# Oscillations Problem Set

**Exercises**

**Free Oscillations**

E1. The displacement $x$ (in metres) of a body in oscillatory motion is given by the equation:

$$x = 3.5 \sin 4.0\,t$$

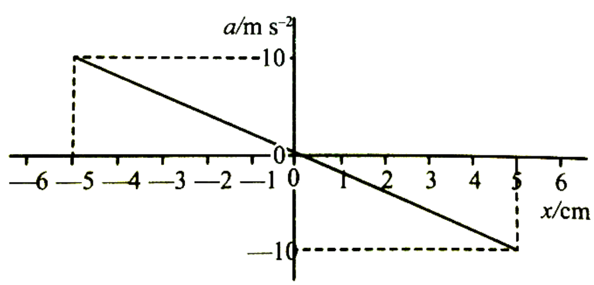
(a) Determine the amplitude of the motion.
(b) Determine the angular frequency.
(c) Determine the displacement 0.20 s after oscillation is begun. [ 2.5 m ]
(d) Find the expression of velocity in terms of time and, [$v$ = 14 cos 4.0$t$ ]
(e) Hence determine the velocity at time $t$ = 0.50 s. [ -5.8 m $s^{-1}$ ]

E2. [N06/1/13] A simple harmonic oscillator has a time period of 10 seconds. Which equation relates its acceleration **a** and displacement **x**?

**A** $a = -(20/2\pi)^2x$ **B** $a = -(20\pi)^2x$
**C** $a = -(20/2\pi)x$ **D** $a = -(2\pi/10)^2x$

**Ans: [D]**

E3. [N08/1/15] The defining equation for a particle moving in simple harmonic motion $a = -\omega^2 x$ where $a$ is the acceleration of the particle, $x$ is the displacement and is the angular frequency. The graph on the right shows how $a$ varies with $x$ for a particle moving in simple harmonic motion.




Which is the amplitude and period of the motion?

| | Amplitude/cm | Period/s |
|---|---|---|
| **A** | 5.0 | 0.44 |
| **B** | 5.0 | 14 |
| **C** | 10 | 0.44 |
| **D** | 10 | 14 |

**Ans: [A]**

E4. A body performs simple harmonic motion with period of 0.063 s.
The maximum speed is 3.0 $ms^{-1}$.
What are the values of the amplitude $x_0$ and the angular frequency $\omega$?

| | $x_0$/m | $\omega$/rad$s^{-1}$ |
|---|---|---|
| **A** | 0.030 | 100 |
| **B** | 0.19 | 16 |
| **C** | 5.3 | 16 |
| **D** | 33 | 100 |

**Ans: [A]**

E5. An oscillating system performs s.h.m. with an amplitude of 2.0 cm. If the period of the motion is 5.0 s, calculate

(a) the magnitude of the velocity of the particle at 0.50 cm from equilibrium position. [2.43 cm $s^{-1}$]

(b) the maximum acceleration of the particle. [3.16 cm $s^{-2}$]

E6. An object moving with simple harmonic motion has an amplitude of 0.020 m and a frequency of 20 Hz. Calculate

(a) the period of oscillation. [0.050 s]
(b) the velocity at the equilibrium point and at the position of maximum negative displacement. [2.51 m $s^{-1}$, 0 m $s^{-1}$]

E7. The motion of a piston in a certain car engine is approximately simple harmonic with amplitude 40 mm. The frequency of oscillation is 120 Hz. Determine
(a) the maximum acceleration, [2.27 x $10^4$ $ms^{-2}$]
(b) the maximum speed, of the piston. [30.1 $ms^{-1}$]

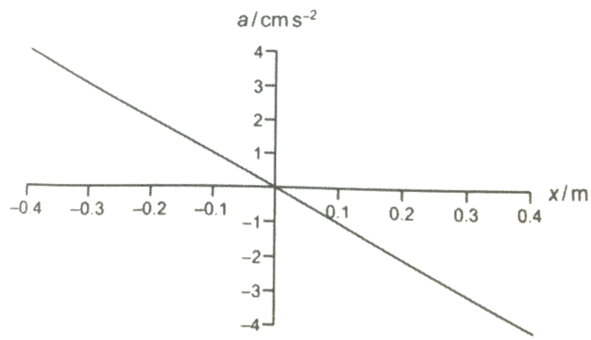
E8. When the mass $M$ on the spring is 0.040 kg, the vertical displacement y of the mass varies with time $t$ according to the relation

$$y = a \cos \omega t$$

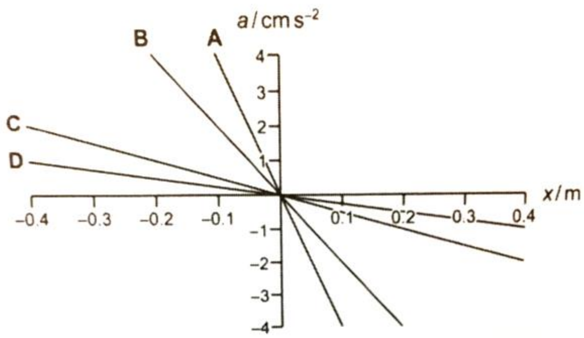
where $a$ = 0.010 m and $\omega$ = 20 rad $s^{-1}$.
Determine
(a) the amplitude of the vibration,
(b) the period $T$ of the vibration. [0.314 s]

**Graphs**

E9. [N11/1/17] The graph below shows the variation of acceleration **a** with displacement **x** for an object undergoing simple harmonic motion.
The simple harmonic motion system is altered so that it has a period of oscillation twice that of before.




Which line could be produced?




**Ans: [D]**

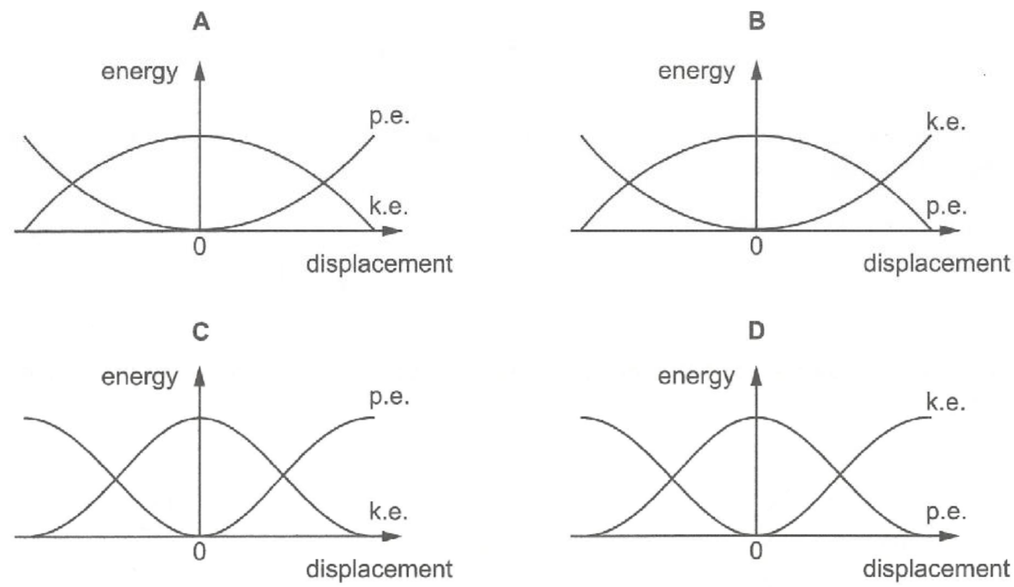
**<u>Energy of Free Oscillations</u>**

E10. [N10/1/17] A mass of 8.0 g oscillate in simple harmonic motion with an amplitude of 5.0 mm at a frequency of 40 Hz. What is the total energy of this simple harmonic oscillator?

