Name	( )	Class	

RIVER VALLEY HIGH SCHOOL
2012 Year 6 Preliminary Examination
Higher 2

MATHEMATICS 9740/01

Paper 1

12 September 2012

3 hours

Additional Materials: Answer Paper
List of Formulae (MF15)
Cover Page

READ THESE INSTRUCTIONS FIRST

Do not open this booklet until you are told to do so.
Write your name, class and index number in the space at the top of this page.

Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphic calculator.
Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, place the cover page on top of your answer paper and fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

1 One root of the equation $z^4 - 2z^3 + 14z^2 + az + b = 0$, where a and b are real, is $z = 1 + 2\mathrm{i}$. Find the values of a and b and the other roots. [5]

Deduce the roots of the equation $z^4 + 2iz^3 - 14z^2 - 18iz + 45 = 0$. [2]

2 Without the use of a calculator, solve the inequality

$$x \geq \frac{9}{x}.$$ [3]

Hence find $\int_n^4 \left| x - \frac{9}{x} \right| \mathrm{d}x$ in terms of n, where $0 < n < 3$. [4]

Describe the behaviour of the value of the integral as $n \to 0$. [1]

3 Relative to the origin O, the points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ given by $\mathbf{a} = p\mathbf{i} + (p-1)\mathbf{j} + 2\mathbf{k}$ and $\mathbf{b} = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$ where $p > 0$. The point Q is such that $OAQB$ is a parallelogram.

(i) Find the position vector of Q when $p = 1$. [2]

(ii) The vector $\mathbf{c}$ is a unit vector in the direction of $\overrightarrow{OB}$. Give the geometrical meaning of $(\mathbf{a} \cdot \mathbf{c})\mathbf{c}$. [1]

(iii) Find the range of values of p if $|\mathbf{a}| < |\mathbf{b}|$. [3]

(iv) When $p = 2$, determine whether $\mathbf{a} + \mathbf{b}$ and $\mathbf{a} - \mathbf{b}$ are perpendicular.
Hence determine the geometrical meaning of $|\mathbf{a} \times \mathbf{b}|$. [3]

4 (For this question, leave all your answers in terms of π.)

It is given that $y = \left(\cos^{-1} x\right)^2$. Show that $(1 - x^2)\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 = 4y$. [2]

By further differentiation of this result, find the Maclaurin's series for y up to and including the term in x^2. [4]

Deduce

(i) the equation of the tangent to the curve $y = \left(\cos^{-1} x\right)^2$ at the point where $x = 0$, [1]

(ii) the first two non-zero terms in the series expansion of $\frac{2\cos^{-1} x}{x^2 - 1}$ by expressing $1 - x^2$ as $\left(\sqrt{1 - x^2}\right)^2$. [2]

5 In order to humidify an air-conditioned bedroom, Victoria decides to place a glass of water in her bedroom. On the first day, she prepares a glass filled with 80 cm^3 of water. It is reckoned that 20% of the water in the glass will be lost at the end of each day due to evaporation. As a result, Victoria decides to pour in an additional 40 cm^3 of water into the glass at the beginning of each day, starting from the second day.

(i) Find the volume of water in the glass at the end of the second day. [1]

(ii) Show that the volume of water in the glass at the end of the nth day is

$$\left(160-120(0.8)^n\right)\ \text{cm}^3.$$ [4]

(iii) Suppose that the maximum capacity of the glass Victoria used is 180 cm^3. Find the earliest possible day such that the addition of 40 cm^3 of water leads to the first case of overflowing of the glass. [3]

(iv) Find the minimum capacity of the glass Victoria should use so that overflowing will not happen. [2]

