1 The curve $y = \mathrm{f}(x)$ is such that $\mathrm{f}'(x) = 2x^2 - 4x + 5$.

(i) Explain why the curve $y = \mathrm{f}(x)$ has no stationary points. [2]

(ii) Given that the curve passes through the point (–1, –6), find an expression for $\mathrm{f}(x)$. [3]

(i)	$\mathrm{f}'(x) = 2\left[x^2 - 2x + 2.5\right]$ $= 2\left[(x-1)^2 + 1.5\right]$ $= 2(x-1)^2 + 3$ Since $(x-1)^2 \geq 0$ for all real x, $(x-1)^2 + 3 > 0$ for all x. $\mathrm{f}'(x) \neq 0$, f(x) has no stationary points. <u>Method 2</u> $2x^2 - 4x + 5 = 0$ $b^2 - 4ac = (-4)^2 - 4(2)(5)$ $= -24 < 0$ No real roots. Hence, $\mathrm{f}'(x) \neq 0$, f(x) has no stationary points.	Note: must conclude with $\mathrm{f}'(x) \neq 0$
(ii)	$\mathrm{f}(x) = \int 2x^2 - 4x + 5 \ dx$ $\mathrm{f}(x) = \frac{2}{3}x^3 - 2x^2 + 5x + c$ Sub (–1, –6), $c = \frac{5}{3}$ $\mathrm{f}(x) = \frac{2}{3}x^3 - 2x^2 + 5x + \frac{5}{3}$	

2 The function f is defined by $\mathrm{f}(x) = x^4 - 2x^3 + kx^2 + 8$, where k is a constant.
It is given that $\mathrm{f}(x) = 0$ has a repeated root 2.

(i) Find the value of k, [2]

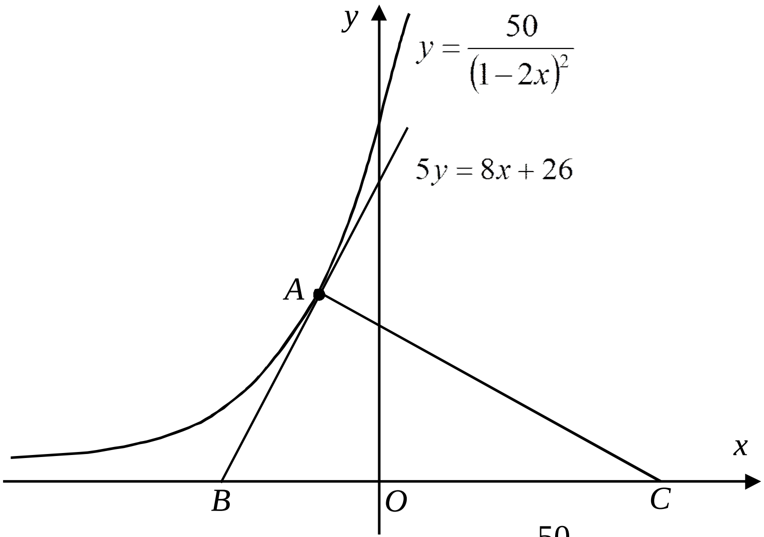
(ii) Determine, showing all necessary working, the number of real solution(s) of the equation $\mathrm{f}(x) = 0$. [4]

(i)	$\mathrm{f}(2) = 2^4 - 2(2)^3 + k(2)^2 + 8 = 0$ $4k = -8$ $k = -2$	Repeated roots means the roots of the equation are $x = 2$ and $x = 2$
(ii)	$\mathrm{f}(x) = x^4 - 2x^3 - 2x^2 + 8$ $\mathrm{f}(x) = (x-2)(x-2)(ax^2 + bx + c)$ By comparing, $a = 1$, $c = 2$, $\mathrm{f}(x) = (x^2 - 4x + 4)(x^2 + bx + 2)$ Compare x^2 term, $2x^2 - 4bx^2 + 4x^2 = -2x^2$ $b = 2$ $\mathrm{f}(x) = (x-2)^2(x^2 + 2x + 2)$ $\mathrm{f}(x) = (x-2)^2(x^2 + 2x + 2) = 0$ $(x-2)^2 = 0$ or $(x^2 + 2x + 2) = 0$ $x = 2$ (repeated) or $x = \dfrac{-2 \pm \sqrt{2^2 - 4(1)(2)}}{2(1)}$ (no real soln) $\therefore$ 1 real solution or 2 real and repeated solutions	Repeated root means $(x-2)(x-2)$ or expand $(x^2 - 4x + 4)$ $x^2 - 4x + 4 \overline{\big) x^4 - 2x^3 - 2x^2 + 8}$ quotient $x^2 + 2x + 2$ $x^4 - 4x^3 + 4x^2$ $2x^3 - 6x^2 + 8$ $2x^3 - 8x^2 + 8x$ $2x^2 - 8x + 8$ $2x^2 - 8x + 8$ 0

