times 4\pi (4)^2$ (M1)

$= 32\pi$ / 100.53

SA $= 20\pi + 32\pi$

$= 52\pi$

$= 163$

Answer ..... 163 (A1) cm^2 [4]

(c) Mr Lim bought the solid for \$25 000. He wants to melt the solid into smaller identical pyramids with base area of 11π cm^2 and vertical height 2 cm. Calculate the minimum price that he must sell each pyramid in order to not make any losses.

Vol pyramid $= \frac{1}{3} \times 11\pi \times 2$

$= \frac{22}{3}\pi$ / 23.038

No. of pyramid $= \frac{176}{3}\pi \div \frac{22}{3}\pi$

$= 8$

Price $= \frac{25\,000}{8}$

$= \$3125$

Answer \$ 3125 [3]

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9 Mr Lim is comparing prices of buffet caterers for his father's 60th birthday celebration that will be held in a chalet at Pasir Ris. He went online to search for the information and below are the details of the two caterers that he had shortlisted.

Caterer	Hanabi Catering	Grandma's Delicacies
Buffet Package Name	Summer Feast	Grandpa's Favourite
Price	$24.00 per person (inclusive of GST) Minimum order 30 pax	$22.50 per person (exclusive of GST) Minimum order 25 pax
Delivery	• $60 ($64.80 inclusive of GST) • Strictly no delivery to offshore areas (Sentosa and Jurong Island)	• $70 ($75.60 inclusive of GST) • Delivery charge is waived for orders above $1000 after GST
Payment Mode	• Cash, Cheque, Bank Transfer or PayNow upon delivery.	• PayNow, Credit Card, Debit Card or Cheque upon delivery.
Remarks	• 6% discount for weekday promotion lunch • All payment is inclusive of 8% GST charge.	• Orders can only be made in multiples of 5 people. • All payment is inclusive of 8% GST charge

(a) Calculate the amount that Mr Lim needs to pay if he placed a buffet order for 40 people from Hanabi Catering on a Wednesday for lunch.

$24 \times 40 = 960$ M1

$960 + 64.8 = 1024.8$ M1

$\frac{94}{100} \times 1024.8 = 963.31$ A1

Answer $ 963.31 [3]

(b) If Mr Lim wants to order from Grandma's delicacies but does not want to pay for the delivery charge. Calculate the minimum number of people that he needs to order.

$\frac{108}{100} \times 22.5 = \$24.30$ (M1)

$\frac{1000}{24.3} = 41.152$

$= 45$ (A1)

Answer 45 [2]

(c) Mr Lim decided to order a lunch buffet catering for 63 people to be delivered on Thursday. He has a DCBC credit card that provides a 4% discount for food related purchases. Which caterer should he choose? Justify your answer with clear workings below.

Answer:

either (M1)

Hanabi

$63 \times 24 = 1512$

$1512 + 64.8 = 1576.80$

$\frac{94}{100} \times 1576.80 = \$1482.19$ (M1)

Grandma

$65 \times 24.30 = 1579.50$

$\frac{96}{100} \times 1579.5 = 1516.32$ (M1)

$1516.32 - 1482.19 = 34.13$

Answer: Hanabi because cheaper by 34.13 (A1)

..... [4]

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