6 The function f is defined by

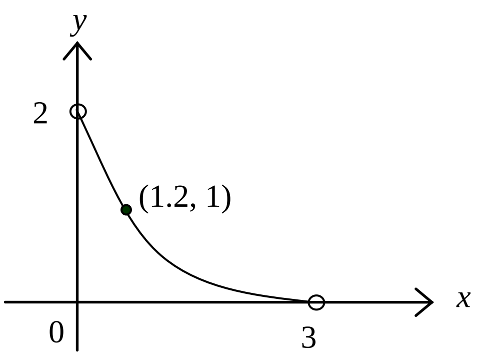
$$\mathrm{f} : x \mapsto x^2+2x-3, \quad x \le a.$$

(i) Explain why f^{-1} does not exist when $a=1$. [1]

(ii) State the largest value of a such that f^{-1} exists. Find f^{-1}, stating its domain. [3]

(iii) Find the exact solution of the equation $\mathrm{f}(x) = \mathrm{f}^{-1}(x)$, using the value of a found in part (ii). [2]

The function g has domain (0, 3) and its graph passes through the point with coordinates (1.2, 1). The graph of g is given below.



For the rest of the question, take $a=1$.

(iv) Give a reason why fg does not exist, where f is the function given above. [1]

(v) State the largest domain of g such that fg exists. Hence, find the range of fg, showing clearly your working. [3]

7 The curve C has equation $y = \dfrac{2x^2 + 13x + 23}{x+3}$.

(i) Prove, using an algebraic method, that C cannot lie between two values which are to be determined. [3]

(ii) State the equations of the asymptotes of C. [2]

(iii) Draw a sketch of C, showing clearly any axial intercepts, asymptotes and stationary points. [3]

(iv) By considering a circle with centre at the point $(-3, 1)$, find the range of values of k such that the equation $(x+3)^2 + \left(\dfrac{2x^2 + 12x + 20}{x+3}\right)^2 = k^2$ has a positive root. [3]

8 The parametric equations of a curve are

$$x = \sec t, \qquad y = \tan t$$

where $0 < t < \dfrac{\pi}{2}$.

(i) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t. [2]

(ii) Show that the equation of the tangent to the curve at the point $P(\sec\theta, \tan\theta)$, is of the form $y = mx - \cot\theta$ where m consists of a single trigonometric term. [3]

(iii) The tangent at the point P intersects the x-axis and the y-axis at the points A and B respectively. Given that $\theta = \dfrac{\pi}{6}$, find the exact area of triangle AOB. [3]

(iv) Find a cartesian equation of the locus of the mid-point of AB as θ varies. [3]

9 (a) (i) Express $\dfrac{r+1}{r+2}-\dfrac{r}{r+1}$ as a single fraction. [1]

(ii) Hence, find $\displaystyle\sum_{r=1}^{10}\left[\frac{1}{r^2+3r+2}-\ln r^2\right]$ in the form $\dfrac{p}{q}-2\ln k$ where p, q and k are integers. [4]

(b) Use the method of mathematical induction to prove that

$$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2 .$$ [4]

Hence, find $\displaystyle\sum_{r=6}^{n+3}(r-4)^3$. [3]

10 The equations of three planes p_1, p_2 and p_3 are

$$2y - z = 0,$$
$$\beta x + z = 2,$$
$$x + \lambda y - 2z = \mu$$

respectively, where β, λ and μ are constants.

Relative to the origin O, the points A and B have position vectors given by $4\mathbf{k}$ and $3\mathbf{j}$ respectively.

(i) Find the acute angle between p_1 and the z-axis. Hence or otherwise, find the exact distance from the point A to p_1. [4]

(ii) A plane p_4 is parallel to the plane p_1 such that the distance of p_4 from the point B is twice that of the distance of p_1 from the point B. Find the two possible vector equations of p_4, in scalar product form. [3]

(iii) Verify that the point with coordinates $(0, 1, 2)$ lies on the planes p_1 and p_2. The planes p_1 and p_2 intersect in a line l. Find the equation of the line l in terms of β. [3]

(iv) Given that $\beta = 2$ and the three planes p_1, p_2 and p_3 have no point in common, what can be said about the values of λ and μ? [3]

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