3 The roots of the equation $2x^2 - 4x + p = 0$, where p is a constant, are α and β.
The roots of the equation $6x^2 + qx + 9 = 0$, where q is a constant, are $\dfrac{\alpha^2}{\beta}$ and $\dfrac{\beta^2}{\alpha}$. Find the value of p and of q. [6]

$2x^2 - 4x + p = 0$ $\alpha + \beta = -\dfrac{-4}{2} = 2 \ldots(1)$ $\alpha\beta = \dfrac{p}{2} \ldots(2)$ $6x^2 + qx + 9 = 0$ $\dfrac{\alpha^2}{\beta} + \dfrac{\beta^2}{\alpha} = -\dfrac{q}{6} \ldots(3)$ $\dfrac{\alpha^2}{\beta} \times \dfrac{\beta^2}{\alpha} = \dfrac{9}{6}$ $\Rightarrow \alpha\beta = \dfrac{3}{2} \ldots(4)$ (2) = (4), $\dfrac{p}{2} = \dfrac{3}{2}$ $p = 3$ From (3), $\dfrac{\alpha^2}{\beta} + \dfrac{\beta^2}{\alpha} = \dfrac{\alpha^3 + \beta^3}{\alpha\beta} = -\dfrac{q}{6}$ Sub (2) and p = 3, $\alpha^3 + \beta^3 = -\dfrac{q}{6} \times \dfrac{3}{2} = -\dfrac{q}{4} \ldots(5)$ $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ $= 4 - 2\left(\dfrac{3}{2}\right)$ $= 1$ $\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)$ $= (2)\left(1 - \dfrac{3}{2}\right)$ $= -1$ From (5), $-\dfrac{q}{4} = -1$ $q = 4$	

4



The diagram shows part of the curve $y=\dfrac{50}{(1-2x)^2}$. The tangent $5y=8x+26$ at the point A on the curve cuts the x-axis at B. The normal at A cuts the x-axis at C. Find the area of triangle ABC. [9]

$\dfrac{dy}{dx}=50(-2)(-2)(1-2x)^{-3}$ $\dfrac{dy}{dx}=\dfrac{200}{(1-2x)^3}$, Gradient of tangent $=\dfrac{8}{5}$ $\dfrac{200}{(1-2x)^3}=\dfrac{8}{5}$ $125=(1-2x)^3$ $1-2x=5$ $x=-2$ $y=2$ A = (–2, 2) Sub y=0 into $5y=8x+26$, x = –3.25 B=(–3.25, 0) Gradient of normal $=-\dfrac{5}{8}$ $\dfrac{0-2}{x-(-2)}=-\dfrac{5}{8}$ $x=1.2$ C = (1.2, 0) Area of ΔABC = $\dfrac{1}{2}\times 2\times 4.45=$ 4.45 sq units	

5 **(a)** **(i)** Write down the general term in the binomial expansion of $\left(\frac{x^2}{4}+\frac{2}{x}\right)^{18}$. [1]

(ii) Write down the power of x in this general term. [1]

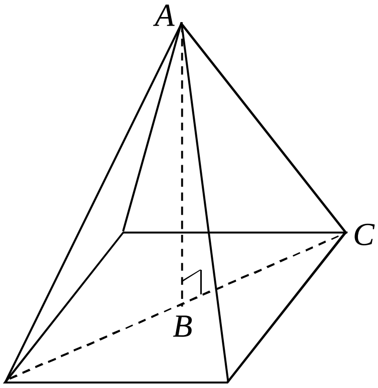
(iii) Hence, or otherwise, determine the term independent of x in the binomial expansion of $\left(\frac{x^2}{4}+\frac{2}{x}\right)^{18}$. [2]