**A** 0.16 mJ **B** 6.3 mJ **C** 13 mJ **D** 640 mJ

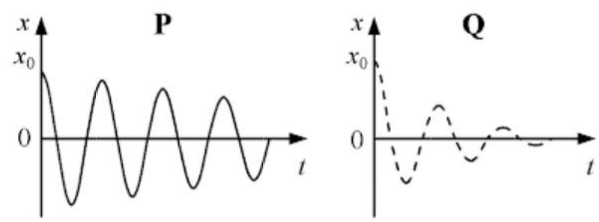
**Ans: [B]**

E11. [N16/1/16] For an object undergoing simple harmonic motion, there is a continuous interchange of potential energy (p.e.) and kinetic energy (k.e.).
Which energy-displacement graph shows this interchange of energy correctly? **Ans: [A]**



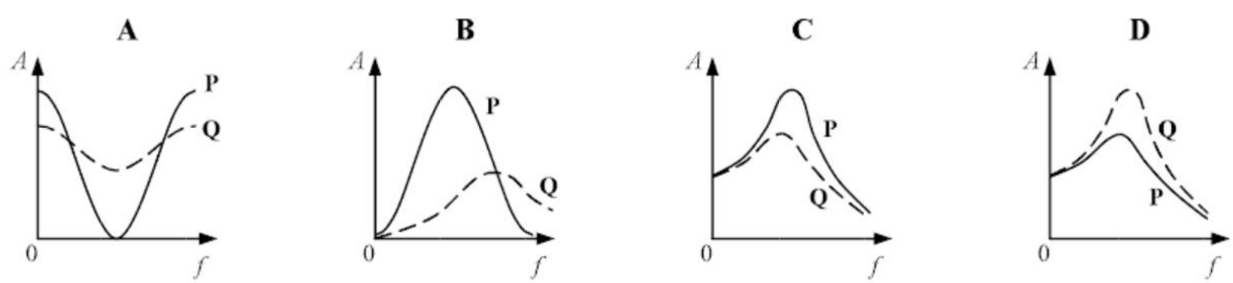

**Damping and Resonance**

E12. Given the same initial displacement and then released, the variation with time $t$ of the displacements $x$ of two objects **P** and **Q** are as shown in the given graphs.



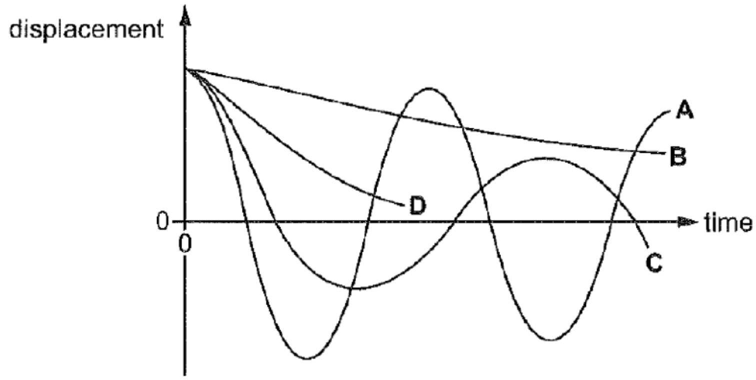

The two objects are then driven by sinusoidal driving forces of the same constant amplitude and of variable frequency $f$.

Which of the following graphs correctly shows how the amplitudes $A$ of **P** and **Q** varies with $f$?



E13. The diagram shows four displacement-time curves for oscillations with various degrees of damping.

Which curve would ideally characterise a good car suspension system?



**Problems (*Various concepts combined*)**

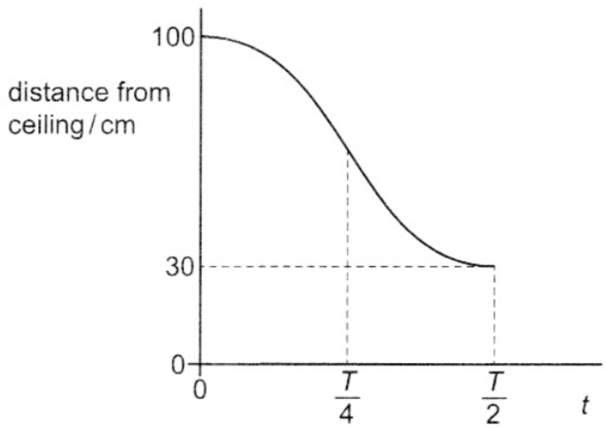
P1. Determine whether or not the following quantities can be in the same direction for a simple harmonic oscillator:

(a) displacement and velocity,
(b) velocity and acceleration,
(c) displacement and acceleration.

P2. 2018 P1 Q15

**15** A mass, hanging from a spring suspended from the ceiling, is pulled down and then released. The mass then oscillates vertically with simple harmonic motion of period $T$.

The graph shows how its distance from the ceiling varies with time $t$.




What can be deduced from this graph?

**A** The amplitude of the oscillation is 70 cm.

**B** The kinetic energy is a maximum at $t = \frac{T}{2}$.

**C** The restoring force on the mass increases between $t = 0$ and $t = \frac{T}{4}$.

**D** The speed is a maximum at $t = \frac{T}{4}$.

P3. 2020 P1 Q16

**16** A pendulum consists of a small object on the end of a light string.

When the object is set in motion it oscillates with simple harmonic motion with negligible damping.

The maximum potential energy of the object at maximum displacement $x$ is $E$.

The pendulum is stopped and then started again.

Now the maximum potential energy of the object is reduced by $\frac{E}{4}$.

What is the change in the amplitude of the oscillation?

**A** $0.134\,x$ **B** $0.250\,x$ **C** $0.500\,x$ **D** $0.866\,x$

P4. 2014 P3 Q2

**2** **(a)** **(i)** Define the *radian*.

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.................................................................................................................................. [2]

**(ii)** State, by reference to simple harmonic motion, what is meant by *angular frequency*.

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**(b)** A sphere of mass 120 g is suspended from a fixed point by an inelastic string, as shown in Fig. 2.1.

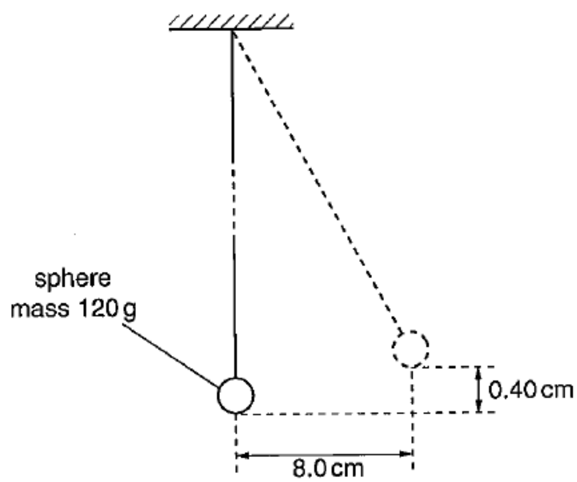



**Fig. 2.1** (not to scale)

The sphere is given a horizontal displacement of 8.0 cm from its rest position. The sphere rises vertically through a distance of 0.40 cm.

When the sphere is released, it swings, performing simple harmonic motion.
For the motion of the sphere,

**(i)** show that the total energy of the sphere is $4.7 \times 10^{-3}$ J,

[2]

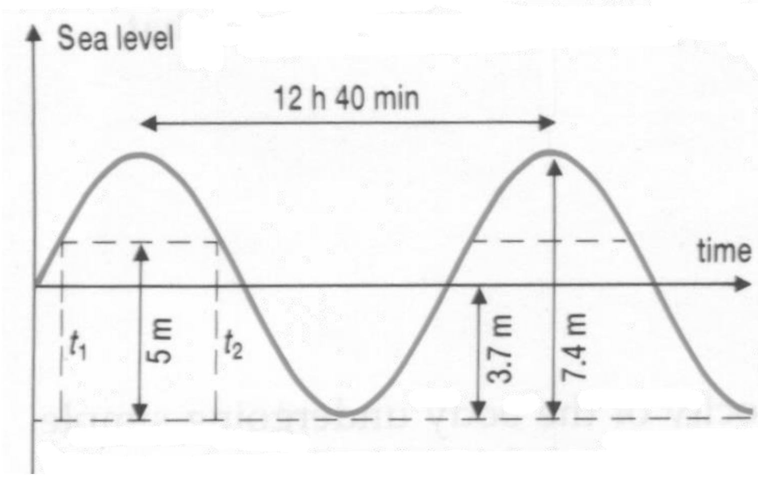
**(ii)** calculate the frequency.

frequency = .................................................... Hz [4]

P5. At Liverpool the tide is nearly sinusoidal and has a period of 12 hours 40 minutes. On a day when the height of high tide above low tide is 7.4 m, determine

*(i)* The length of time within a cycle during which the water is more than 5.0 metres above low tide. A ship's captain always knows the depth of water his ship requires and he needs to be able to do this type of calculation from tide charts.

