AHMAD IBRAHIM SECONDARY SCHOOL
END-OF-YEAR EXAMINATION 2021

SECONDARY 3 EXPRESS

Name: MARKING SCHEME	Class:	Register No.:

MATHEMATICS **4048/01**
Paper 1 **29 September 2021**
2 hours

Candidates answer on the Question Paper**.**

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

For Examiner's Use
/80

This document consists of **20** printed pages.

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Answer **all** the questions.

1 **(a)** Calculate $\dfrac{56.8}{\left(\sqrt[3]{18.22}-1.4\right)^2}$.

Write down the first five digits on your calculator display.

37.459	B1

Answer ……………………………….. [1]

(b) Write your answer to **part (a)** correct to 3 significant figures.

37.5	B1

Answer ……………………………….. [1]

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2 A pond contains 5.62×10^6 drops of water and 2.91×10^9 micro-organisms.
Let N be the average number of micro-organism per drop of water.
Find the value of N.
Give your answer in standard form.

$N=\dfrac{2.91\times10^9}{5.62\times10^6}$ M1

$=517.79$

$=5.18\times10^2$ A1

Answer $N=$……………...……………..[2]

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3 A 4-litre orange juice mixture (water and orange juice syrup) has 16% of orange juice syrup.
How much water must be added to reduce the concentration of orange juice syrup to 8%?

Amount of orange juice syrup

$=0.16\times4l$

$=0.64l$

let the amount of water to be added be x litres,

$\dfrac{0.64}{4+x}\times100\%=8\%$ M1

$0.08x+0.32=0.64$

$x=4$

$4l$ of water must be added. A1

Alternative Method

8% represents 0.64 ℓ M1
1% represents 0.08 ℓ
92% represents 7.36 ℓ

Amount of water to be added = 7.36 – 3.36
= 4 ℓ A1

Answer ...…………...…………….. *l* [2]

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4 Without using a calculator, show that $7^{2021}-7^{2020}$ is a multiple of 3.

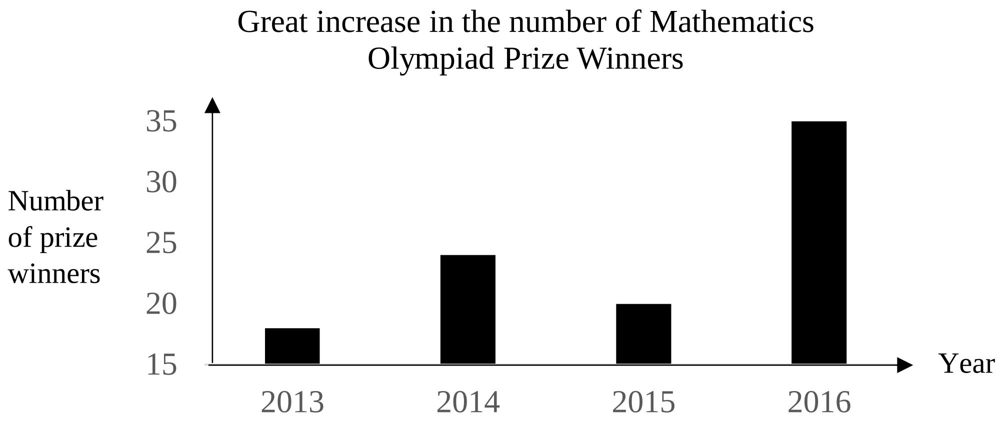
Answer

$7^{2021}-7^{2020}$
$=7^{2020}(7-1)$
$=6(7^{2020})$
$=3(2\times7^{2020})$ M1

Since 3 is a factor of 2×7^{2020}, $7^{2021}-7^{2020}$ is a multiple of 3. A1

[2]

5



The graph shows the number of Mathematics Olympiad Prize wineners over a number of years.

(a) State one misleading feature of the graph.

……………………………………………………………………………………………

The title of the graph is biased. OR

The vertical axis does not start with 0.

B1

…………………………………………………………………………………………….. [1]

(b) Explain how this feature affects the reader's interpretation of the graph.

