Candidate Index Number

Anglo - Chinese School (Independent)

FINAL EXAMINATIONS 2017
YEAR 3 INTEGRATED PROGRAMME
CORE MATHEMATICS
PAPER 1

WEDNESDAY **4th OCTOBER 2017** **1 h 30 min**

INSTRUCTIONS TO CANDIDATES

- Write your index number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Answer all questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.
- The maximum mark for this paper is 80.

For Examiner's Use

This paper consists of 16 printed pages.

[Turn over

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

*Answer **all** the questions in the spaces provided.*

1 *[Maximum mark: 3]*

(i) Simplify $(p+a)(p-a)-p^2$. *[1 mark]*

(ii) Hence, evaluate $48919 \times 48913 - 48916^2$. *[2 marks]*

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2 *[Maximum mark: 7]*

(a) Evaluate $\left(2-\frac{1}{2}\right)^{2}-\left(1-\frac{1}{3}\right)^{-1}$. *[2 marks]*

(b) Simplify the following

(i) $\left[\left(2x^{-1}\right)^{-2}\right]^{-3}$, leaving your answer in the positive power. *[2 marks]*

(ii) $\frac{x^{2}y^{-3}}{y^{2}z^{-4}}\times\frac{x^{3}y^{7}}{\left(xy^{2}z\right)^{-1}}$, giving your answer in the from $x^{p}y^{q}z^{r}$, where p, q and r are rational numbers. *[3 marks]*

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3 *[Maximum mark: 9]*

(a) Given that $-4 \le x \le -\frac{1}{2}$ and $-1 \le y \le 7$, find

(i) the smallest possible value of $y - x^3$, *[2 marks]*

(ii) the greatest possible value of $\frac{y}{2x}$. *[2 marks]*

(b) Solve $x - \frac{3}{2} < \frac{5-6x}{4} \le x + \frac{1}{2}$ and state the integer that satisfies the inequality. *[5 marks]*

4 *[Maximum mark: 10]*

Three points *A*, *B* and *C* have coordinates $(2, -1)$, $(6,\ 3)$ and $(-2, -3)$ respectively.
If *D* is the mid-point of *AB*, find

(i) the coordinates of *D*, *[1 mark]*

(ii) the equation of the line L_1 which passes through *D* and is parallel to *BC*, *[3 marks]*

(iii) the coordinates of *E*, the point where L_1 cuts $2y + 4 = x$. *[3 marks]*

(iv) the area of *BCED*. *[3 marks]*

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5 *[Maximum mark: 3]*

It is given that x and y are related by $x = \ln\left(3x - \frac{5}{\sqrt{y}}\right)$. If a straight line PQ is obtained by plotting $e^x\sqrt{y}$ against $x\sqrt{y}$, find the gradient of the line PQ.

6 *[Maximum mark: 8]*

(i) Solve the simultaneous equations $y = x^3$ and $y = \frac{16}{x}$. *[3 marks]*

(ii) On the same axes, draw the graphs of $y = x^3$ and $y = \frac{16}{x}$, labelling the points of intersection clearly on your sketch. *[3 marks]*

(iii) Find the distance between the points of intersection, leaving your answer in the form $a\sqrt{b}$. *[2 marks]*

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7 *[Maximum mark: 10]*

Solve the following equations

(i) $9^{3x+1} - 5\left(9^{3x}\right) = 108$ *[3 marks]*

(ii) $3^{2x+1} + 4\left(3^{x}\right) = 39$ *[4 marks]*

(iii) $64^{\log_8 x} = 9$ *[3 marks]*

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8 *[Maximum mark: 7]*

Solve the simultaneous equations

$$3^{x+2} = 27\left(9^{\frac{3}{2}y}\right)$$

$$\log_3 9 - \log_3(15y - 3x) = 1$$

9 *[Maximum mark: 10]*

(a) Given that the roots of the quadratic equation $4x^2 - 8x - 3 = 0$ are α and β, find the quadratic equation, whose roots are $\alpha^2\beta$ and $\alpha\beta^2$. *[4 marks]*

(b) Given that $x + y = 5$ and $\frac{6}{x} + \frac{6}{y} = 5$, find the value of $x^2 - y^2$, where $x < y$. *[6 marks]*