*(ii)* The rate at which the tide is rising or falling when the water is 5.0 metres above low tide.



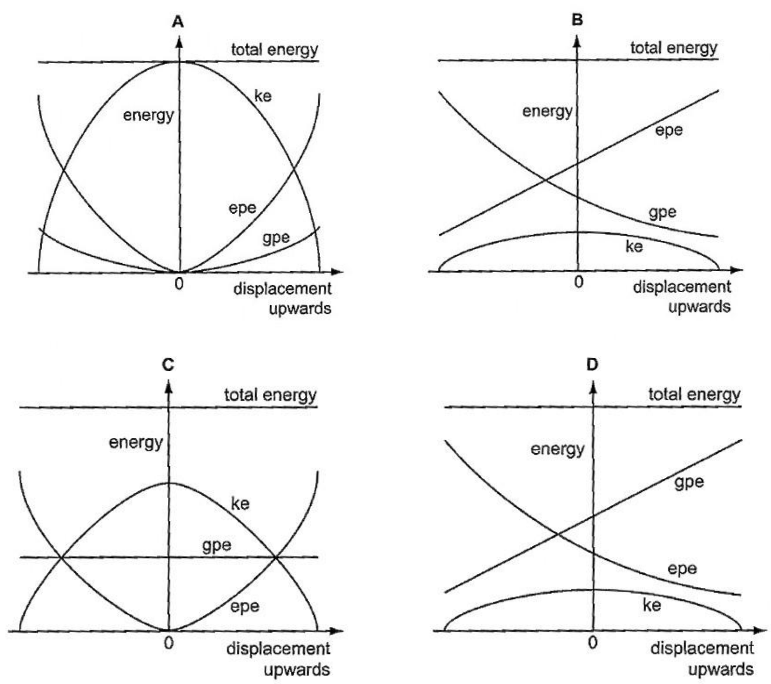

The graph shows how the sea level varies with time at Immingham. The zero of the sea level axis is taken at the mid-tide level.

P6. 2013 P1 Q15

**15** A mass attached to a spring is undergoing simple harmonic oscillations vertically.

Interchange between kinetic energy (ke), gravitational potential energy (gpe) and elastic potential energy (epe) takes place.

Which graph of energy against displacement shows these changes?



P7. 2015 P1 Q15

**15** A mass of 2.5 kg is undergoing vertical simple harmonic motion on the end of a long spring. Its maximum velocity is $1.2\,m\,s^{-1}$ and its maximum displacement from its central position is 0.15 m.

Which table correctly gives at the top, centre and bottom of the movement,

1 the kinetic energy (k.e.) of the mass

2 the change in the gravitational potential energy (g.p.e.) of the mass, compared with the value at the centre

3 the change in the elastic potential energy (e.p.e.) of the spring, compared with the value at the centre?

**A**

| | k.e. / J | g.p.e. / J | e.p.e. / J |
|---|---|---|---|
| top | 0 | +3.7 | −1.9 |
| centre | 1.8 | 0 | 0 |
| bottom | 0 | −3.7 | +5.5 |

**B**

| | k.e. / J | g.p.e. / J | e.p.e. / J |
|---|---|---|---|
| top | 0 | +3.7 | −1.9 |
| centre | 1.8 | 0 | 0 |
| bottom | 0 | −3.7 | +1.9 |

**C**

| | k.e. / J | g.p.e. / J | e.p.e. / J |
|---|---|---|---|
| top | 0 | +3.7 | −0.1 |
| centre | 3.6 | 0 | 0 |
| bottom | 0 | −3.7 | +7.3 |

**D**

| | k.e. / J | g.p.e. / J | e.p.e. / J |
|---|---|---|---|
| top | 0 | +3.7 | +0.1 |
| centre | 3.6 | 0 | 0 |
| bottom | 0 | −3.7 | +3.7 |

P8. 2020 P3 Q3

**3** The piston in a cylinder of a car engine is shown in Fig. 3.1.

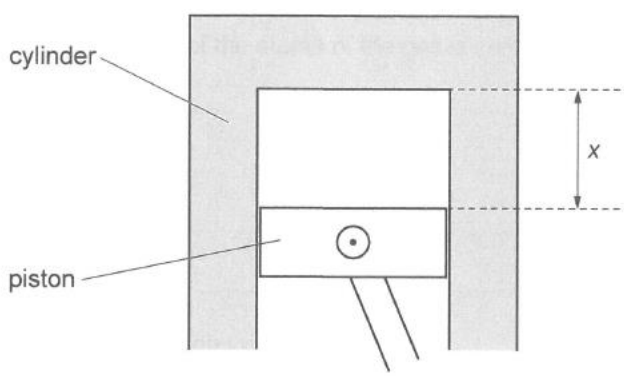



**Fig. 3.1**

The piston moves in the cylinder with simple harmonic motion.

The distance $x$ between the top of the cylinder and the top of the piston varies from 1.0 cm to 6.0 cm.

At one speed of the engine, the engine completes 4200 revolutions in 1.0 minute. During one revolution of the engine, the piston moves from the position where $x$ is minimum to where $x$ is maximum and then back to the minimum position.

The variation with time of the distance $x$ is shown in Fig. 3.2.

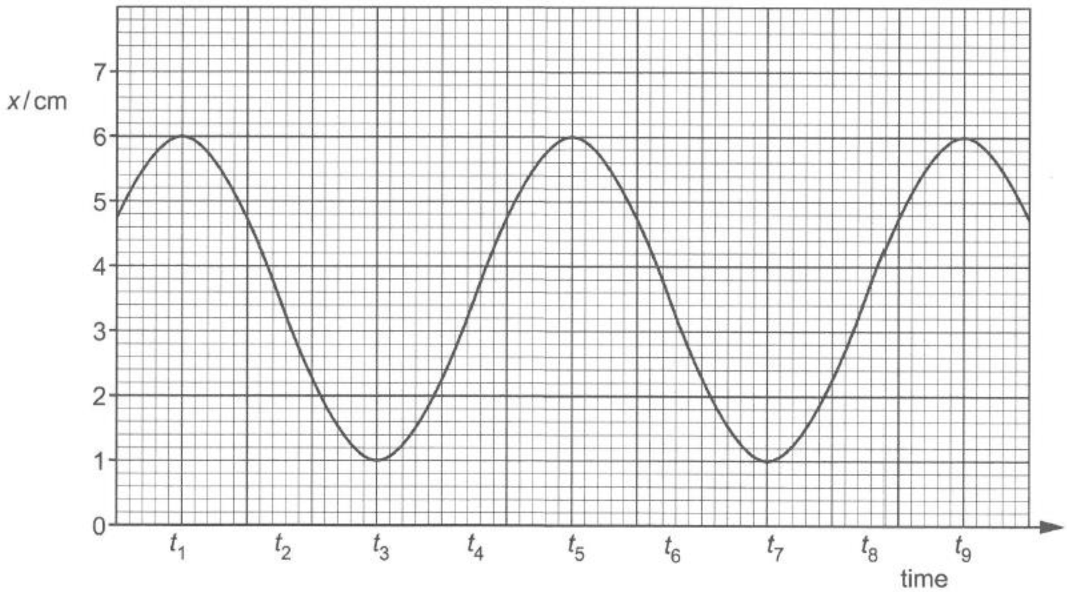



**Fig. 3.2**

**(a)** Use Fig. 3.2 to state two times at which the piston is:

**(i)** at its highest point near the top of the cylinder

time ............................... and time ............................... [1]

**(ii)** moving away from the top of the cylinder with maximum speed.

time ............................... and time ............................... [1]

**(b)** Determine, for the piston:

**(i)** the frequency of oscillation

frequency = ..................................................... Hz [1]

**(ii)** the maximum acceleration.

acceleration = ................................................ $m s^{-2}$ [3]

**(c)** Use your answer to **(b)(ii)** to sketch, on the axes of Fig. 3.3, a graph to show quantitatively how the acceleration of the piston varies with the displacement from the equilibrium position.