It does not allow the reader to make his own judgement. OR

It exaggerates the difference in the number of winners over the years.

B1

[1]

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6 Simplify $\left(\dfrac{625x^{12}}{y^8}\right)^{-\frac{1}{4}}$.

$\left(\dfrac{625x^{12}}{y^8}\right)^{-\frac{1}{4}}$

$=\left(\dfrac{y^8}{625x^{12}}\right)^{\frac{1}{4}}$ M1

$=\dfrac{y^2}{5x^3}$ A1

Answer …………………………….. [2]

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7 Simplify $\dfrac{8x^2-2y^2}{12x^2+6xy}$

$$\frac{8x^2-2y^2}{12x^2+6xy}$$
$$=\frac{2(4x^2-y^2)}{6x(2x+y)}$$
$$=\frac{2(2x+y)(2x-y)}{6x(2x+y)}$$

M2 Factorise numerator and denominator

$$=\frac{2(2x-y)}{6x}$$
$$=\frac{2x-y}{3x}$$

A1

Answer ……………………………….. [3]

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8 Factorise completely $a^2-a^4-b+a^2b$.

$a^2-a^4-b+a^2b$

$=a^2(1-a^2)-b(1-a^2)$ M1

$=(a^2-b)(1-a^2)$ M1

$=(a^2-b)(1-a)(1+a)$ A1

Answer ……………………………….. [3]

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9 A bank guarantees an interest of \$$x$ at the end of 2 years with a compound interest of 4% per annum, compounded half yearly, for a principal amount of \$5000.
Calculate the amount of guaranteed interest earned at the end of 2 years.

Total amount = $\$5000\,(1+\frac{2}{100})^4$ M1

Interest

$=\$5000\,(1+\frac{2}{100})^4-\$5000$ M1

$=\$412.16$ A1

Answer \$……………………………… [3]

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10 5 men are hired to paint a house.
If an additional man is hired, the painting can be completed 4 days earlier.
Calculate the number of additional men to be hired if the painting is to be completed 18 days earlier.

$D = \frac{k}{M}$

$D_1 - D_2 = 4$

$\frac{k}{5} - \frac{k}{6} = 4$

$6k - 5k = 120$ M1

$k = 120$

$D_{original} - D_{new} = 18$

$\frac{120}{5} - \frac{120}{M} = 18$ M1

$6 = \frac{120}{M}$

$M = 20$

Additional no. of men = 15 A1

Alternative Method:

5 men – x days

6 men – $(x-4)$ days

$5x = 6(x-4)$

$x = 24$

5 men – 24 days

20 men –6 days

Additional no. of men = 15

Answer …………………………….. [3]

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11 It is given that $p = \sqrt[3]{\frac{1-2r^2}{4q+r^2}}$.
Express r in terms of p and q.

$p = \sqrt[3]{\frac{1-2r^2}{4q+r^2}}$

$p^3 = \frac{1-2r^2}{4q+r^2}$ M1

$4p^3q + p^3r^2 = 1-2r^2$

$p^3r^2 + 2r^2 = 1-4p^3q$

$r^2(p^3+2) = 1-4p^3q$

$r^2 = \frac{1-4p^3q}{p^3+2}$ M1

$r = \pm\sqrt{\frac{1-4p^3q}{p^3+2}}$ A1

Answer $r =$………………………….. [3]

[Turn over

12 Solve the equation $\dfrac{2}{x-5} - 3 = \dfrac{4x+1}{3}$.

$$\frac{2}{x-5} - 3 = \frac{4x+1}{3}$$
$$2(3) - 3(3)(x-5) = (4x+1)(x-5)$$
$$6 - 9x + 45 = 4x^2 - 20x + x - 5$$
$$4x^2 - 10x - 56 = 0 \qquad \boxed{\text{M1}}$$
$$2x^2 - 5x - 28 = 0$$
$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(-28)}}{2(2)} \qquad \boxed{\text{M1}}$$
$$x = 5.19(3sf)$$
or
$$x = -2.69(3sf) \qquad \boxed{\text{A1, both answers}}$$

Answer $x =$ ………… or ………….. [3]

