Check your Understanding (P&C)

Section 1: Combinations

1) A committee of five members is to be chosen from seven men and five women. Find the number of ways which this committee can be formed if

(i) the committee consists of three men and two women, **[2]**

(ii) the man Aaron or the woman Beatrice or both of them must be in the committee. **[3]**

(i)	Number of ways $= {}^{7}C_3 \times {}^{5}C_2$ $= 35 \times 10$ $= 350$
(ii)	**Method 1 (using complement)** Total number of ways = Number of ways of forming committee – Number of ways of forming committee without Aaron and Beatrice $= {}^{12}C_5 - {}^{10}C_5$ $= 792 - 252$ $= 540$ **Method 2** **Case 1 – Aaron is in the committee** Number of ways $= {}^{11}C_4 = 330$ **Case 2 – Beatrice is in the committee** Number of ways $= {}^{11}C_4 = 330$ Number of ways $= 330 + 330 - {}^{10}C_3$ $= 660 - 120$ $= 540$ **Method 3** **Case 1 – Only Alan is in the committee** Number of ways $= {}^{10}C_4 = 210$ **Case 2 – Only Beatrice is in the committee** Number of ways $= {}^{10}C_4 = 210$ Number of ways $= 210 + 210 + {}^{10}C_3$ $= 420 + 120$ $= 540$

2) Five men and five women took part in a competition. Of these ten people, three prize winners are chosen at random. Find the number of ways that the prize winners can be chosen such that there are

(i) no restrictions, [1]

(ii) at least two women. [2]

(i)	No. of ways to choose 3 prize winners without restriction = ${}^{10}C_3 = 120$
(ii)	No. of ways all three winners are women = ${}^{5}C_3 = 10$ No. of ways two women and one man = ${}^{5}C_2 \times {}^{5}C_1 = 50$ Number of ways required = 60

3) A group of 18 students are to be selected from 12 boys and 15 girls to go for an Overseas Community Service Project. Find the number of ways in which the group can be formed if

(i) an equal number of boys and girls is selected, [2]

(ii) a particular boy and a particular girl cannot be selected together, [2]

(iii) no more than 5 boys are selected. [3]

3(i)	No of ways for 9 boys and 9 girls selected $= {}^{12}C_9 \times {}^{15}C_9$ $= 1101100$
(ii)	Required no of ways $= {}^{27}C_{18} - {}^{25}C_{16}$ $= 2643850$
(iii)	Case 1: 3 boys and 15 girls No of ways = ${}^{12}C_3 \times {}^{15}C_{15} = 220$ Case 2: 4 boys and 14 girls No of ways = ${}^{12}C_4 \times {}^{15}C_{14} = 7425$ Case 3: 5 boys and 13 girls No of ways = ${}^{12}C_5 \times {}^{15}C_{13} = 83160$ Total of ways = 220 +7425+83160 = 90805

4) A school is required to send a delegation of 10 teachers to attend the Teachers' Conference 2018. This group of 10 teachers are to be selected from a pool of 7 Mathematics teachers, 5 Humanities teachers and 4 Science teachers.

(i) How many different delegations can be formed? [1]

One of the Mathematics teachers is the brother of a particular Humanities teacher.

(ii) How many different delegations can be formed such that the siblings cannot be in the same delegation? [2]

4	(i) No of different delegations $= {}^{16}C_{10} = 8008$ (ii) <u>Method 1</u> No of different delegations $= {}^{14}C_9 \times 2 + {}^{14}C_{10}$ $= 5005$ <u>Method 2</u> No of different delegations $= {}^{16}C_{10} - {}^{14}C_8$ $= 5005$

<u>Section 2: Permutations</u>

1. Ken has a mobile phone which allows him to set a password consisting of 5 characters. The characters are to be chosen from {1, 2, 3, 4, 5, 6, 7, A, B, C, D, E}.

(i) Find the number of possible passwords if repetitions are not allowed. [1]

(ii) Ken has set a password which he could not recall. However, he is certain that he uses 3 distinct digits and 2 distinct letters. He attempts to recall the password. Find the maximum number of failed possible attempts he need to make before he can recall the password correctly. [2]

1(i)	Number of possible passwords = ${}^{12}C_5 \times 5! = 95040$
(ii)	Total possible passwords = ${}^{7}C_3 \times {}^{5}C_2 \times 5! = 42000$ Maximum number of failed attempts = $42000 - 1 = 41999$

2) Find the number of arrangements which can be formed from all 8 letters of the word *QUESTION* if

(i) there are no restrictions, [1]

(ii) the letters *O* and *N* are together, [2]

(iii) the letters *O* and *N* are separated. [1]

2 (i)	Total number of arrangements: 8! = 40 320 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1
(ii)	Total number of arrangements: 7! x 2! = 10080 ON
(iii)	No of ways with no restriction – No of ways when O&N are consecutive = 8!-7!2!=30240 <u>Alternative method:</u> Total number of arrangements: $6! \times {}^{7}C_{2} \times 2! = 30240$ Insert : *O, N*

3) In a students' council phototaking session, committee members consisting of five girls, six boys and a male teacher are to arrange themselves in two rows of seats shown below.