(b) In the binomial expansion of $(2-kx)^n$, where $n \geq 3$ and k is a constant, the coefficients of x and x^2 are equal. Express k in terms of n. [4]

(a)(i)	General term ${}^{18}C_r\left(\frac{x^2}{4}\right)^{18-r}\left(\frac{2}{x}\right)^r$ or ${}^{18}C_r\left(\frac{x^2}{4}\right)^{r}\left(\frac{2}{x}\right)^{18-r}$	
(a)(ii)	Power of $x = 36-3r$ or $3r-18$	
(a)(iii)	$36 - 3r = 0$ $r = 12$ term independent of x = ${}^{18}C_{12}\left(\frac{x^2}{4}\right)^{18-12}\left(\frac{2}{x}\right)^{12}$ $=18564$	or r = 6
(b)	$(2-kx)^n$ Term x = ${}^nC_1(2)^{n-1}(-kx)^1$ $= -nk(2)^{n-1}x$ Term x^2 = ${}^nC_2(2)^{n-2}(-kx)^2$ $= \frac{n(n-1)}{2}k^2(2)^{n-2}x^2$ $\therefore -nk(2)^{n-1} = \frac{n(n-1)}{2}k^2(2)^{n-2}$ $k = \frac{-n(2)^{n-1}}{n(n-1)(2)^{n-3}}$ $k = \frac{-(2)^{n-1-n+3}}{(n-1)}$ $k = \frac{4}{1-n}$	or $-\frac{4}{n-1}$, or $\frac{-4}{n-1}$

6 Do not use a calculator in this question.

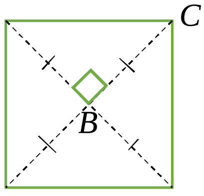
(i) Express $\dfrac{26\sqrt{3}}{3\sqrt{3}+1}$ in the form $a+b\sqrt{3}$, where a and b are integers. [3]



A toy is modelled in the shape of a right pyramid with a square base as shown in the diagram. The vertical height AB of the pyramid is $\left(\sqrt{3}+4\right)$ cm. Given that the length of the slant edge AC is $\dfrac{26\sqrt{3}}{3\sqrt{3}+1}$ cm,

(ii) find an expression for BC^2 in the form $c+d\sqrt{3}$, where c and d are integers, [3]

(iii) express the volume of the pyramid in the form $\dfrac{2}{3}\left(k-39\sqrt{3}\right)$ cm^3, where k is an integer. [2]

(i)	$\dfrac{26\sqrt{3}}{3\sqrt{3}+1}\times\dfrac{3\sqrt{3}-1}{3\sqrt{3}-1}$ $=\dfrac{26\left(9-\sqrt{3}\right)}{27-1}$ $=9-\sqrt{3}$	
(ii)	$BC^2=\left(9-\sqrt{3}\right)^2-\left(\sqrt{3}+4\right)^2$ $BC^2=81-18\sqrt{3}+3-3-8\sqrt{3}-16$ $BC^2=65-26\sqrt{3}$	
(iii)	Volume = $\dfrac{1}{3}l^2\left(\sqrt{3}+4\right)$ $=\dfrac{1}{3}\left[2(BC)^2\right]\left(\sqrt{3}+4\right)$ $=\dfrac{2}{3}\left(\sqrt{3}+4\right)\left(65-26\sqrt{3}\right)$ $=\dfrac{2}{3}\left(65\sqrt{3}+260-78-104\sqrt{3}\right)$ $=\dfrac{2}{3}\left(182-39\sqrt{3}\right)$	 Base area = ½ × BC × BC × 4

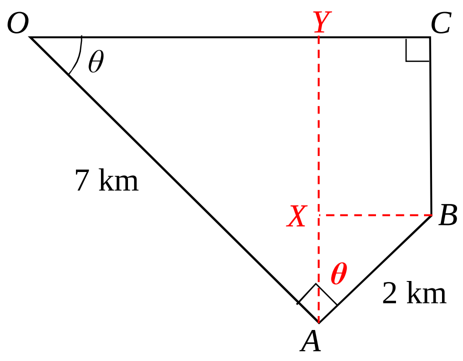
7 **(i)** Differentiate $\ln(2x^2+1)$ with respect to x. [2]

(ii) Express $\dfrac{4x^2+2x+1}{(x+1)(2x^2+1)}$ in partial fractions. [4]