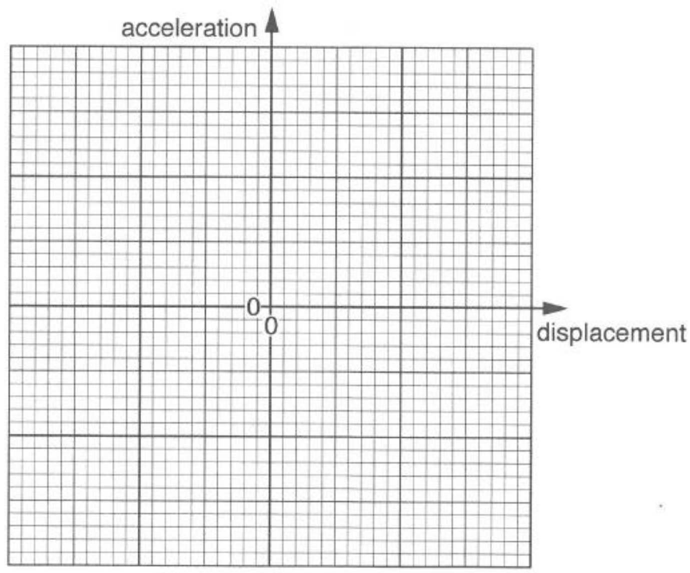



**Fig. 3.3**

[3]

P9. 2018 P3 Q9

**9** **(a)** **(i)** State what is meant by *simple harmonic motion*.

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**(ii)** The variation with displacement *x* of the acceleration *a* of a mass is shown in Fig. 9.1.

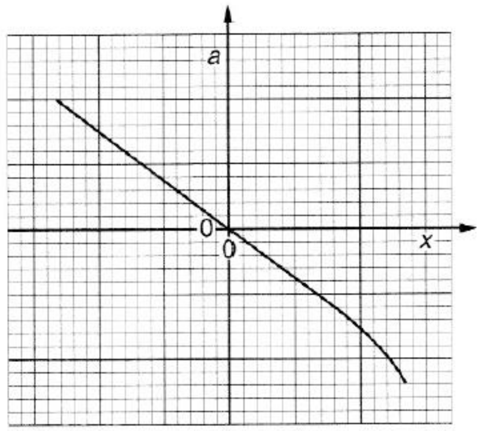


**Fig. 9.1**

Use Fig. 9.1 to explain how it is known that

**1.** the mass is oscillating

..........................................................................................................................................

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**2.** the oscillations are **not** simple harmonic.

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[2]

**(b)** A flat horizontal plate is attached to an oscillator, as shown in Fig. 9.2. Sand is sprinkled on to the plate.

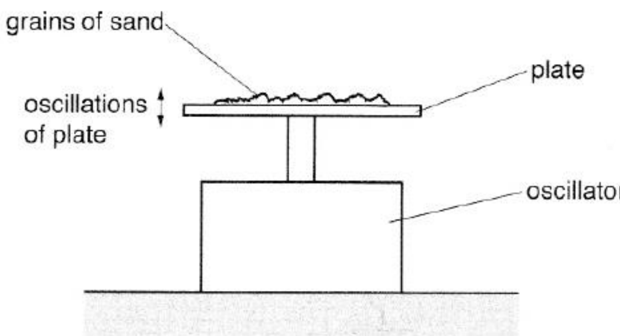


**Fig. 9.2**

The plate is made to oscillate vertically with small amplitude oscillations at a frequency of 13 Hz. The oscillations of the plate are simple harmonic.

The amplitude of oscillation is gradually increased until the sand first loses contact with the surface of the plate.

**(i)** For the plate, as the sand first loses contact with the plate,

**1.** state the position of the plate

..................................................................................................................[1]

**2.** calculate the amplitude of the oscillations.

amplitude = .................................................. mm [3]

**(ii)** The sand consists of small grains.
The sand is replaced by larger pebbles.

Suggest and explain whether, for the pebbles to lose contact with the plate, the minimum amplitude of the oscillations would be different.

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..................................................................................................................[2]

**(c)** A spring hangs vertically from a fixed point.
A mass of 1.2 kg is attached to the free end of the spring, as shown in Fig. 9.3.

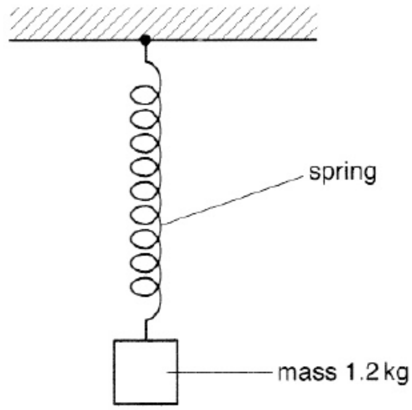


**Fig. 9.3**

The mass undergoes vertical oscillations with frequency 2.5 Hz and amplitude 3.4 cm.

**(i)** For the oscillations of the mass, determine

**1.** the total energy $E_T$

$E_T$ = .................................................... J [2]

**2.** the displacement $d$ at which the potential energy $E_P$ and the kinetic energy $E_K$ of the oscillations are equal.

$d$ = .................................................. cm [2]