1^{st} row 1 2 3 4 5

2^{nd} row 1 2 3 4 5 6 7

Find the number of ways the committee members can be arranged in two rows if

(i) there is no restrictions, [1]

(ii) the 5 girls are seated together and the 6 boys are seated together. [2]

3(i)	12! = 479,001,600
(ii)	5! 6! x2! = 172,800

4) A student from Happy College has been told to reset her password for the school's portal for security reasons. The password must consist of 7 characters. The first 3 characters of the password consist of 3 letters chosen from $\{A, B, C, D, E, F\}$. The last 4 characters of the password consist of 4 digits chosen from $\{1, 2, 3, 4, 5, 6, 7, 8\}$.

(i) How many different passwords can be formed if repetitions are not allowed? [2]

(ii) Given that the last 4 characters is an even number greater than 5000, find the number of different passwords that can be formed if repetitions are not allowed. [4]

4(i)	**<u>Method 1:</u>** No. of ways $=\left({}^{6}C_{3}\times 3!\right)\times\left({}^{8}C_{4}\times 4!\right)$ $=201\,600$ **<u>Method 2:</u>** No. of ways $={}^{6}P_{3}\times{}^{8}P_{4}$ $=201\,600$ **<u>Method 3:</u>** No. of ways $=(6\times5\times4)\times(8\times7\times6\times5)$ $=201\,600$
4(ii) **[4]**	**<u>Method 1:</u>** <u>Case 1: last digit is '2' or '4'</u> No. of ways $=\left({}^{6}C_{3}\times 3!\right)\times(4\times6\times5\times2)$ $=28800$ <u>Case 2: last digit is '6' or '8'</u> $=\left({}^{6}C_{3}\times 3!\right)\times(3\times6\times5\times2)$ $=21600$ Total number of ways = 28800 + 21600 = 50400 **<u>Method 2:</u>** <u>Case 1: first digit is '5' or '7'</u> No. of ways $=\left({}^{6}C_{3}\times 3!\right)\times(2\times6\times5\times4)$ $=28800$ <u>Case 2: first digit is '6' or '8'</u> $=\left({}^{6}C_{3}\times 3!\right)\times(2\times6\times5\times3)$ $=21600$ Total number of ways = 28800 + 21600 = 50400

5) 5 male students, 4 female students and 1 teacher are going for a class photoshoot. All 10 of them are involved and they have to sit in a single row of 10 seats.

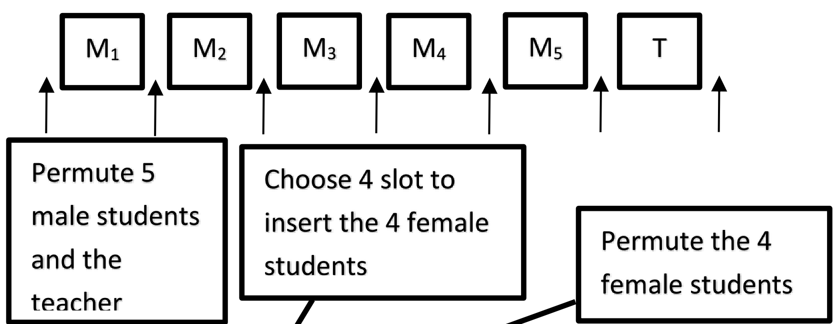
Find the number of ways they can be seated if

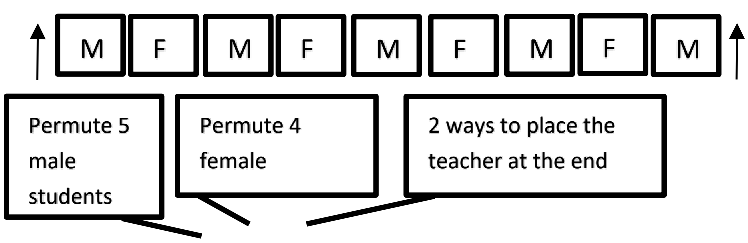
(i) there is no restriction, [1]

(ii) no two female students are to sit next to each other, [2]

(iii) all the students are seated together such that the male and female students must alternate, [2]

(iv) the teacher must be seated between a male student and a female student. [3]

<table>
<tr><td>(i)</td><td>No of ways = $10! = 3628800$</td></tr>
<tr><td>(ii)</td><td>


No of ways = $6! \times {}^7C_4 \times 4! = 604800$</td></tr>
<tr><td>(iii)</td><td>


No of ways = $5! \times 4! \times 2! = 5760$</td></tr>
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(iv)	Group MTF as a unit. 5 units in total $\boxed{\text{MTF}}\ \boxed{\text{M}}\ \boxed{\text{M}}\ \boxed{\text{M}}\ \boxed{\text{M}}\ \boxed{\text{F}}\ \boxed{\text{F}}\ \boxed{\text{F}}$ Permute 8 units Choose 1M and 1F for group MTF Permute MF for group MTF No of ways $= 8! \times \left({}^5C_1 \times {}^4C_1 \times 2! \right) = 1612800$

6) The word BINOCULARS has ten distinct letters.

(a) Find the number of ways in which all the letters can be arranged if

(i) there are no restrictions, [1]

(ii) the first and last letters are vowels. [2]

(b) 6 letters are randomly selected from the ten distinct letters to form a codeword. Find the number of 6-letter codewords which have exactly one vowel. [3]

(ai)	The number of ways $= 10! = 3628800$
(aii)	The vowels are I, O, U, A The number of ways $= \binom{4}{2} 2! \times 8!$ = 483840
(b)	**Step 1** ${}^4C_1$: choose vowel ${}^6C_5$: choose consonants **Step 2** 6! choose and arrange the 6 chosen letters The number of 6-letter codewords which have exactly one vowel $= \binom{4}{1}\binom{6}{5} 6!$ = 17280