SPRINGFIELD SECONDARY SCHOOL

Preliminary Examination 2021

STUDENT NAME						
CLASS	**S**		**-**		INDEX NUMBER	

ADDITIONAL MATHEMATICS

Secondary 4 Express

4049/02
30 Aug 2021
2 hours 15 minutes

Candidates answer on the question paper
Additional Materials: Graph paper

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams, graphs or rough working.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use	
Total	**/90**

Do not turn over this question paper until you are told to do so.

This question paper consists of 13 printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}\, b + \binom{n}{2} a^{n-2}\, b^2 + \ldots + \binom{n}{r} a^{n-r}\, b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\text{Area of triangle} = \frac{1}{2} bc \sin A$$

1 Express $\frac{2x^3 - 1}{x^3 - x^2}$ in partial fractions. [6]

2 The table shows experimental values of two variables x and y.

x	0	1	2	3	4	5
y	10	25	50	280	260	480

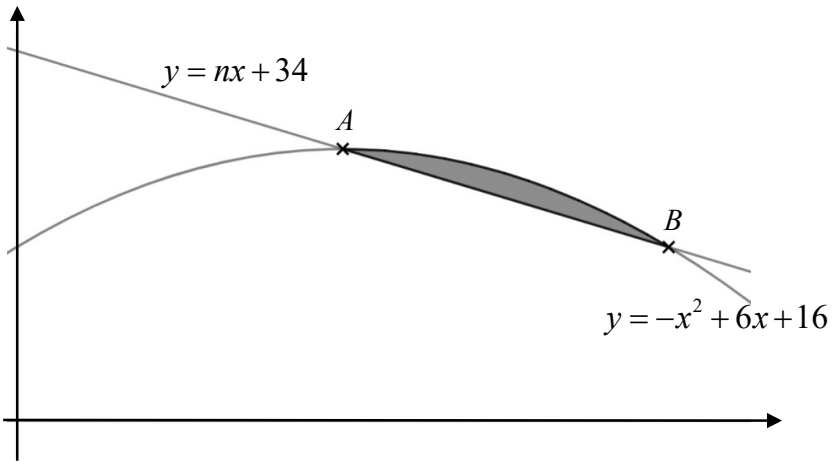
(a) Variables x and y are related by the equation $y = ma^x$, where m and a are constants. Explain how m and a can be calculated when a graph of $\lg y$ against x is drawn. [3]

(b) On the graph paper, draw a straight line graph to illustrate the experimental values. [3]

(c) Hence, determine the values of a and m. [2]

(d) One of the pairs of values is wrong. What should be the correct x value or y value? [2]

3



The diagram above shows a straight line $y = nx + 34$, where n is a constant, intersecting the curve $y = -x^2 + 6x + 16$ at two points A and B respectively. Point A is the highest point of the curve $y = -x^2 + 6x + 16$.

(a) Find the value of n. [4]

(b) Show that the coordinates of B is (6 , 16). [3]

(c) Find the area of the shaded region. [3]

4 **(a)** Determine the range of m if $\dfrac{x^2 - x + 1}{x^2 + x - 1} = m$ has no real roots. [4]

(b) Find the range of values of x for which $1 - x^2 \geq \dfrac{x^2 + 5x}{-3}$. [3]

(c) It is given $\{x : x < -2 \text{ or } x > 5\}$ is the solution set of the quadratic inequality $\dfrac{1}{2}x^2 + px + q > 0$. Find the values of p and q. [3]

5 **(a)** Given that $\int_0^3 f(x)\,dx = \int_3^4 f(x)\,dx = 5$.

Find

(i) $\int_0^4 2f(x)\,dx + \int_4^3 f(x)\,dx$, [2]

(ii) the value of k for which $\int_0^3 \left[f(x) - kx\right]\,dx = 8$ [3]

(b) Find $\frac{d}{dx}(x\sin x)$. Hence evaluate $\int_0^{\frac{\pi}{2}} x\cos x\,dx$. [5]

6 **(a)** Use the substitution $u = 2^x$ to solve the equation $2^{2x} - 7\left(2^x\right) + 12 = 0$. [4]

(b) Solve the equation $\log_4 x - \log_x 8 = \frac{1}{2}$. [8]

7 The equation of the curve is $y = \frac{4x}{\sqrt{x-1}}$.

(a) Find an expressions for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [6]

(b) Find the coordinates of the stationary point. [3]

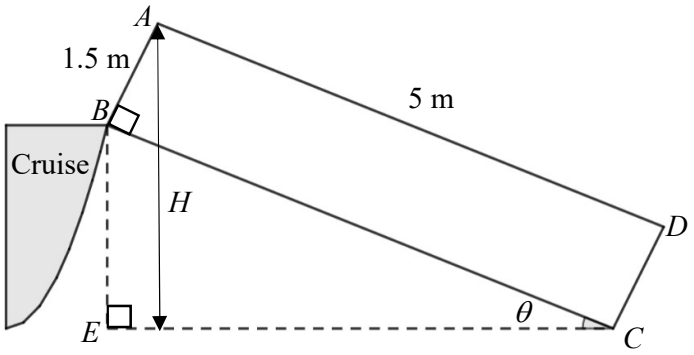
(c) Find the nature of the stationary point [2]

8 **(a)** A line M passes through $P(-3 , -1)$ intersects at right angle with another line L, whose equation is $y + x = 4$ at point C. Find the equation of the line M. [3]

(b) A circle C_1 has P as its centre and PC as its radius. Find the equation of the circle. [5]

(c) Circle C_2 is a reflection of circle C_1 along line L. Find coordinates of the highest point in Circle C_2. [4]

9



The diagram above shows a portion of the cruise ship with an inclined bridge BC for tourist to board the ship. The bridge BC has a handrail AD and tilted at an acute angle, θ from the horizontal ground EC. The handrail AD is parallel to the bridge BC and $\angle ABC = \angle BEC = 90°$.
Given that $AB = 1.5$ m, $AD = 5$ m and H is the height of A above the horizontal ground.

(a) Show that $H = 5\sin\theta + 1.5\cos\theta$. [2]

(b) Express H in the form $R\sin(\theta + \alpha)$, where $R > 0$ and α is an acute angle. [3]

(c) Hence find the value of θ for which $H = 4.8$ m. [2]

(d) Find the maximum value of H and the corresponding value of θ. [2]

End of Paper