(iii) Hence evaluate $\int_0^{\frac{1}{2}} \dfrac{4x^2+2x+1}{(x+1)(2x^2+1)}\,dx$. [4]

(i)	$\dfrac{d}{dx}\ln(2x^2+1)=\dfrac{4x}{2x^2+1}$	
(ii)	$\dfrac{4x^2+2x+1}{(x+1)(2x^2+1)}=\dfrac{A}{x+1}+\dfrac{Bx+C}{2x^2+1}$ $4x^2+2x+1=A(2x^2+1)+(Bx+C)(x+1)$ sub $x=-1$, $3=3A$, $A=1$ sub $x=0$, $1=1+C$, $C=0$ Sub $x=1$, $7=3+2B$, $B=2$ $\dfrac{4x^2+2x+1}{(x+1)(2x^2+1)}=\dfrac{1}{x+1}+\dfrac{2x}{2x^2+1}$	
(iii)	$\int_0^{\frac{1}{2}} \dfrac{4x^2+2x+1}{(x+1)(2x^2+1)}\,dx$ $=\int_0^{\frac{1}{2}} \left(\dfrac{1}{x+1}+\dfrac{2x}{2x^2+1}\right)dx$ $=[\ln(x+1)]_0^{\frac{1}{2}}+\left(\dfrac{1}{2}\right)[\ln(2x^2+1)]_0^{\frac{1}{2}}$ $=\ln\dfrac{3}{2}-\ln 1+\dfrac{1}{2}\ln\dfrac{3}{2}-\dfrac{1}{2}\ln 1$ $=0.608$	Note: Use reverse $\dfrac{d}{dx}\ln(2x^2+1)=\dfrac{4x}{2x^2+1}$ $\int \dfrac{4x}{2x^2+1}dx=\ln(2x^2+1)+c1$ $\int \dfrac{2x}{2x^2+1}dx=\dfrac{1}{2}\ln(2x^2+1)+c$

8



The diagram shows a cycling circuit formed from four straight roads *OA*, *AB*, *BC*, and *CO*. *OA* = 7 km, *AB* = 2 km, angle *OAB* = angle *BCO* = 90°, and angle *COA* = θ where $0° \le \theta \le 90°$. A cyclist cycled along the circuit *OABCO*.

(i) Show that $OA + AB + BC + CO = 9\sin\theta + 5\cos\theta + c$, where c is a constant to be found. [2]

(ii) Express $OA + AB + BC + CO$ in the form $R\cos(\theta - \alpha) + c$. [4]

(iii) Find the values of θ for which the cyclist cycled a total distance of 19 km. [3]

(iv) State the maximum possible value for the total distance of the circuit. [1]

(i)	$\sin\theta = \frac{AY}{7} \Rightarrow AY = 7\sin\theta$, $\quad \cos\theta = \frac{OY}{7} \Rightarrow OY = 7\cos\theta$ $\sin\theta = \frac{XB}{2} \Rightarrow XB = 2\sin\theta$, $\quad \cos\theta = \frac{AX}{2} \Rightarrow AX = 2\cos\theta$ $OC = OY + XB = 7\cos\theta + 2\sin\theta$ $BC = AY - AX = 7\sin\theta - 2\cos\theta$ $\text{Total} = 7 + 2 + 7\sin\theta - 2\cos\theta + 7\cos\theta + 2\sin\theta$ $= 9\sin\theta + 5\cos\theta + 9$	Note: use vertical and horizontal lines to divide diagram. The sides 7 cm and 2 cm are usually the hypotenuse of the right-angled triangle.
(ii)	$5\cos\theta + 9\sin\theta = R\cos(\theta - \alpha)$ $= R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$ $5 = R\cos\alpha$ $9 = R\sin\alpha$ $R = \sqrt{5^2 + 9^2} = \sqrt{106}$ $\tan\alpha = \frac{9}{5}$ $\alpha = 60.945°$ $\sqrt{106}\cos(\theta - 60.9°) + 9$	
(iii)	$\sqrt{106}\cos(\theta - 60.9°) + 9 = 19$ $\qquad 0° \le \theta \le 90°$ $\cos(\theta - 60.945°) = \frac{10}{\sqrt{106}}$ $\qquad -60.95° \le \theta - 60.95° \le 90° - 60.95°$ Ref $\angle = 13.763°$ $\qquad -60.9° \le \theta - 60.9° \le 29.1°$ $\theta - 60.945° = 13.763°, \ -13.763°$ $\theta = 74.7°, \quad 47.2°$	