**(ii)** Use your answers in **(c)(i)** to sketch, on the axes of Fig. 9.4, the variation with displacement *x* of

**1.** the total energy (label this line $E_T$)

**2.** the kinetic energy (label this line $E_K$)

**3.** the potential energy (label this line $E_P$).

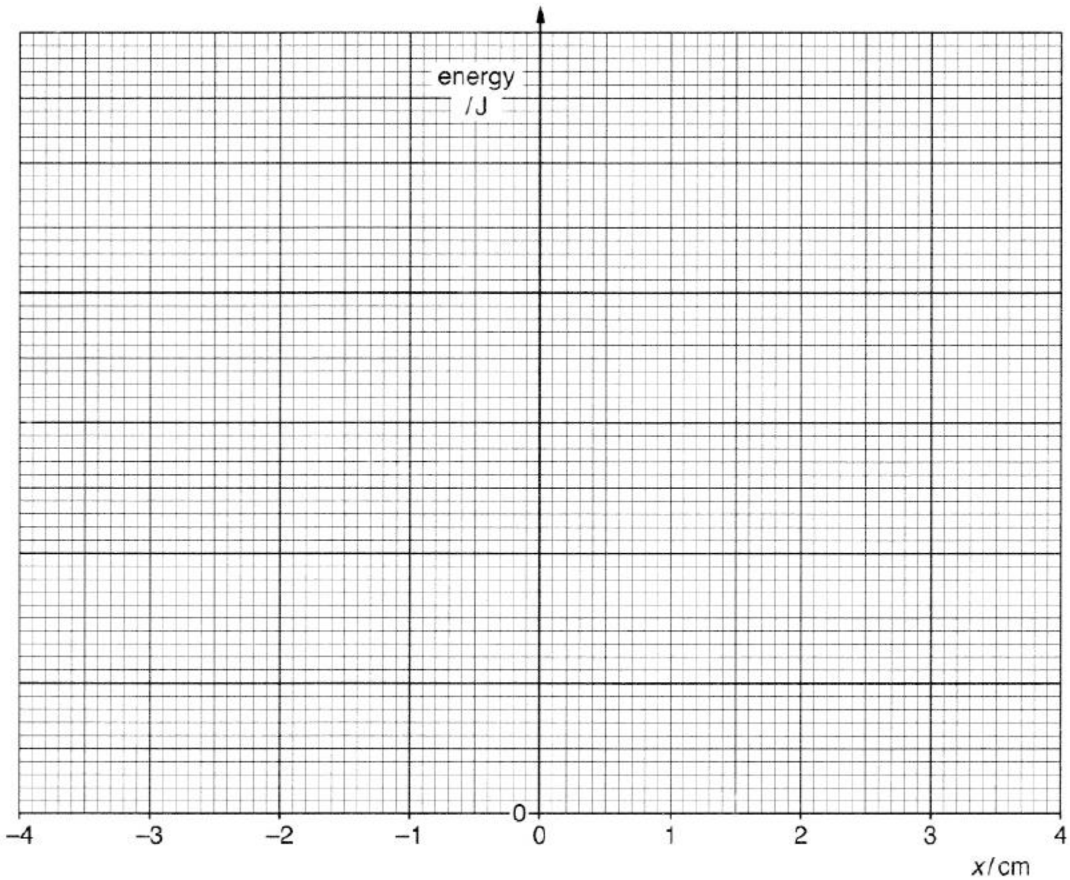


**Fig. 9.4**

[6]

[Total: 20]

P10. [N13/3/7]

(a) State the origin of upthrust acting on a body in a fluid. [2]

(b) A tube sealed at one end, has a uniform area of cross-section *A*. Some sand is placed is placed in the tube so that it floats upright in a liquid density *ρ*, as shown in Fig. 1.

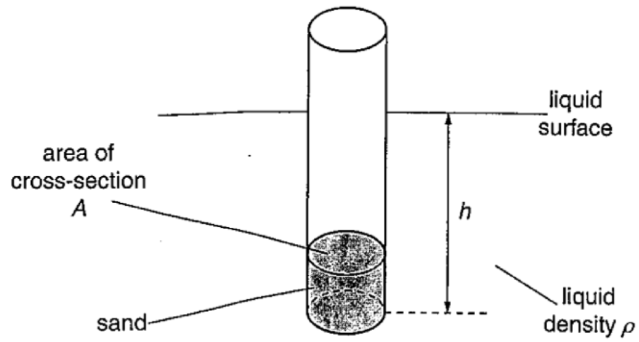


Fig. 1

The total mass of the tube and the sand is *m*.
The tube floats with its base a distance *h* below the surface of the liquid.

Derive an expression relating *m* to *h*, *A* and *ρ*. Explain your working. [3]

(c) The tube in **(b)** is displaced vertically and then released. For a displacement *x*, the acceleration *a* of the tube is given by the expression $a = -\left(\frac{\rho A g}{m}\right)x$ where *g* is the acceleration of free fall.

(i) Explain why the expression leads to the conclusion that the tube is performing simple harmonic motion. [3]

(ii) The tube has a total mass *m* of 32 g and area *A* of its cross-section is 4.2 $cm^2$.
It is floating in liquid of density *ρ* of $1.0 \times 10^3$ kg $m^{-3}$.
Show that the frequency of oscillation of the tube is 1.8 Hz. [3]

(d) The tube in **(b)** is now placed in a different liquid.
The tube oscillates vertically. The variation with time *t* of the vertical displacement *x* of the tube is shown in Fig. 2.

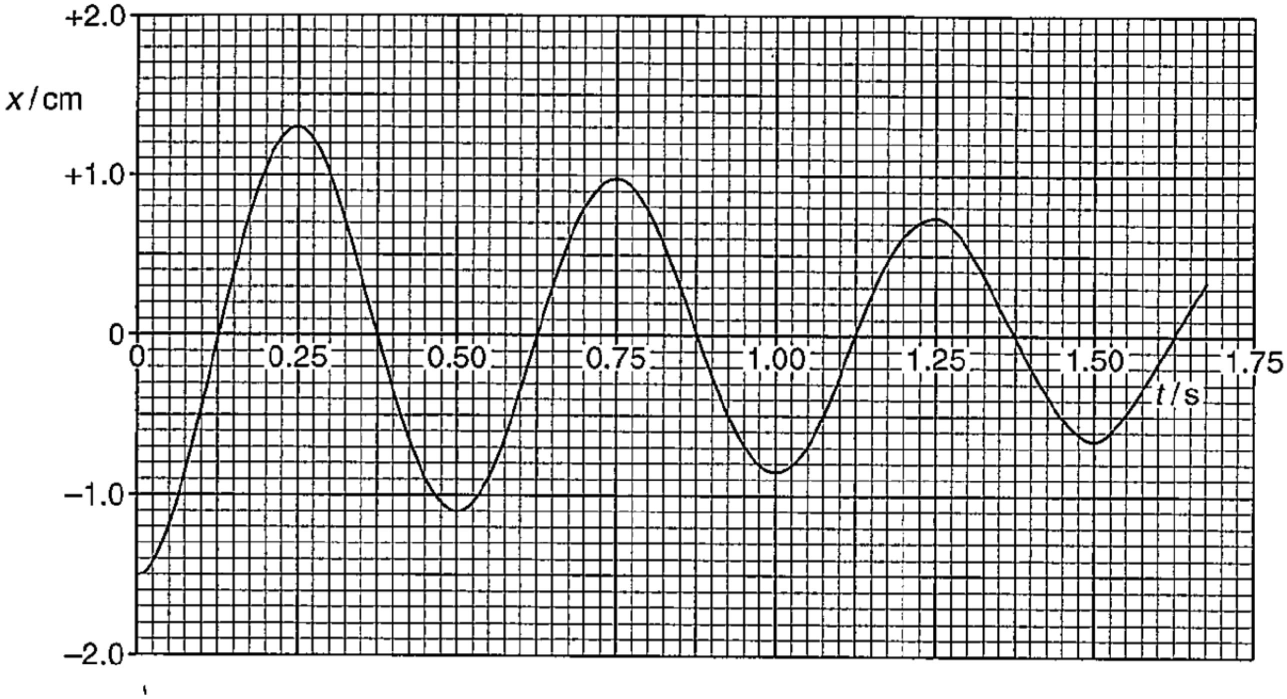


**Fig. 2**

(i) Use Fig. 2 to

1. Determine the frequency of oscillation of the tube, [2]
2. Calculate the density of the liquid. [2]

(ii) 1. Suggest two reasons why the amplitude of the oscillation decreases with time. [2]

2. Calculate the decrease in energy of the oscillation during the first 1.0 s. [3]

P11. 2016 P3 Q7

**7** **(a)** A mass undergoes simple harmonic motion.

**(i)** State, for the motion of the mass, what is meant by

**1.** the *displacement*,

..............................................................................................................................

.............................................................................................................................. [1]

**2.** the *amplitude*.

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.............................................................................................................................. [1]

**(ii)** State, by reference to displacement, what is meant by *simple harmonic motion*.

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**(b)** The bob of a simple pendulum undergoes simple harmonic motion, as illustrated in Fig. 7.1.

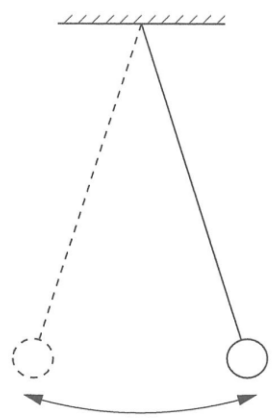

**Fig. 7.1**

A block of wood, floating in water, undergoes vertical oscillations, as illustrated in Fig. 7.2.

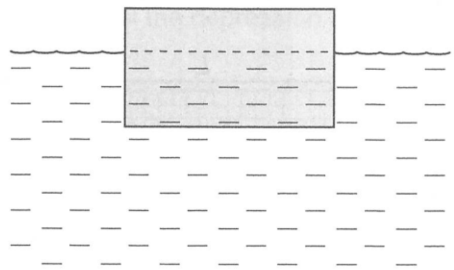

Fig. 7.2

For each system, describe the restoring force that gives rise to the oscillations.

simple pendulum ..........................................................................................................