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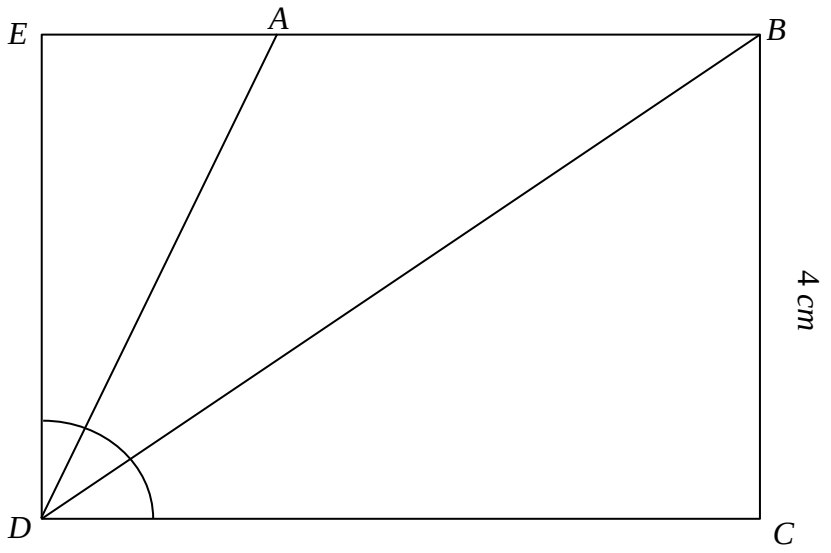
10 *[Maximum mark: 4]*

In rectangle *BCDE*, *CB* = 4 cm. Lines *AD* and *BD* divides $\angle EDC$ into 3 equal parts. Given that $\sin 30° = \frac{1}{2}$ and $\cos 30° = \frac{\sqrt{3}}{2}$, what is the area of ΔABD?



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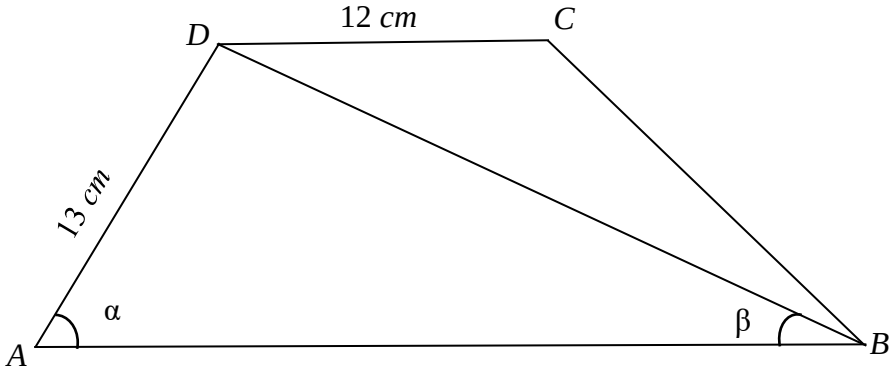
11 *[Maximum mark: 5]*

Figure *ABCD* below is a trapezium such that $AD = 13cm$, $CD = 12cm$, $\cos\alpha = \frac{5}{13}$ and $\tan\beta = \frac{3}{4}$.

Find BC^2.



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12 *[Maximum mark: 4]*

The sale price of a square towel \$$y$ is related to the length of each side of the towel x cm by the equation $y = (k-5)x^2 - 8x + k$. Given that the minimum sale price must be \$5 for all sizes of the towel, find the range of values of k.

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ANSWER KEY

1. (i) $-a^2$ (ii) -9

2 (a) $\frac{3}{4}$ (b) (i) $\frac{64}{x^6}$ (ii) $x^6y^4z^5$

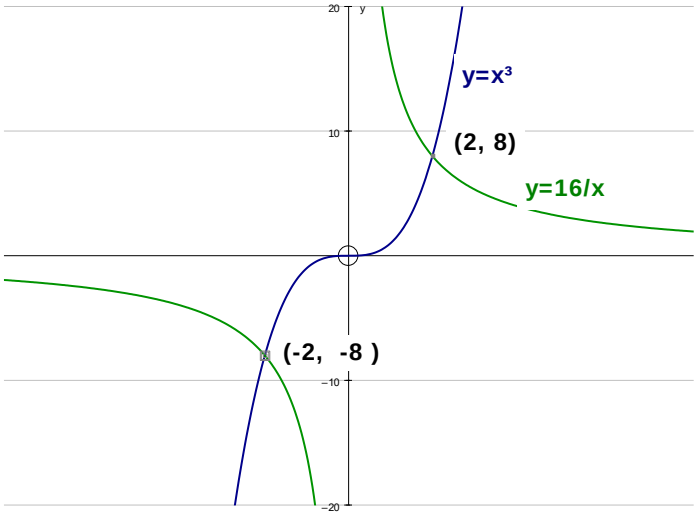
3. (a) (i) $-\frac{7}{8}$ (ii) 1 (b) $\frac{3}{10} \le x < 1\frac{1}{10}$, 1

4. (i) $(4, 1)$ (ii) $4y = 3x - 8$ (iii) $E = (0,-2)$ (iv) 3 unit²

5. 3

6. (i) $(-2,-8)$ and $(2, 8)$ (iii) $4\sqrt{17}$

6 (ii)



7. (i) $\frac{1}{2}$ (ii) 1 (iii) 3

8. $x = 4$, $y = 1$

9. (a) $64x^2 + 96x - 27 = 0$ (b) (-5)

10. $\frac{16\sqrt{3}}{3}$ units²

11. 160 cm²

12. $k \ge 9$