(iv)	Max = $\sqrt{106}+9=$ 19.3 km	

9 The equation of a curve is $y=(1+2x)^3(3-2x)$.

(i) Find the coordinates of the stationary points of the curve. [6]

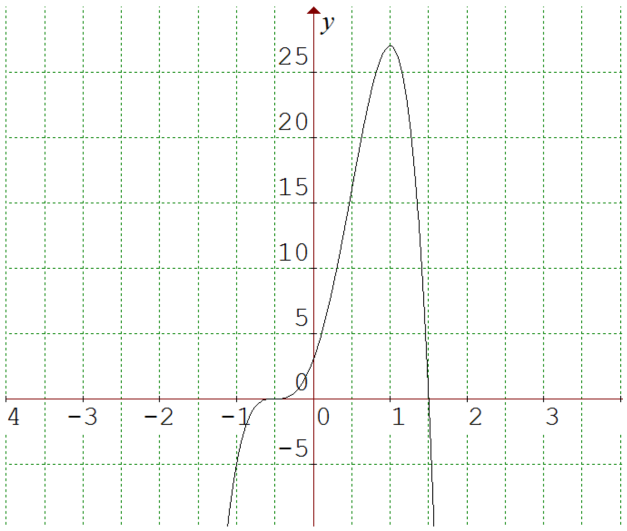
(ii) Determine the nature of these stationary points. [3]

(iii) Hence, sketch the curve. [2]

<table>
<tr><td>(i)</td><td>$y=(1+2x)^3(3-2x)$
$\frac{dy}{dx}=(1+2x)^3(-2)+(3-2x)(3)(2)(1+2x)^2$
$=(1+2x)^2[-2(1+2x)+6(3-2x)]$
$=(1+2x)^2[-2-4x+18-12x]$
$=(1+2x)^2[16-16x]$

$(1+2x)^2[16-16x]=0$
$x=-0.5$, $x=1$
$y=0$ $y=27$
Coordinates are (–0.5, 0) and (1, 27)</td><td></td></tr>
<tr><td>(ii)</td><td><table><tr><td>x</td><td>–0.6</td><td>–0.5</td><td>–0.4</td><td>0.9</td><td>1</td><td>1.1</td></tr><tr><td>$\frac{dy}{dx}$</td><td>+</td><td>0</td><td>+</td><td>+</td><td>0</td><td>–</td></tr></table>(–0.5, 0) is a point of inflexion.
(1, 27) is a maximum point.

Alternative:
$\frac{d^2y}{dx^2}=16[4(1+2x)(1-x)+(1+2x)^2(-1)]$
$=16(1+2x)[4(1-x)+(1+2x)(-1)]$
$=16(1+2x)[4-4x-1-2x]$
$=48(1+2x)[1-2x]$
Sub x = –0.5, $\frac{d^2y}{dx^2}=0$
$\Rightarrow$no conclusion do dy/dx sign test (see above)
(–0.5, 0) is a point of inflexion.
Sub x = 1, $\frac{d^2y}{dx^2}=-144<0$
(1, 27) is a maximum point.</td><td>When $\frac{d^2y}{dx^2}=0$, no conclusion, must do dy/dx sign test</td></tr>
</table>

(iii)		Show coordinates (0, 3), (–0.5, 0), (1, 27)

10 A circle, C_1, passes through the points $A(-3,-25)$ and $B(7,-25)$. The centre and radius of the circle are (a,b) and r respectively. The x-axis is a tangent to the circle.

