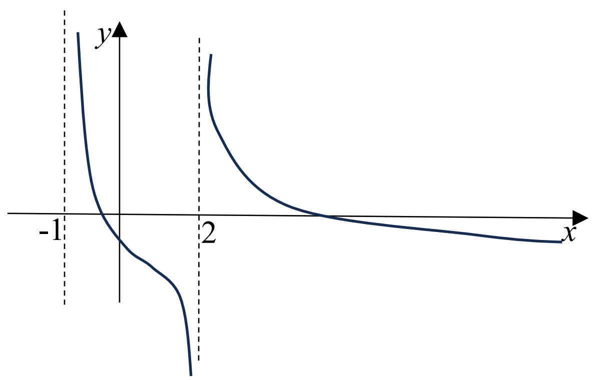
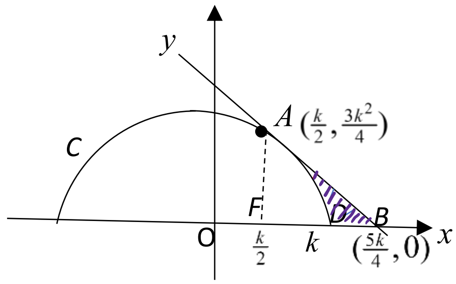
ACJC 2024 H1 Preliminary Exam

1	Let selling price of an electric car be \$ e Let selling price of a gas-powered car be \$ g Let selling price of a hybrid car be \$ h. $4e \quad -3g = 172800 \quad \ldots\ldots \quad (1)$ $h = 1.2g \quad \ldots\ldots \quad (2)$ $3h \quad +2g \quad +2e = 716880 \quad \ldots\ldots \quad (3)$ By GC, Selling price of an electric car is \$109 800.	Some interpreted eqn (2) wrongly. Some students attempted to solve algebraically not knowing that GC can solve simultaneous equations involving 3 eqns and 3 unknowns.
2	Discriminant =D $=(k-6)^2-4(k^2+36)(0.5)$ $=k^2-12k+36-2(k^2+36)$ $=k^2-12k+36-2k^2-72$ $=-(k^2+12k+36)=-(k+6)^2$ Since coefficient of $x^2=k^2+36>0$ the curve has a minimum point. $(k^2+36)x^2+(k-6)x+0.5\geq 0$ if $D\leq 0$ $-(k+6)^2\leq 0$ $\therefore (k+6)^2\geq 0$ $\therefore k\in R$ For $\ln[(k^2+36)x^2+(k-6)x+0.5)]$ to be defined, $(k^2+36)x^2+(k-6)x+0.5)>0$ for all real x. Hence, D < 0 $-(k+6)^2<0$ $\therefore (k+6)^2>0$ $k\in R,\ k\neq -6$	Most students considered the discriminant but did not proceed to solve the correct inequality. Those who arrived at this step: $(k+6)^2\geq 0$, didn't conclude correctly. Students should note that since this inequality is always true for any values of k, hence k is the set of real numbers. Students should note that the calculation will result in $(k+6)^2>0$ and hence k can be any real numbers except -6.
3a.	$\int \frac{(25x^4+4)}{\sqrt{x}}\,dx$ $=\int 25x^{\frac{7}{2}}+4x^{-\frac{1}{2}}\,dx$ $=\frac{50}{9}x^{\frac{9}{2}}+8x^{\frac{1}{2}}+c$	Students should attempt to simply their answer and not leave it as: $\frac{25x^{\frac{9}{2}}}{\frac{9}{2}}+\frac{4x^{\frac{1}{2}}}{\frac{1}{2}}+c$
3b.	$\frac{4x-3}{4-x}=-4+\frac{13}{4-x}$	Long division is an assumed knowledge for H1 students.

	$\int \frac{4x-3}{4-x}\ \mathrm{dx} = \int -4+\frac{13}{4-x}\ \mathrm{dx}$ $= -4x-13\ln\lvert 4-x\rvert + c$	
4i	$y=-\ln(x+1)+\frac{5}{x-2}+2$ $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{x+1}-\frac{5}{(x-2)^2}$ To show no stationary points Method 1 For $y=-\ln(x+1)$ to be defined $x+1>0$ Hence $-\frac{1}{x+1}<0$ for all real x Also $-\frac{5}{(x-2)^2}<0$ for all real x $\therefore \frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{x+1}-\frac{5}{(x-2)^2}<0$ for all real x. Hence curve C has no stationary points. Method 2 $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{x+1}-\frac{5}{(x-2)^2}$ $=\frac{-(x-2)^2-5(x+1)}{(x+1)(x-2)^2}$ $=\frac{-(x^2+4-4x)-5x-5}{(x+1)(x-2)^2}=\frac{-(x^2+x+9)}{(x+1)(x-2)^2}$ For stationary points $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ $x^2+x+9=0$ Discriminant = $(-1)^2-4(1)(9) = -35 <0$ No solution. Hence curve C has no stationary points. Method 3 $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{x+1}-\frac{5}{(x-2)^2}$ $=\frac{-(x-2)^2-5(x+1)}{(x+1)(x-2)^2}$	Many students attempted to find $\frac{\mathrm{d}y}{\mathrm{d}x}$ and didn't make any progress thereafter. They should show $\frac{\mathrm{d}y}{\mathrm{d}x}\neq 0$.