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floating block ..........................................................................................................

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.......................................................................................................... [4]

**(c)** A uniform strip of steel is clamped at one end. A metal block of mass $M$ is fixed to the strip, a distance $L$ from the clamp, as illustrated in Fig. 7.3.

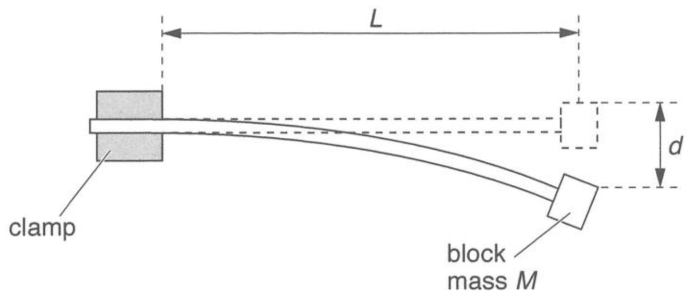


Fig. 7.3

The mass causes the end of the steel strip to be depressed by a distance $d$.

Explain why part of the strip is compressed and part of the strip is extended.

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.......................................................................................................... [3]

**(d)** The free end of the strip in **(c)** is given a small vertical displacement and then released.

The variation with displacement $x$ from the equilibrium position of the acceleration $a$ of the end of the strip is given by the expression

$$a = -\frac{CE}{L^3 M}x$$

where $C$ is a constant for the strip and $E$ is a constant for the material of the strip.

Show that the end of the strip is undergoing simple harmonic motion.

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.......................................................................................................................................... [2]

**(e)** The constant $E$ for steel is $2.0 \times 10^{11}\,\text{N m}^{-2}$.
When a mass $M$ of 150 g is clamped to the steel strip so that the distance $L$ on Fig. 7.3 is 0.80 m, the variation with time $t$ of the depression $d$ of the end of the strip is shown in Fig. 7.4.

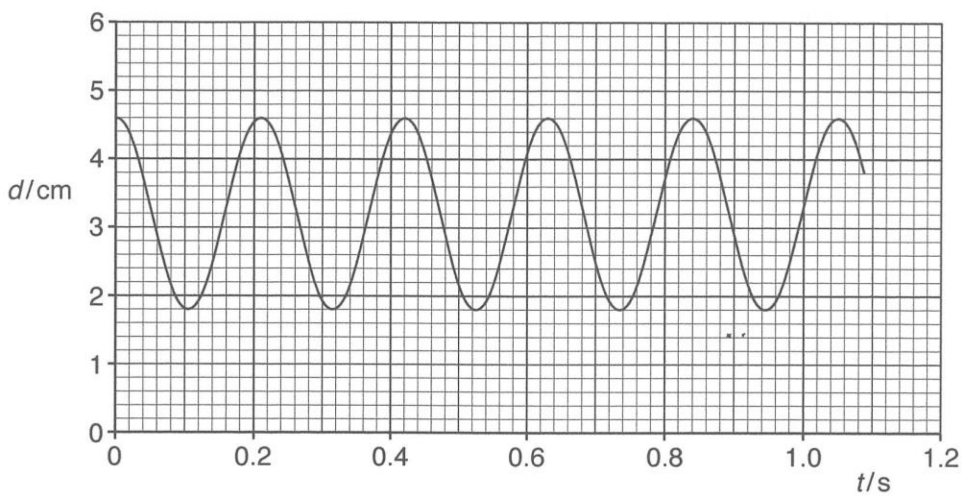



**Fig. 7.4**

(i) the angular frequency $\omega$ of the oscillations,

$\omega$ = ............................................. rad $s^{-1}$ [2]

(ii) the value (without the unit) of the constant $C$ for this strip.

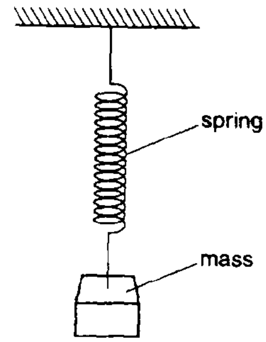
value of $C$ = .......................................................... [2]

(f) A strip of aluminium has the value of $C$ calculated in **(e)(ii)**.
The constant $E$ for aluminium is $7.1 \times 10^{10}$ N $m^{-2}$.

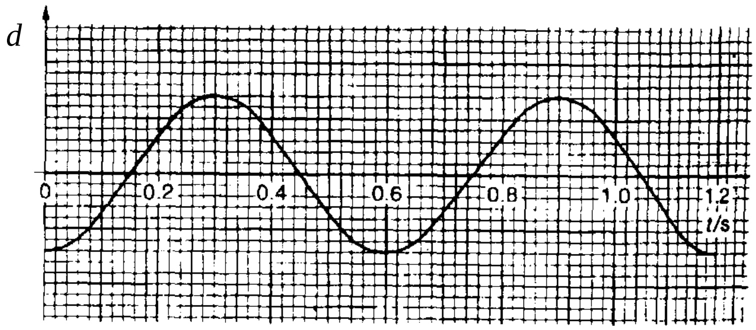
Using data given in **(e)** and the expression in **(d)**, calculate the mass, positioned on the aluminium strip such that $L$ = 0.80 m, so that the steel and the aluminium strips have the same frequency of oscillation.

mass = .................................................... kg [3]

P12. A light spring hangs vertically from a fixed support and a mass is attached to its free end as illustrated in the figure.




The mass is displaced vertically and then released. The variation with time *t* of the displacement *d* of the mass from its equilibrium position is as shown in the figure below.




(a) Use the figure to determine, for the oscillation of the mass,
  (i) the period,
  (ii) the angular frequency

(b) The mass-spring system is used to demonstrate the effects of damping on the oscillations of the mass
  (i) Explain what is meant by *damping*.
  (ii) Suggest how
    1. light damping of the oscillations may be achieved,
    2. the degree of damping may be increased.

P13. 2012 P2 Q1

**1** A spring has an unextended length of 12.0 cm. The force $F$ required to extend the spring to a length $\ell$ is measured.

The variation with the length $\ell$ of the force $F$ is shown in Fig. 1.1.

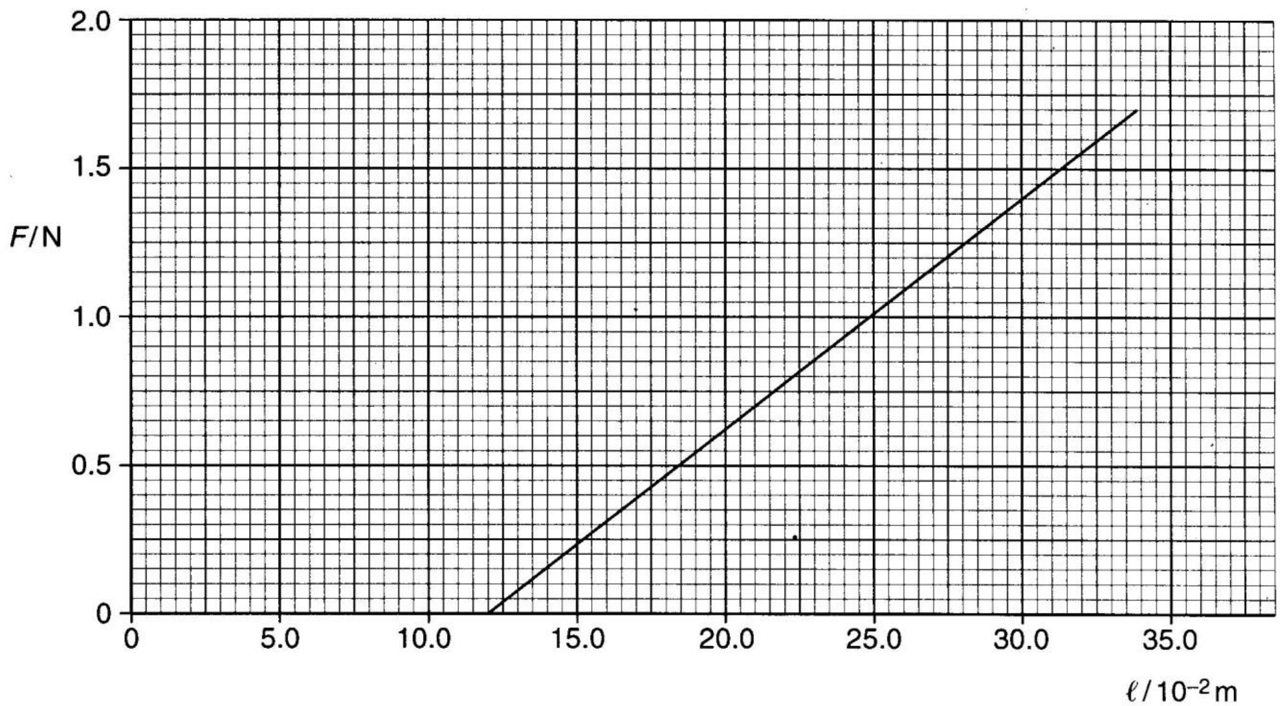



**Fig. 1.1**

**(a)** State and explain whether the spring obeys Hooke's law.

.............................................................................................................................