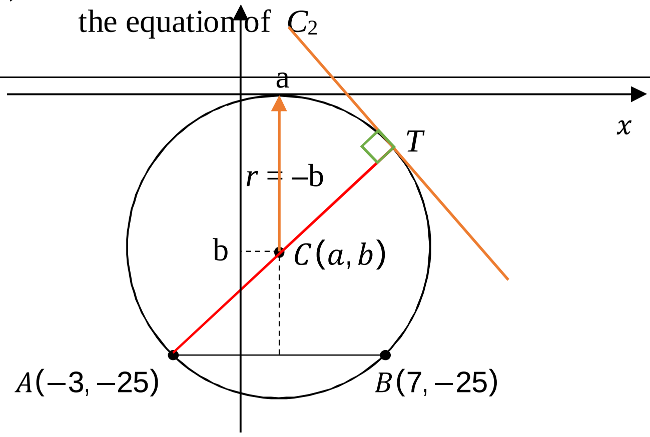
(i) Find the value of a. [1]

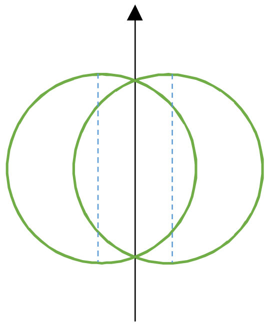
(ii) Show that $b=-13$. [3]

(iii) Find the equation of the circle C_1. [1]

(iv) AT is a diameter of the circle. Find the equation of the tangent to the circle at T. [5]

(v) Given that a second circle, C_2, is a reflection of C_1 in the y-axis, find the equation of C_2 [2]

(i)	 $a=\dfrac{7-3}{2}=2$	Note: perpendicular bisector of chord passes through centre
(ii)	Centre is $(2,b)$ $\sqrt{(-3-2)^2+(-25-b)^2}=-b$ $(-3-2)^2+(-25-b)^2=b^2$ $25+625+50b+b^2=b^2$ $50b=-650$ $b=-13$	

(iii)	Centre is $(2,-13)$ $r=13$ Equation of C_1: $(x-2)^2+(y+13)^2=169$	
(iv)	Gradient $=\dfrac{-13-(-25)}{2-(-3)}$ $=\dfrac{12}{5}$ Gradient of tangent at T $=-\dfrac{5}{12}$ Let T = (x, y) $\left(\dfrac{x-3}{2},\dfrac{y-25}{2}\right)=(2,-13)$ $x=7$, $y=-1$ T = (7, –1) Sub (7, –1) into $y=-\dfrac{5}{12}x+c$ $-1=-\dfrac{5}{12}(7)+c$ $c=\dfrac{23}{12}$ $y=-\dfrac{5}{12}x+\dfrac{23}{12}$	
(v)	Centre of C_2 = (–2, –13) Radius = 13 Eqn: $(x+2)^2+(y+13)^2=13^2$	

11 A student records the population, y, of a certain type of bacteria, x hours after the start of her experiment, in the table below. It is believed that an error was made in recording one of the values. The student wants to use the equation $y=ab^x$, where a and b are constants, to model the population of this type of bacteria.