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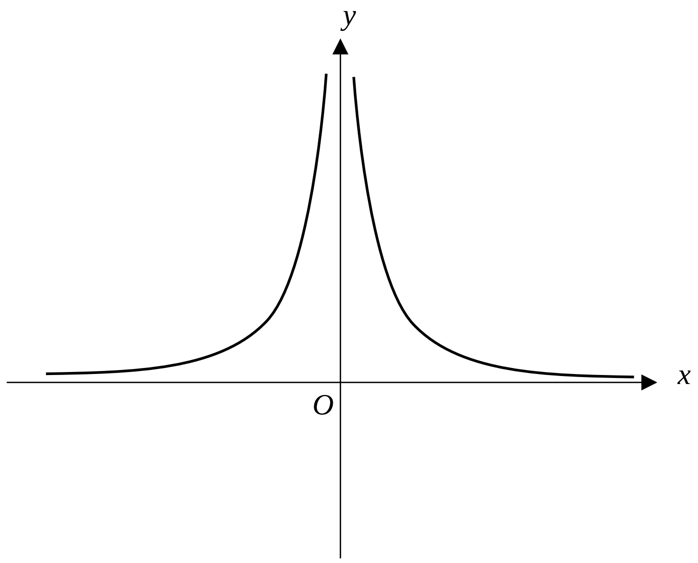
13 **(a)** The sketch shows the graph of $y = kx^n$.



Write down a possible value of *n*.

$\boxed{-2}$ $\boxed{\text{B1}}$

Answer $n=$………………………….. [1]

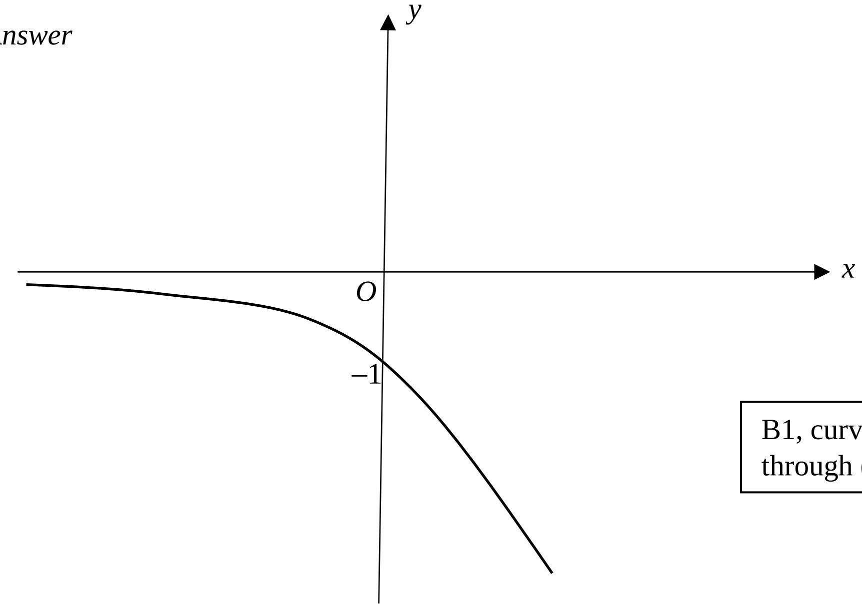
(b) State the range of values of *k*.

$\boxed{k > 0}$ $\boxed{\text{B1}}$

Answer ...………………………….. [1]

(c) Sketch the graph of $y = -3^x$

Answer



B1, curve must pass through $(0, -1)$

[1]

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14 Two geometrically similar figurines are made of the same material.
The base areas of the smaller and larger figurines are 210 cm^2 and 680.4 cm^2 respectively.