	$=\dfrac{-(x^2+4-4x)-5x-5}{(x+1)(x-2)^2}=\dfrac{-(x^2+x+9)}{(x+1)(x-2)^2}=$ $=\dfrac{-((x+\frac{1}{2})^2+9-\frac{1}{4})}{(x+1)(x-2)^2}=\dfrac{-((x+\frac{1}{2})^2+\frac{35}{4})}{(x+1)(x-2)^2}$ <0 for all real x. Hence curve C has no stationary points	
4ii	 When $y = 0$, x = -0.222 or x = 11.5 Asymptotes x = -1 and x = 2.	$y=-\ln(x+1)+\dfrac{5}{x-2}+2$ Students should note how the asymptotes are obtained: $x+1=0 \quad \therefore x=-1$ $x-2=0 \quad \therefore x=2$
4iii	$\int_3^5 -\ln(x+1)+\dfrac{2x+1}{x-2}\,dx$ =6.29	Some students aren't aware that numerical area implies that the integral can be evaluated using the GC.
5i	When $y=\dfrac{3}{4}k^2$ $\dfrac{3}{4}k^2=k^2-x^2$ $x^2=\dfrac{1}{4}k^2$ $x=\dfrac{1}{2}k$ $\dfrac{dy}{dx}=-2x$ $\dfrac{dy}{dx}=-2(\dfrac{1}{2}k)=-k$ $y-\dfrac{3}{4}k^2=-k(x-\dfrac{k}{2})$ $y=\dfrac{5}{4}k^2-kx$	To find tangent, students should find 1) The coordinates of the point (,) and 2) The gradient in terms of k.

5ii	 Area of $\Delta AFB = \frac{1}{2}(\frac{3k^2}{4})(\frac{5k}{4} - \frac{k}{2}) = \frac{9k^3}{32}$ Area $AFD = \int_{\frac{k}{2}}^{k}(k^2 - x^2)dx = [k^2x - \frac{x^3}{3}]_{\frac{k}{2}}^{k}$ $= k^3 - \frac{k^3}{3} - (\frac{k^3}{2} - \frac{k^3}{24}) = \frac{5k^3}{24}$ Area bounded by the curve C, the tangent to the curve at point A and the x-axis = $\frac{5k^3}{24} - \frac{9k^3}{32} = \frac{7k^3}{96}$	Most students identified the correct region but made mistakes in the limits of the integrals. Students should firstly compute the x-intercepts of the curve and the tangent.
6i	**Method 1** $P = -(e^{0.2t} - 4)^2 + 9$ $\frac{dP}{dt} = -2(e^{0.2t} - 4)(0.2e^{0.2t})$ For stationary points $\frac{dP}{dt} = 0$ $\frac{dP}{dt} = -2(e^{0.2t} - 4)(0.2e^{0.2t}) = 0$ $(e^{0.2t} - 4) = 0$ $e^{0.2t} = 4$ $0.2t = \ln 4$ $t = 5\ln 4$ **Method 2** $P = -(e^{0.2t} - 4)^2 + 9$ $= -(e^{0.4t} + 16 - 8e^{0.2t}) + 9$ $= -e^{0.4t} - 7 + 8e^{0.2t}$ $\frac{dP}{dt} = -0.4e^{0.4t} + 1.6e^{0.2t}$ For stationary points $\frac{dP}{dt} = 0$	Many students solved $\frac{dP}{dt} = 0$ using the GC, instead of doing differentiation as stated in the question.

