

CATHOLIC JUNIOR COLLEGE General Certificate of Education Advanced Level Higher 2 JC2 Preliminary Examination

MATHEMATICS

Paper 2

9740/02

31 Aug 2015

3 hours

Additional Materials: List of Formulae (MF15) Graph Paper

Name: _____

Class: _____

READ THESE INSTRUCTIONS FIRST

Write your name and class on all the work you hand in.

Write in dark blue or black pen on both sides of the paper.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, arrange your answers in NUMERICAL ORDER.

The number of marks is given in brackets [] at the end of each question or part question.

Section A: Pure Mathematics [40 marks]

1 A Singaporean tourist is visiting three countries, Denmark, England and Russia and is planning to buy souvenirs back for her loved ones. She plans to spend SGD\$84 on cheese, SGD\$85 on chocolate, and SGD\$77 on candy. She can only buy in souvenir packs consisting of 1kg cheese, 1kg chocolate and 1kg candy.

The exchange rates to the three countries are as follows:

SGD\$1 – 5 Danish Krone SGD\$1 – 0.5 British Pound SGD\$1 – 35 Russian Ruble



The prices of the commodities in the various countries' currency are given in the following table.

	Cheese/kg	Chocolate/kg	Candy/kg
Price in Danish Krone	20	30	30
Price in British Pound	4	5	2
Price in Russian Ruble	280	175	245

Find the total number of souvenir packs she should buy so that she spends all her money. [4]

2 The diagram below shows the graph of y = f(x). The curve has a minimum point at $\left(1, -\frac{1}{2}\right)$ and crosses the x-axis at (-2, 0), (0, 0) and (2, 0). The lines x = -1, x = 3 and y = -3 are the asymptotes of the curve.



- (i) Sketch, on separate diagrams, the graphs of
 - (a) $y^2 = -f(x)$, [4]
 - **(b)** y = f'(x), [3]

stating clearly in each case the equations of asymptotes, the coordinates of turning points and axial intercepts whenever possible.

- (ii) State the number of distinct real roots of the equation $|f(x)| \frac{1}{2} = 0$. [1]
- (iii) Describe fully a sequence of transformations which would transform the graph of y = f(x) to the graph of y = f(2x+1)-3. [3]

3 A calculator is **not** to be used in answering this question.

The complex numbers a and b are given by $\frac{1+i}{1-i}$ and $\frac{\sqrt{2}}{1-i}$ respectively. (i) Find the moduli and arguments of a and b.

- (ii) In an Argand diagram, the points A, B and C represent the complex numbers a, b and a+b respectively. The origin is denoted by O. By considering the quadrilateral OACB and the argument of a+b, show that $\tan\left(\frac{3\pi}{8}\right) = 1 + \sqrt{2}$. [3]
- (iii) Using a single Argand diagram, sketch the loci (a) |z-a|=2,

(b)
$$\arg(z-b)=\frac{\pi}{2}$$
.

Find the exact complex number z, in the form x + iy, that satisfies parts (a) and (b). [5]

4 In a research project, the population is modelled by the following *logistic* differential equation,

$$\frac{\mathrm{d}P}{\mathrm{d}t} = 0.64P \left(1 - \frac{P}{10}\right),$$

where P is the population function of time t.

(i) Solve the differential equation by expressing P in terms of t, given that P = 1 when t = 0. [5]

Sketch the solution curve for $t \dots 0$. Comment on the population in the long run. [2]

An alternative model for the population is the *Gompertz* function, which is the solution to the following differential equation,

$$\frac{\mathrm{d}P}{\mathrm{d}t} = 0.4P \left(\ln 10 - \ln P\right).$$

(ii) By solving the differential equation, show that the general solution is $P = 10e^{-Ae^{-0.4t}}$, where *A* is a constant. [3]

Given the same initial condition that P = 1 when t = 0, sketch the solution curve of the particular solution for $t \dots 0$ on the same diagram in part (i). Comment on the similarity and difference between the two models. [4]

[3]

Section B: Statistics [60 marks]

- 5 A class consists of 15 female and 10 male students. The form teacher needs to select 5 students from this class to attend a school function. The teacher wrote each student's name (all the students' names are distinct) on a small piece of paper of the same size, folded it into half and placed it in a large bowl. He then shook the bowl and took out five pieces of paper one by one without replacement.
 - (i) State the name of this method of sampling and explain a disadvantage of this method in the context of the question.
 [2]
 - (ii) Find the probability that 3 female students and 2 male students are selected. [3]

Amy is one of the 15 female students and Bertrand is one of the 10 male students.