.............................................................................................................................

............................................................................................................................. [2]

**(b)** Calculate the work done on the spring when it is extended from $\ell = 12.0 \times 10^{-2}$ m to $\ell = 30.0 \times 10^{-2}$ m.

work done = ........................ J [2]

**(c)** One end of the spring is fixed and an object M of weight 1.40 N is hung vertically from the other end. The object M is pulled down and then released. The object oscillates vertically.

The variation with time $t$ of the length $\ell$ of the spring is shown in Fig. 1.2.

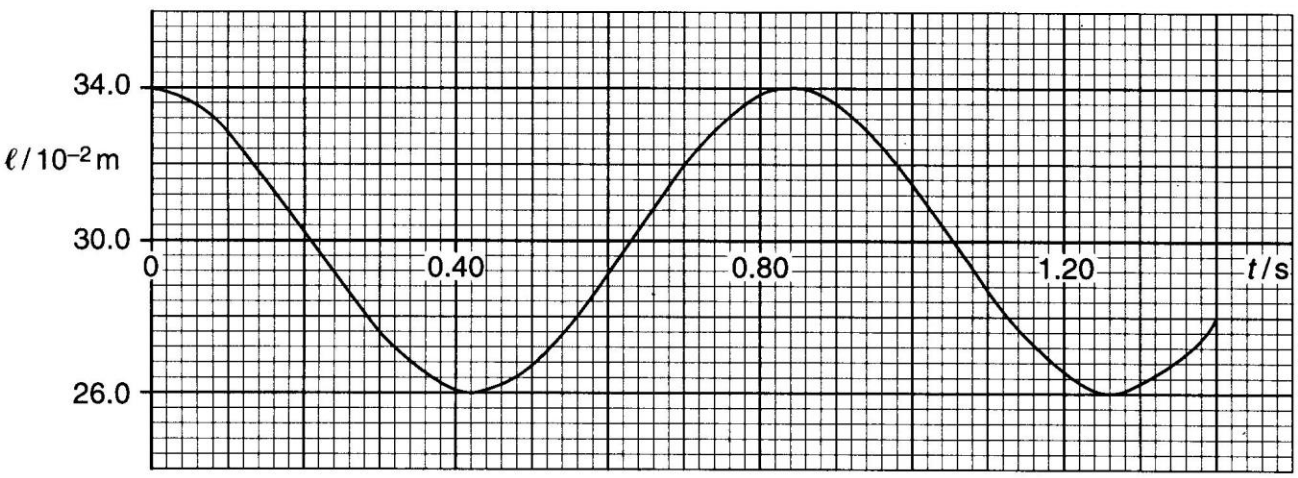



**Fig. 1.2**

**(i)** Use Fig. 1.2 to calculate the maximum speed of M.

maximum speed = ....................................... m s$^{-1}$ [2]

**(ii)** On Fig. 1.3, show the variation with time $t$ of the velocity $v$ of M. Include a suitable scale on the velocity axis.

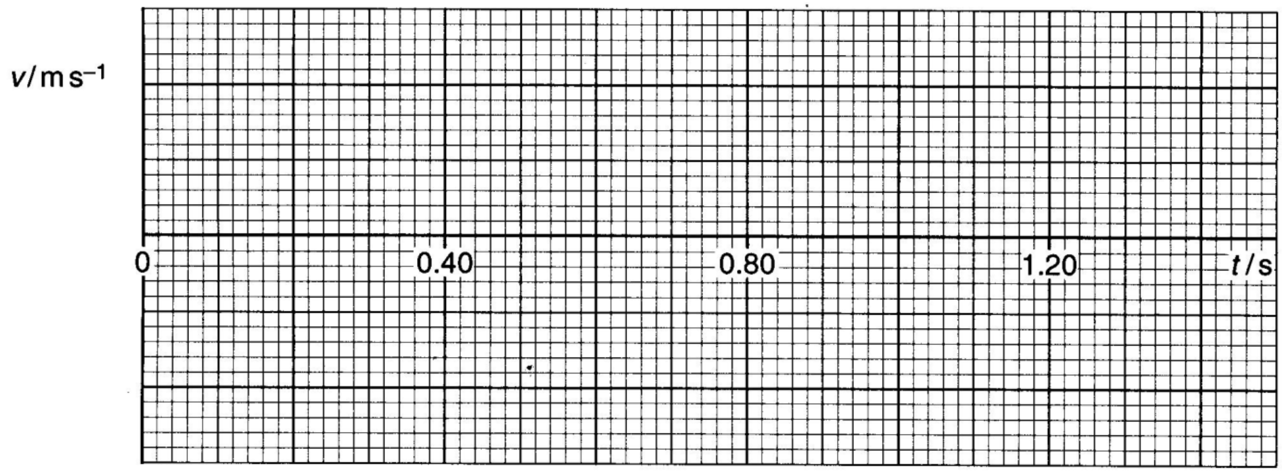



**Fig. 1.3**

[2]

**(d)** The object M consists of two parts connected by a stiff wire of negligible mass and volume. The lower part of M is immersed in a liquid as shown in Fig. 1.4.

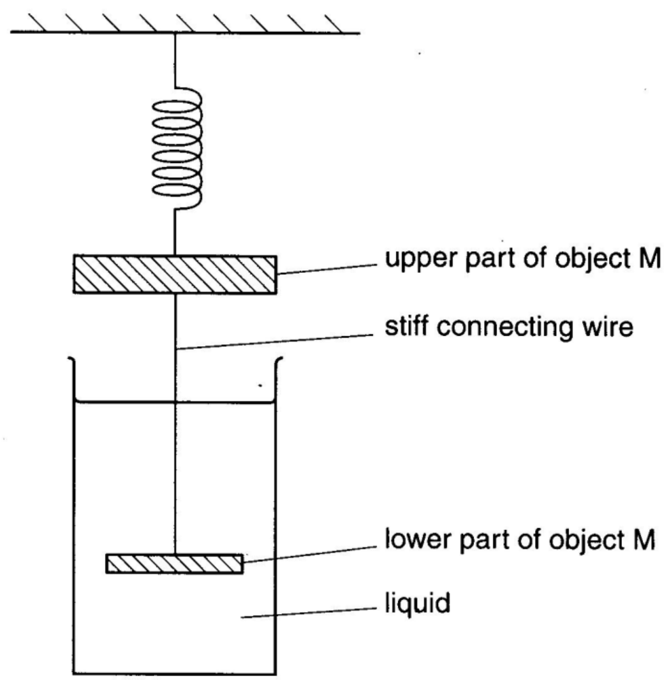



**Fig. 1.4**

The liquid has a density of $1000\,\text{kg}\,\text{m}^{-3}$. The volume of the part of M that is immersed in the liquid is $20\,\text{cm}^3$.