(a) Find the ratio of the height of the smaller figurine : the height of the larger figurine.

$$\frac{\text{height of smaller figurine}}{\text{height of larger figurine}} = \sqrt{\frac{210}{680.4}}$$

$$= \sqrt{\frac{25}{81}}$$

$$= \frac{5}{9}$$

ratio= 5:9

B1

Answer …………… : …………….. [1]

[Turn over

(b) The mass of the smaller figurine is 750 g.
Find the mass of the larger figurine.

$$\frac{750}{\text{mass of larger figurine}} = \left(\frac{5}{9}\right)^3$$ M1

$$\text{mass of larger figurine} = \frac{750}{\left(\frac{5}{9}\right)^3}$$

$$= 4374g$$ A1

Answer ………………………… g [2]

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15 **(a)** Solve the inequalities $\frac{3x-2}{3} \le 4x-1 < 16-\frac{x}{4}$.

$$\frac{3x-2}{3} \le 4x-1 < 16-\frac{x}{4}$$

$$\frac{3x-2}{3} \le 4x-1$$

$$3x-2 \le 12x-3$$

$$1 \le 9x$$

$$\frac{1}{9} \le x$$ M1

and

$$4x-1 < 16-\frac{x}{4}$$

$$16x-4 < 64-x$$

$$17x < 68$$

$$x < 4$$ M1

$$\therefore \frac{1}{9} \le x < 4$$ A1

Answer …………………………….. [3]

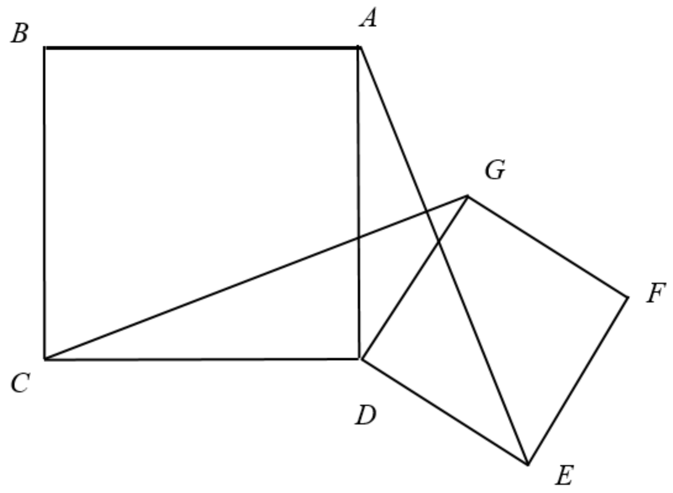
(b) Write down the smallest rational number which satisfies $\frac{3x-2}{3} \le 4x-1 < 16-\frac{x}{4}$.

B1

Smallest rational number$=\frac{1}{9}$

Answer …………………………….. [1]

16



ABCD and *DEFG* are squares.
Show that triangle *ADE* is congruent to triangle *CDG*.
Give a reason for each statement you make.

Answer

$AD = CD$ (Sides of square)
$\angle EDG = \angle CDA = 90°$ ($\angle$ of square)
$\angle GDA$ is a common angle.
$\angle EDG + \angle GDA = \angle CDA + \angle GDA$
$\angle ADE = \angle CDG$
$DE = DG$ (Sides of square)
$\therefore \Delta ADE \equiv \Delta CDG$ (SAS)

B2 for 3 correct statements with reasoning

B1 for 2 correct statement with reasoning

B1 for correct congruency test

[3]

17 Given that the coordinates of Q is $(1, -2)$, the gradient of PQ is -1 and the length of PQ is $\sqrt{50}$ units, find two possible coordinates of P.

Let coordinate of P be (x, y).

$$\frac{y-(-2)}{x-1} = -1$$
$$y+2 = -x+1$$
$$y = -x-1 - - - - - - -(1)$$

M1 Using gradient to form first equation

$$\sqrt{(x-1)^2 + (y-(-2))^2} = \sqrt{50}$$
$$(x-1)^2 + (y+2)^2 = 50 - - - - - -(2)$$

M1 Using length to form second equation

sub (1) into (2),

$$(x-1)^2 + (-x+1)^2 = 50$$
$$x^2 - 2x + 1 + x^2 - 2x + 1 = 50$$
$$2x^2 - 4x - 48 = 0$$
$$x^2 - 2x - 24 = 0$$
$$(x+4)(x-6) = 0$$
$$x = -4 \quad \text{or} \quad 6$$

M1 Correct quadratic equation

when $x = -4$, $y = -(-4) - 1$
$= 3$

when $x = 6$, $y = -6 - 1$
$= -7$

$\therefore P(-4, 3)$ or $P(6, -7)$

A1 Correct coordinates of P

Answer (……………. , …………….) or

(……………. , …………….)[4]

18 **(a)** Given that $3a\times10^5+5b\times10^4=c\times10^6$, find b in terms of a and c.
Give your answer in its simplest form.