$0.4e^{0.4t} = 1.6e^{0.2t}$
$e^{0.2t} = 4$
$0.2t = \ln 4$
$t = 5\ln 4$

To justify whether it is a maximum or minimum point

<u>Method 1</u>

$$\frac{\mathrm{d}^2P}{\mathrm{d}t^2} = -0.4e^{0.4t} + 1.6e^{0.2t}$$

When $t = 5\ln 4$, $\frac{\mathrm{d}^2P}{\mathrm{d}t^2} = -1.28$

At $t = 5\ln 4$, the stationary point is a maximum point

<u>Method 2</u>

$t = 5\ln 4$=6.93 to 3 significant figures

t	6.5	5ln4	7
$\frac{dp}{dt}$	0.485	0	-0.0895
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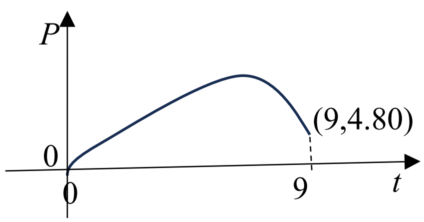
At $t = 5\ln 4$, the stationary point is a maximum point.

For all maxima/minima questions, students are expected to determine the nature of the point unless otherwise stated in the question.

6ii



Students have to indicate the endpoints of all curve sketching questions.

6iii

$$\int -(e^{0.2t}-4)^2+9 \,\mathrm{d}t = = \int -e^{0.4t}+8e^{0.2t}-7) \,\mathrm{d}t$$

$$= -2.5e^{0.4t}+40e^{0.2t}-7t+C$$

$$\int_0^9 -(e^{0.2t}-4)^2+9 \,\mathrm{d}t = [-2.5e^{0.4t}+40e^{0.2t}-7t]_0^9$$

$= -2.5e^{0.4(9)}+40e^{0.2(9)}-7(9)+2.5-40$

= 49.9903= 50 (to the nearest integer)

Or using GC

$\int_0^9 -(e^{0.2t}-4)^2+9 \,\mathrm{d}t$ = 49.9903= 50 (to the nearest integer)

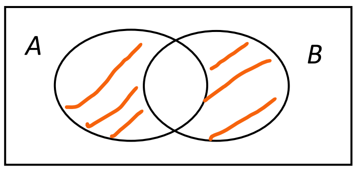
Over a span of **9 months** Mr Lim made **a total profit** of **$50000**.

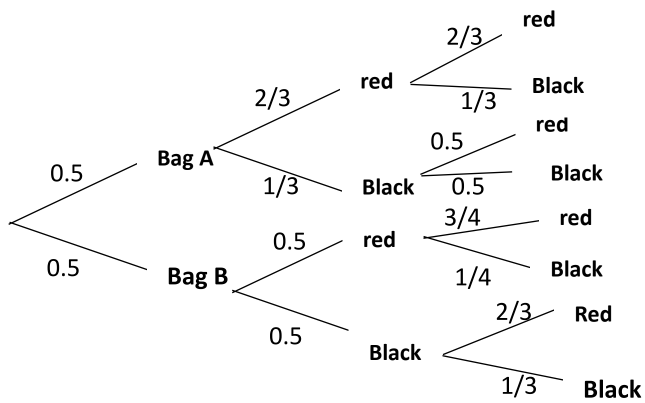
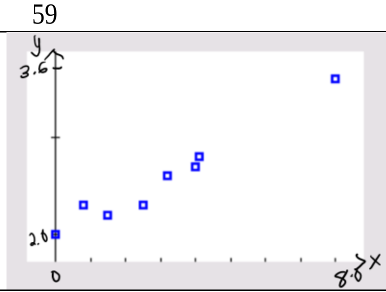
Most students tried to expand but made careless mistakes. Students have to note that:

$$\left(e^{0.2t}\right)^2 = \left(e^{0.2t}\right).\left(e^{0.2t}\right)$$
$$= \left(e^{0.2t+0.2t}\right) = \left(e^{0.4t}\right)$$

Some students aren't aware that they can evaluate the integral using GC to get its numerical value to check their answers.