- (iii) Find the probability that Amy and Bertrand are selected.
- (iv) Given that 3 females and 2 males are selected, find the probability that Amy and Bertrand are selected.[4]

The school management decided that the students selected must be representative of the class gender make-up.

- (v) Describe how the form teacher could select the 5 students. Write down the probability that Amy and Bertrand are selected using this method of sampling. [3]
- 6 On average 8% of cherries sold in supermarkets are rotten. A customer randomly selected 26 cherries from a large number of cherries.
 - State, in context, two assumptions for the number of rotten cherries in the sample to be well-modelled by a binomial distribution. [2]
 - (ii) Find the most likely number of rotten cherries that the customer could have picked. [2]
 - (iii) Another customer randomly selected n cherries such that the probability of having at most one rotten cherry is less than 0.1. Express this information as an inequality in n, and hence find the smallest possible integer value of n. [3]
 - (iv) The cherries are packed in boxes, each containing 60 cherries. Using a suitable approximation, find the probability that a randomly chosen box contains at most one rotten cherry.[3]

On average, 1 box of rotten cherries will be discarded every month.

(v) Using a suitable approximation, find the probability that in a year, the number of boxes of rotten cherries that will be discarded is between 2 and 5 inclusive. [3]

[2]

7 The masses, in grams, of carrots and onions are normally distributed with means and standard deviations as shown in the table below.

	Mean (g)	Standard deviation (g)		
Carrot	С	5		
Onion	75	3		

(i) The probability that twice the mass of a randomly chosen carrot exceeds the total mass of 5 randomly chosen onions is more than 0.9. Find the range of values of *c*.
 State an assumption needed for your calculation. [5]

It is given that c = 200g.

- (ii) Find the probability that the average mass of two carrots and three onions exceeds 130g.
- (iii) Carrots are sold at \$1.80 per kg and onions at \$1.50 per kg. Find the probability that the price difference between 3 carrots and 4 onions is less than \$0.60. [4]
- 8 A manufacturer claims that the mean mass of peanut butter in a jar is μ_0 g. A shopkeeper suspects that the manufacturer is overstating the mean mass. He takes a random sample of 10 jars of peanut butter and weighs the content, x g, in each jar. The results are summarized as follows:

$$\sum x = 2695$$
 and $\sum x^2 = 726313$.

(i) It is given that $\mu_0 = 270$.

Test at 10% level of significance, whether the shopkeeper's suspicion is justifiable. Find the p-value of the test and give the conclusion.

State, giving a reason, whether any assumption about the population is needed in order for the test to be valid.

The shopkeeper wants to test if the mean mass differs from μ_0 at 10% level of significance. Using the p-value obtained above, explain whether the conclusion would change. [6]

(ii) The manufacturer establishes that the mass of peanut butter in a jar is normally distributed with standard deviation 1.1 g. Using the same data from the shopkeeper's sample, find the range of possible values of μ_0 so that the shopkeeper's suspicion that the mean mass differs from μ_0 is not justified at the 5% level of significance. Give your answer correct to one decimal place. [4]

[3]

9 A type of fertilizer is applied to a small plot of land. The amount of fertilizer, w mg, retained by the soil after a period of t days is measured and the results are shown in the table below.

t days	5	10	15	20	25	30	35	40
w mg	89	82	60	42	45	21	21	18

- (i) Draw a scatter diagram for the data. [2] Find the product moment correlation coefficient between t and w, and comment on its value (ii) in the context of this question. [2] (iii) Find the equation of the least squares regression line of w on t in the form w = a + bt. Sketch this line on your scatter diagram. [2] (iv) Explain the meaning of b as found in part (iii) in the context of the question. [1] Find the estimated value of t when w is 30.7 mg. Comment on the reliability of your **(v)** estimate. [2] (vi) When an additional data pair (t_1, w_1) is added to the existing data, the correlation coefficient
 - found in part (**ii**) and the regression line found in part (**iii**) will not change. Find this data pair and plot it on your scatter diagram in part (**i**). [2]

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