$3a\times10\times10^4+5b\times10^4=c\times10^2\times10^4$ M1

$30a+5b=100c$

$5b=100c-30a$

$b=20c-6a$ A1

Answer ………………………………. [2]

(b) Two integers, A and B, can be written as product of prime factors.

$A=2^{m+2}\times3^n$ $B=2^m\times3^{n+1}\times5$

(i) Find the lowest common multiple of A and B.
Give your answer as a product of its prime factors in terms of m and n.

$\text{LCM}=2^{m+2}\times3^{n+1}\times5$ B1

Answer ………………………….......... [1]

(ii) Hence find the smallest value of the lowest common multiple of A and B.

LCM

$=2^{1+2}\times3^{1+1}\times5$

$=360$ B1

Answer ………………………….......... [1]

19 **(a)** Express $4-6x+x^2$ in the form $p+(x-q)^2$.

$4-6x+x^2$
$=(x-3)^2-(-3)^2+4$
$=-5+(x-3)^2$

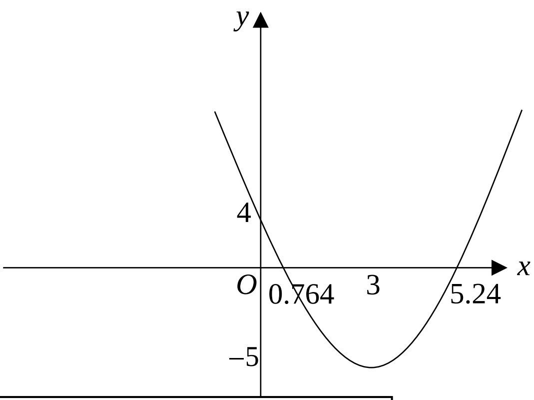
M1 Completing the square

A1

Answer ……………………………….. [2]

(b) Hence sketch the graph of $y=4-6x+x^2$.
Indicate clearly the values where the graph crosses the coordinate axes.

Answer



B1 Parabola passing through (3, −5) and (0,4)

B1 Shape

[2]

(c) Write down the equation of the line of symmetry of the graph.
$x=3$ B1

Answer ………………………………..[1]

(d) Explain why the equation $4-6x+x^2=k$ does not have solutions for some values of *k*.

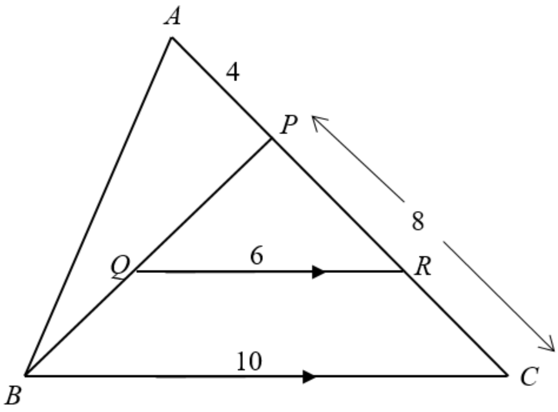
Answer ………………………………………………………………………………...

…………………………………………………………………………………………

…………………………………………………………………………………………..[1]

From the graph, the minimum value of $y=x^2-6x+4$ occurs at $y=-5$.
Hence the equation $4-6x+x^2=k$ does not have solutions for $k<-5$. B1

20



BQP and *APRC* are straight lines and *QR* is parallel to *BC*.

(a) Show that triangle *PQR* and triangle *PBC* are similar.
Give a reason for each statement you make.