6iv	$A=\frac{200}{3}(-20t+75+\frac{375}{2t+5})$ $\frac{\mathrm{d}A}{\mathrm{d}t}=\frac{200}{3}(-20-\frac{375(2)}{(2t+5)^2})$ $=\frac{200}{3}(-20-\frac{750}{(2t+5)^2})$ When $t = 3$, $\frac{\mathrm{d}A}{\mathrm{d}t}=\frac{200}{3}(-20-\frac{750}{11^2})$ $\frac{\mathrm{d}A}{\mathrm{d}t}=-1746$ (to the nearest integer) When $t = 3$, the remaining budget is decreasing at the rate of \$1746 per month	Many students evaluated the derivative using GC despite the question asking them to use differentiation.
7a	$X\sim \mathrm{B}(30,\frac{p}{100})$ $np=18$ $30\left(\frac{p}{100}\right)=18$ $p=60$ $\therefore \mathrm{P}(X\geq 20)=1-P(X\leq 19)=0.29147\approx 0.291$ (3 sf)	Students have to note that the probability was given as *p*%=*p*/100. Many wrote *p* incorrectly as 0.6. Hence, they were penalized. Students to note that they have to show this step before they embarked on their GC calculations: $\mathrm{P}(X\geq 20)=1-P(X\leq 19)$
7b	Let Y be the random variable representing the number of elderly patients with diabetes. $Y\sim \mathrm{B}(40,\frac{k}{100})$ $\mathrm{P}(9\leq Y\leq 20)=0.25$ $\mathrm{P}(Y\leq 20)-\mathrm{P}(Y\leq 8)=0.25$ Using GC, $k=17.367$ or $k=56.512$ Since $k<50$, $\therefore\ k=17.4$ (3 s.f)	Many students interpreted the question wrongly. As GC only supports binompdf or binomcdf, students need to show this step: $\mathrm{P}(9\leq Y\leq 20)=0.25$ $\mathrm{P}(Y\leq 20)-\mathrm{P}(Y\leq 8)=0.25$

8a	$\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)} = 0.7$ $\mathrm{P}(A \cap B) = 0.7\left(\dfrac{1}{3}\right) = \dfrac{7}{30}$ $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ $\therefore \mathrm{P}(A) = \dfrac{3}{5} - \dfrac{1}{3} + \dfrac{7}{30} = \dfrac{1}{2}$	
8b	 Method 1 Required answer $= \mathrm{P}(A) - \mathrm{P}(A \cap B) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ $= \dfrac{1}{2} - \dfrac{7}{30} + \dfrac{1}{3} - \dfrac{7}{30}$ $= \dfrac{11}{30}$ Method 2 Required answer $= \mathrm{P}(A \cup B) - \mathrm{P}(A \cap B)$ $= \dfrac{3}{5} - \dfrac{7}{30}$ $= \dfrac{11}{30}$	Students are encouraged to use the venn diagram to indicate the required region before they start their calculations. There is no need to apply any new formulas to do this question.
8c	Method 1 $\mathrm{P}(A \mid B) = \dfrac{7}{10}$ $\mathrm{P}(A) = \dfrac{1}{2}$ Not independent. Method 2 $\mathrm{P}(A \cap B) = \dfrac{7}{30}$ $\mathrm{P}(A).\mathrm{P}(B) = \dfrac{1}{6}$ Not independent.	Students have to write down the value of both probabilities and make comparison.

9a		But some did not do the first branch to merge the diagram as one tree.
9b	$\frac{1}{2}\cdot\frac{2}{3}\cdot\frac{2}{3}+\frac{1}{2}\cdot\frac{1}{3}\cdot\frac{1}{2}+\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{3}{4}+\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{3}$ $=\frac{83}{144}$	
9c	$\frac{\text{P(first ball from Bag A and both balls are red)}}{\text{P(both balls are red)}}$ $=\frac{\text{P(first ball from Bag A and both balls are red)}}{\text{P(first ball from Bag A and both red)+P(first ball from Bag B and both red)}}$ $=\frac{\frac{1}{2}\cdot\frac{2}{3}\cdot\frac{2}{3}}{\frac{1}{2}\cdot\frac{2}{3}\cdot\frac{2}{3}+\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{3}{4}}$ $=\frac{32}{59}$	Some students are confused in the application of the conditional prob formula.
10i		some did not label the min and max values of x and y on the axes.
10ii	$r = 0.980$ (3 s.f) There is a strong positive linear correlation between the amount of fertiliser used and the average crop yield.	Some answers were not given in context and hence full credit was not given.
10 iii	$y = 1.9673+0.19578\,x$ $y = 1.97+0.196\,x$	Answers to be in 3 s.f.
10 iv	$2.5 = 1.9673+0.19578\,x$ $x = 2.69 = 2.7$ g/m^2 Since $y = 2.5$ is within $2 \le y \le 3.6$ and $r = 0.980$ is close to 1, hence interpolation gives a reliable estimate.	Students should note that there are TWO points for a reliable estimate. Since they have substituted y=2.5, they