**(i)** Determine the new length of the spring.

length = .......................................... m [3]

**(ii)** The object M is pulled down $4.0 \times 10^{-2}$ m and is then released. The lower part of M remains immersed in the liquid at all times.
State and explain two differences that would be seen in the oscillations when compared with those shown in Fig. 1.2.

1. ..........................................................................................................................

..........................................................................................................................

2. ..........................................................................................................................

..........................................................................................................................
[2]

P14. A student sets up the apparatus illustrated in Fig. 5.1 in order to investigate the oscillations of a metal cube suspended on a spring.

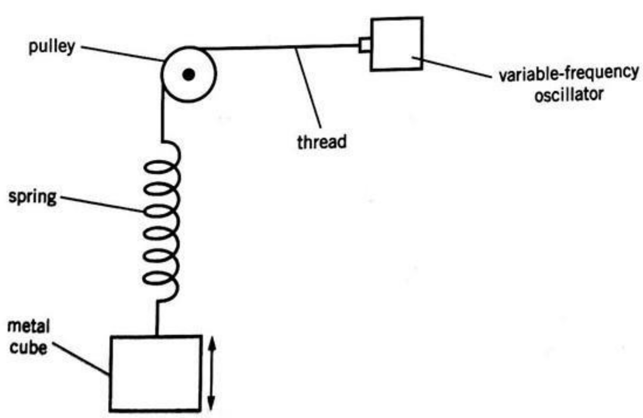


Fig. 5.1

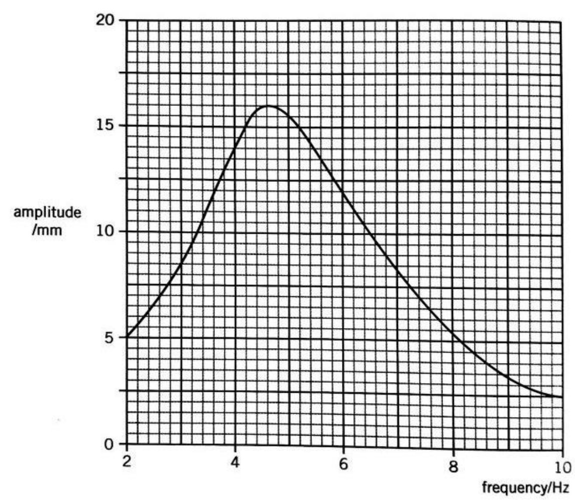


Fig. 5.2

The amplitude of the vibrations produced by the oscillator is constant.
Fig. 5.2 shows the variation with frequency of the amplitude of the oscillations of the metal cube.

i. State the phenomenon illustrated in Fig. 5.2.

ii. For the maximum amplitude of vibration, state the magnitudes of the amplitude and the frequency.

(a) The oscillations of the metal cube of mass 150 g may be assumed to be simple harmonic. Use your answers in (a)ii to determine, for the metal cube,

i. its maximum acceleration,

ii. the maximum resultant force on the cube

(b) Some very light feathers are attached to the top surface of the cube so that the feathers extend outwards, beyond the vertical sides of the cube.
The investigation is now repeated.
On Fig. 5.2, draw a line to show the new variation with frequency of the amplitude of vibration for the frequencies between 2 Hz to 10 Hz.

P15. When a driver of mass 80 kg, steps into the car of mass 920 kg, the vertical height of the car above the road decreases by 2.0 cm. If the car is driven over a series of equally spaced bumps, the amplitude of vibration becomes much larger at one particular speed. Explain why this occurs and calculate the separation of the bumps if it occurs at a speed of 15 $ms^{-1}$. (Given that the expression for *f* is $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ )

**Challenging**

C1. 2016 P1 Q17

**17** A simple pendulum is 1.0 m long and oscillates with simple harmonic motion of angular amplitude 0.050 rad and period 2.0 s.

What is the angular speed of the pendulum bob when the angular displacement is 0.030 rad?

**A** 0.0010 rad $s^{-1}$

**B** 0.020 rad $s^{-1}$

**C** 0.050 rad $s^{-1}$

**D** 0.13 rad $s^{-1}$

C2. 2012 P3 Q3

**3** Two helical springs are attached to a trolley of mass 590 g, as shown in Fig. 3.1.

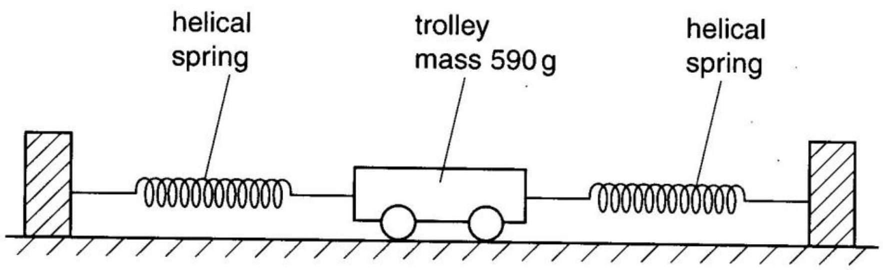



**Fig. 3.1**

The trolley is displaced along the axis of the springs and then released. The trolley undergoes simple harmonic motion. The variation of the kinetic energy $E_K$ of the trolley with displacement $x$ from its equilibrium position is shown in Fig. 3.2.

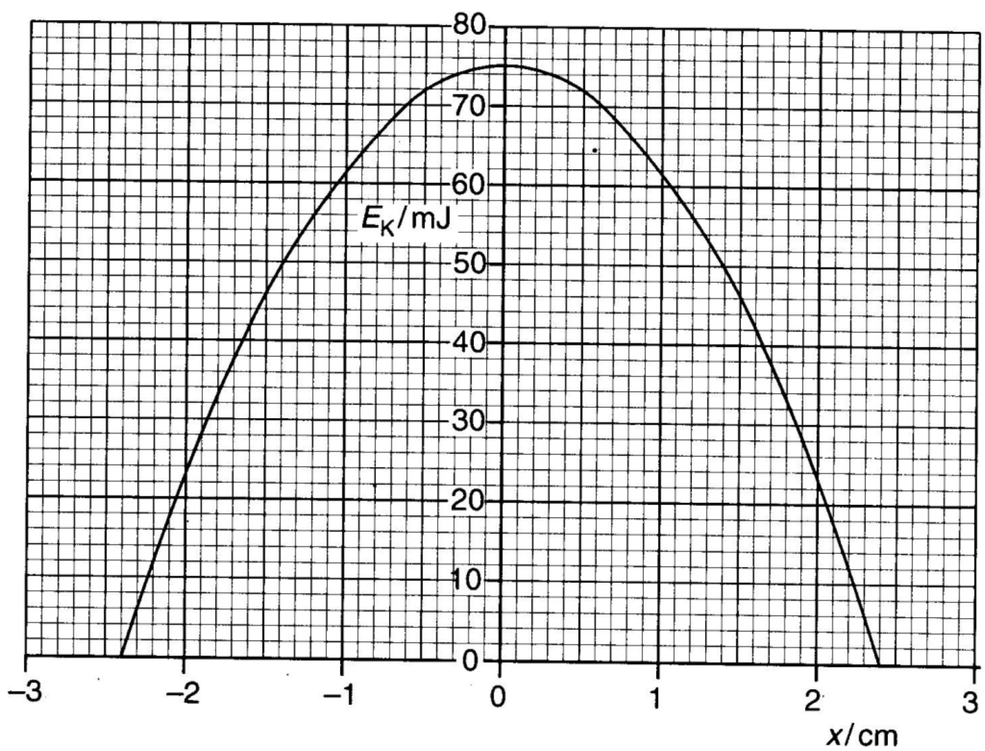



**Fig. 3.2**

**(a)** Use data from Fig. 3.2 to calculate the frequency of oscillation of the trolley.

frequency = .......................................... Hz [4]

**(b)** The trolley loses energy so that its maximum kinetic energy is reduced **by** 40 mJ.

Use Fig. 3.2, without any further calculation, to determine the amplitude of the oscillations.
Show your construction on Fig. 3.2.

amplitude = .......................................... cm [2]