Answer

B2 for 3 correct statements with reasoning

B1 for 2 correct statement with reasoning

$\angle PQR = \angle PBC$ (Corr. $\angle$s, $QR // BC$)
$\angle QRP = \angle BCP$ (Corr. $\angle$s, $QR // BC$)
Triangle *PQR* and triangle *PBC* are similar. (2 pairs of corr. angles are equal) [2]

(b) Given that *QR* = 6 cm, *BC* = 10 cm, *PC* = 8 cm and *AP* = 4 cm.
Calculate *PR*.

$$\frac{PR}{6} = \frac{8}{10}$$
$PR = 4.8$ cm B1

Answer PR =……………………cm [1]

(c) Find the ratio area of triangle *PQR*: area of triangle *ABP*.

$$\frac{\text{Area of } \Delta ABP}{\text{Area of } \Delta BPC} = \frac{1}{2} \text{ (Common height)}$$

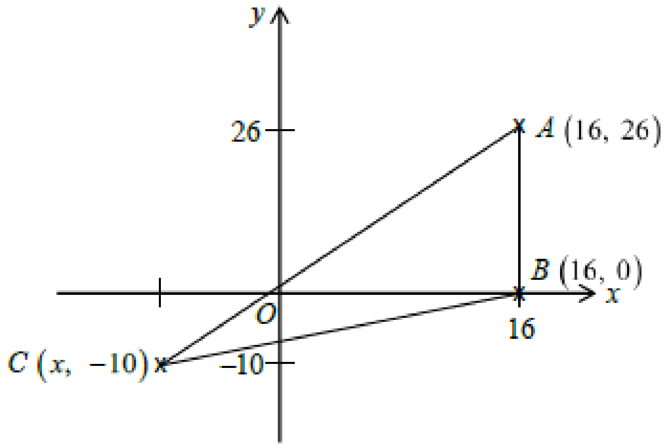
$$\frac{\text{Area of } \Delta PQR}{\text{Area of } \Delta PBC} = \left(\frac{6}{10}\right)^2$$ B1

$$\frac{\text{Area of } \Delta PQR}{\text{Area of } \Delta ABP} = \frac{\text{Area of } \Delta PQR}{\text{Area of } \Delta PBC} \times \frac{\text{Area of } \Delta PBC}{\text{Area of } \Delta ABP}$$
$$= \frac{9}{25} \times \frac{2}{1}$$
$$= \frac{18}{25}$$ B1

Answer ..………………………… [2]

[Turn over

21



A, B and C are points (16, 26), (16,0) and $(x, -10)$.

(a) Given that $\cos\angle ABC = -\frac{5}{13}$, find the x-coordinate of C.

Extend line AB downward and let the point below B be D,

$\cos\angle CBD = \frac{5}{13} = \frac{10}{26}$ — B1

Length of C to AB produced = $\sqrt{26^2 - 10^2} = 24$ units — M1

x-coordinate of C = $16 - 24 = -8$ — A1

Answer x=………………………… [3]

(b) D is a point such that $ACDB$ is a trapezium where AB is parallel to CD.
The area of $ACDB$ is 648 units^2.
Find the coordinates of point D.

Let the y-coordinate of D be y,

$\frac{1}{2}(-10 - y + 26) \times 24 = 648$ — M1 Formulating equation using area of trapezium

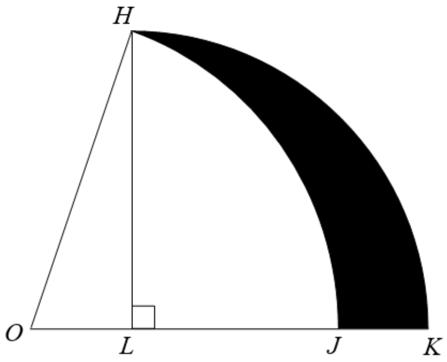
$-y + 16 = 54$

$y = -38$

Coordinates of D $= (-8, -38)$ — A1

Answer D (………… , …………..) [2]

22



HLK is a quadrant, centre *L*, and $LK = 4$ cm.
J is a point on *LK* such that *OHJ* is a sector of a circle centre *O*.
$OL = 2$ cm.