		should state the data range of y, to illustrate interpolation.
10v	There is no change in r as r is a measure of scatter and hence it is not affected by any change of units.	Students should note the correct explanation.
11i (a)	$4! \times 2! \times 3! = 288$	Most did the grouping method but have forgotten to do ordering amongst the sisters and ordering amongst the brothers.
11i (b)	Method 1 $6! + 6! - 5! = 1320$ Father is first + Mother is last – both father is first, and mother is last. Method 2 $6! + 5! \times 5! = 1320$	Students can follow the idea of P(AUB) formula in probability.
11ii	$\dfrac{4! \times {}^5C_3 \times 3!}{7!} = \dfrac{2}{7}$	Many did PnC instead of probability.
12a	Let X be heights of boys. $X \sim \mathrm{N}(172,\ \sigma^2)$ $\mathrm{P}(X > 156) = 0.948$ $\mathrm{P}(Z > \dfrac{156-172}{\sigma}) = 0.948$ $\dfrac{156-172}{\sigma} = -1.6257$ $\sigma = 9.8415 = 9.84$ (3 s.f)	
12bi	$\bar{X} \sim \mathrm{N}(172, \dfrac{9.84^2}{100})$ $\mathrm{E}(\bar{X})$ $= 80 \times \mathrm{P}(\bar{X} > 172)$ $= 80 \times 0.5$ $= 40$	Students to note that expected number of samples means to apply np formula, where n is the number of samples.
12b ii	Let Y be the number of samples with $\bar{X} > 172$. $Y \sim \mathrm{B}(80, 0.5)$ $\mathrm{P}(Y \leq 35)$ $= 0.15715$ $= 0.157$ (3 s.f)	Some students did not state the parameters for the binomial distribution and hence did not get full credit.

12c	Let W be heights of girls. $W \sim \mathrm{N}(165,\ 8^2)$ $X - W \sim \mathrm{N}(172-165,\ 9.84^2+8^2)$ $X - W \sim \mathrm{N}(7,\ 160.8256)$ $\mathrm{P}(-5 < X - W < 5) = 0.265$	Many students interpreted the question incorrectly not knowing that difference can mean that the boy is taller, or the girl is taller. Students should show their intermediate steps to compute the mean and variance.
12d	$W_1+W_2+W_3-2X \sim \mathrm{N}(165(3)-172(2),\ 8^2(3)+2^2\left(9.84^2\right))$ $W_1+W_2+W_3-2X \sim \mathrm{N}(151,\ 579.3024)$ $\mathrm{P}(W_1+W_2+W_3-2X \le 149) = 0.467$ (3 s.f)	Some students interpreted the question incorrectly by using 3W to represent the total height of 3 randomly chosen girls. Students should show their intermediate steps to compute the mean and variance.
13a	$\bar{x} = \dfrac{75}{200} + 18.5 = 18.875$ $s^2 = \dfrac{1}{200-1}\left(1968.04 - \dfrac{75^2}{200}\right) = 9.7483$ Let μ represent the ***population mean lifetime*** of the tyres To test $H_0: \mu = 18.5$ against $H_1: \mu \neq 18.5$ at 5% level of significance Under H_0, since $n = 40$ is large, $\bar{X} \sim \mathrm{N}(18.5, \dfrac{9.7483}{200})$ approximately by Central Limit Theorem. From GC, p-value = 0.0894 > 0.05, we do not reject H_0 There is insufficient evidence to conclude at 5% significance level that the population mean lifetime of tyres is not 18.5 km. Hence the manager's claim is supported by the data.	Students should note that as $\bar{x} = 18.875$ is exact, they shouldn't round it off to 3 s.f. Presentations for hypothesis testing were not satisfactory. Students to note that the concluding statement should be written in the context of the question for the alternative hypothesis. Then a second statement maybe required to answer the QP.
13b	0.05 is the probability of wrongly concluding that the population mean lifetime of tyres is not 18.5 km when in fact it is 18.5 km.	

13c	Let Y represent the ***<u>lifetime</u>*** of the tyres produced by the new process. To test $H_0 : \mu = \mu_0$ against $H_1 : \mu > \mu_0$ at 1% level of significance Under H_0, $\bar{Y} \sim N(\mu_0, \frac{9.5}{400})$ $z = \frac{\bar{y} - \mu_0}{\sqrt{\frac{\sigma^2}{n}}} = \frac{20 - \mu_0}{\sqrt{\frac{9.5}{400}}}$ Since it is given that the new process is effective, we reject H_0 $\frac{20 - \mu_0}{\sqrt{\frac{9.5}{400}}} > 2.3263$ $\mu_0 < 19.641$ $0 \le \mu_0 < 19.6$	Most students did not read the question carefully and hence wrote the wrong alternative hypothesis. Many students aren't aware that the 9.5 given was population variance. They interpreted it incorrectly as the sample variance. As lifetime of the tyres can't be negative, the lower bound is necessary.