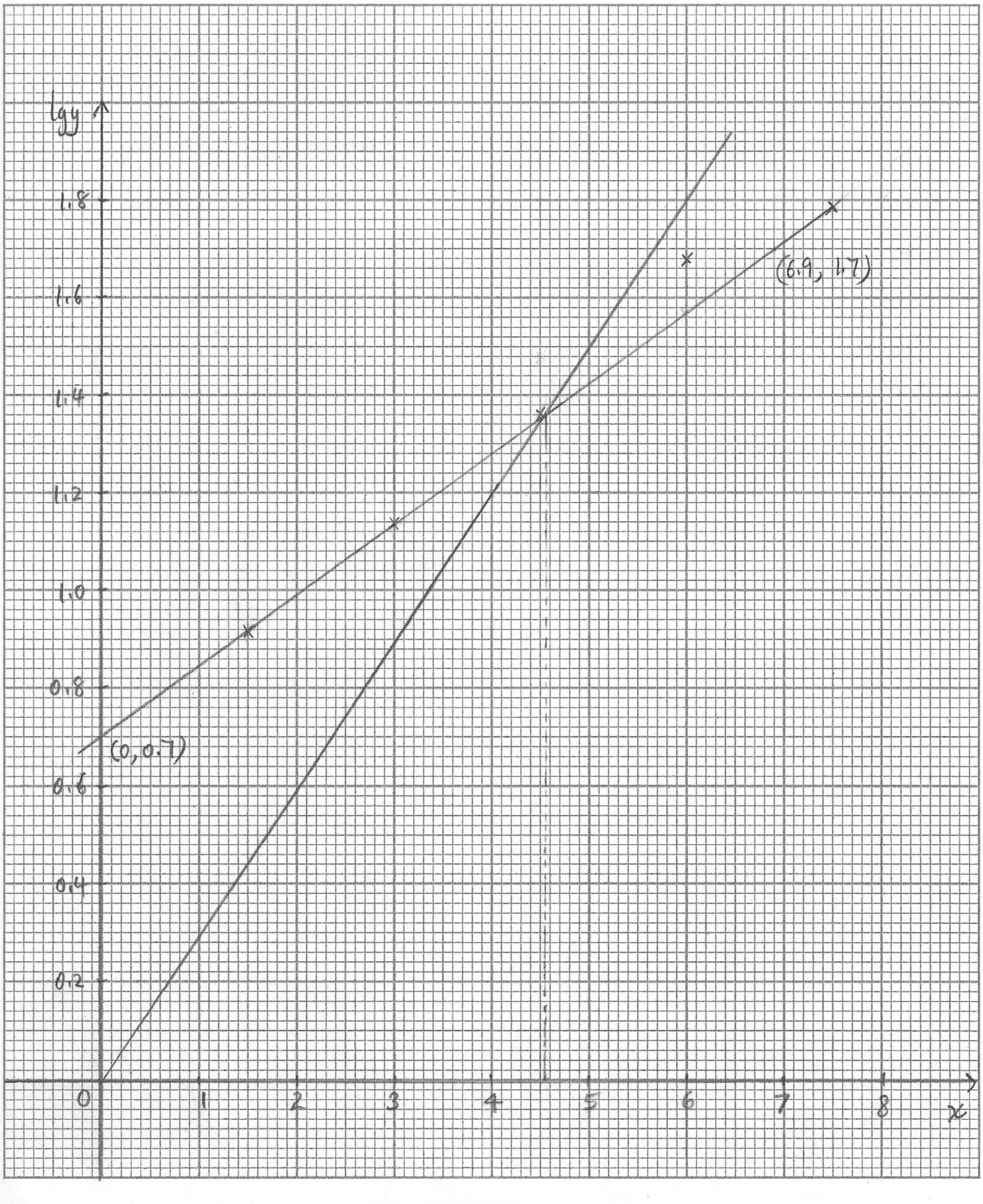
x	1.5	3	4.5	6	7.5
y	8.37	13.8	22.8	47.5	62.0

(i) Plot $\lg y$ against x and hence determine which value of y in the table above is the incorrect recording, and estimate the correct value of y. [5]

(ii) Hence, estimate the value of each of the constants a and b. [5]

(iii) The population of a second type of bacteria is modelled by the equation $y=2^x$. Using your values of a and b, calculate the value of x for which $ab^x=2^x$. [2]

(iv) Explain how another straight line drawn on your diagram can lead to an estimate of the value of x for which the populations of the two types of bacteria are equal. Draw this line and hence verify your value of x found in part **(iii)**. [3]



2 mm Square 20 cm x 24 cm

<table>
<tr><td>(i)</td><td>
<table>
<tr><td>x</td><td>1.5</td><td>3</td><td>4.5</td><td>6</td><td>7.5</td></tr>
<tr><td>$\lg y$</td><td>0.92</td><td>1.14</td><td>1.36</td><td>1.68</td><td>1.79</td></tr>
</table>

Incorrect $y = 47.5$
Correct $\lg y = 1.57$
$y = 37.2$ (3sf)</td><td></td></tr>
<tr><td>(ii)</td><td>$y = ab^x$
$\lg y = \lg a + x \lg b$
$Y = mX + c$
$m = \dfrac{1.7 - 0.7}{6.9 - 0} = 0.14492$
$\lg b = 0.14492$
$b = 1.396 = 1.40$ (3sf)
$c = 0.7$
$\lg a = 0.7$
$a = 5.011 = 5.01$ (3sf)</td><td></td></tr>
<tr><td>(iii)</td><td>$5.011 \times 1.396^x = 2^x$
$\left(\frac{2}{1.396}\right)^x = 5.011$
$x = \lg 5.011 \div \lg \frac{2}{1.396}$
$= 4.48$ (3sf)</td><td></td></tr>
<tr><td>(iv)</td><td>$ab^x = 2^x$
$y = 2^x$
$\lg y = x \lg 2$
Draw the line $\lg y = x \lg 2$.
The solution of the equation is the x-coordinate of the intersection point of the two graphs.
From the graphs,
$x = 4.55$</td><td></td></tr>
</table>