(a) Calculate angle *HOK* in degree.

$\angle HOK$
$= \tan^{-1}\left(\frac{4}{2}\right)$
$= 63.435°$
$= 63.4°$ (1 d.p.) B1

Answer Angle $HOK =$ ………………………… [1]

(b) Calculate the length of the *JK*.

OH
$= \sqrt{2^2 + 4^2}$
$= \sqrt{20}$ cm

LJ
$= (\sqrt{20} - 2)$ cm M1 Finding both *OH* and *LJ*

JK
$= 4 - (\sqrt{20} - 2)$
$= (6 - \sqrt{20})$ cm
$= 1.53$ cm (3 s.f.) M1

Answer $JK =$ …………………………….. cm [2]

(c) Calculate the area of the shaded region.

Area of region HLJ

= Area of sector OHJ − Area of ΔHOL

$$= \frac{63.435°}{360°} \times \pi \times \left(\sqrt{20}\right)^2 - \frac{1}{2} \times 2 \times 4$$ M1

$= 7.07149611 \text{ cm}^2$

Area of shaded region

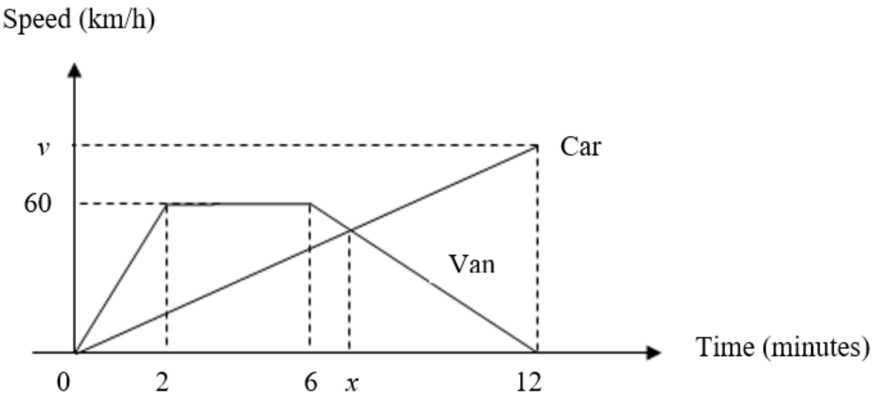
= Area of quadrant HLK − Area of region HLJ

$$= \frac{90°}{360°} \times \pi \times 4^2 - 7.07149611$$ M1

$= 5.49 \text{ cm}^2$ (3 s.f.) A1

Answer ………………………………………cm² [3]

23



The diagram shows the speed-time graphs of a car's and a van's journey during a period of 12 minutes.
The car and van start from the same point at the same time and travel in the same direction.

(a) Calculate the acceleration of the van at the first minute of the journey.

Acceleration

$= \dfrac{60}{\frac{2}{60}}$

$= 1800 \text{ km/h}^2$ B1

Answer ……………………………………km/h^2 [1]

(b) Given that the two vehicles travelled the same distance in the 12 minutes journey, find the speed, *v* km/h, of the car at the end of the 12 minutes.

$\frac{1}{2} \times \frac{12}{60} \times v = \frac{1}{2} \times (\frac{4}{60} + \frac{12}{60}) \times 60$ M1 Formulating equation using area

$v = 80$ A1

Answer …………………………………….km/h [2]

(c) The car and van are travelling at the same speed when time is *x* minutes.
Find the value of *x*.

Equation for Car's journey: $y = 400x - - - (1)$ M1 Finding equation of line

Equation for Van's journey from 6-12 minutes:

$\text{Gradient} = \dfrac{60-0}{\frac{6}{60} - \frac{12}{60}} = -600$

$y = -600x + c$

$60 = -600(\frac{6}{60}) + c$ M1 Finding equation of line

$c = 120$

$y = -600x + 120 - - - (2)$

$400x = -600x + 120$

$1000x = 120$

$x = \frac{3}{5}\text{h}$

$= 7.2\,\text{min}$

M1 Solving simultaneous equations

A1

Answer $x =$ ……………………………………....[4]

End of Paper