Name: ___________________________ (   )   Class:______________

PRELIMINARY EXAMINATION
GENERAL CERTIFICATE OF EDUCATION ORDINARY LEVEL

MATHEMATICS Student Copy **4052/02**

Paper 2 **Friday 18 August 2023**

2 hours 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, register number, and class on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue, correction fluid or correction tape.

Answer **all** questions.
The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The total of the marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142.

Q1		**Q4**		**Q7**	
Q2		**Q5**		**Q8**	
Q3		**Q6**		**Q9**	

Paper 1	**/ 90**
Paper 2	**/90**
Total	**/100**

This document consists of **22** printed pages.

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Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of a triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1 **(a)** Solve the inequality $\frac{2x-1}{3} \geq \frac{x+5}{4}$.

$$\frac{2x-1}{3} \geq \frac{x+5}{4}$$
$$4(2x-1) \geq 3(x+5)$$
$$8x-4 \geq 3x+15$$
$$5x \geq 19$$
$$x \geq 3.8 \text{ or } 3\frac{4}{5} \text{ or } \frac{19}{5}$$

Answer ………………….………….. [2]

(b) It is given that $3d = b^2(c+5)$.

(i) Find d when $b = -2$ and $c = -6.2$.

$$d = \frac{(-2)^2(-6.2+5)}{3}$$
$$d = -1.6$$

Answer $d =$ ………………..………….. [1]

(ii) Express b in terms of c and d.

$$3d = b^2(c+5)$$
$$b^2 = \frac{3d}{c+5}$$
$$b = \pm\sqrt{\frac{3d}{c+5}}$$

Answer $b =$ ………………..………….. [2]

(c) Solve the equation $\dfrac{x}{2x-1}-\dfrac{3}{x}=2$.

Give your solutions correct to 2 decimal places.

$$\frac{x}{2x-1}-\frac{3}{x}=2$$
$$\frac{x^2-3(2x-1)}{x(2x-1)}=2$$
$$x^2-3(2x-1)=2x(2x-1)$$
$$x^2-6x+3=4x^2-2x$$
$$3x^2+4x-3=0$$
$$x=\frac{-4\pm\sqrt{52}}{6}$$
$$x=0.54,\ -1.87\ \text{(2dp)}$$

Answer x = ……………… or ……………….. [5]

2 **(a)** The speed of light is 3×10^{8} m/s.

Find the distance in kilometres travelled by light in 100 nanoseconds, giving your answer in standard form.
[1 nanosecond $=10^{-9}$ seconds]

$$\text{Distance } = \frac{3\times10^{8}}{1000}\times10^{-9}\times100$$
$$=3\times10^{-2}\text{ km (standard form)}$$

Answer ………………….………….. km [2]

(b) The mass of an ant is 5 milligrams.
A cat has a mass of 720 000 times that of the ant.

Calculate the mass of the cat in grams in standard form.
[I milligram $=10^{-3}$ gram]

$$\text{Mass of cat } = \frac{720000\times5}{1000}\text{ g}$$
$$=3600\text{ g}$$
$$=3.6\times10^{3}\text{ g (standard form) A1}$$

Answer ………………….………….. g [2]

(c) In 2023, the Government increased GST to 8%.
At Budget 2023, the Government announced enhancements to the GST Voucher (GSTV) scheme to help Singaporeans with the GST rate increase.

Mrs Lee intends to buy an item that has GST of $7.

Find the price of the item with GST.

$$\text{price} = \frac{7}{8}\times108$$
$$=\$94.50$$

Answer $ …………..……..………….. [2]

(d) June buys an item in Korea costing ₩115000.

The exchange rate between Singapore dollars and Korean Won is $1 = ₩967.54 .
The same item is on sale in Singapore costing $100.

How much would June have saved if she bought the item in Singapore?

Cost June paid in total $= \dfrac{115000}{967.54}$

$\approx \$118.858$

Savings $\approx 118.858 - 100 = \$18.86$

or

$\$100 = \text{W } 96754$

$\text{Save} = \text{W } 115000 - 96754$

$= \text{W } 18246$

$= \$\dfrac{18246}{967.54}$

$= \$18.86$

Answer $ …………..……..………….. [2]

3 J is a point $(-2, 7)$ and K is a point $(10, 1)$.

(a) Find the length of JK.

$$\text{Distance} = \sqrt{(-2-10)^2 + (7-1)^2}$$
$$= 13.41$$
$$\approx 13.4 \text{ units (3sf)}$$

Answer ……………..…..…………… [2]

(b) Find the equation of JK.

$$m_{JK} = \frac{7-1}{-2-10} \qquad 1 = -\frac{1}{2}(10) + c$$
$$= -\frac{1}{2} \qquad c = 6$$
$$\therefore y = -\frac{1}{2}x + 6$$

Answer ……………..…..………...…… [2]

(c) The line with equation $6y - 7x = 16$ intersects the line JK at point L.
Find the coordinates of L.

$y = -\frac{1}{2}x + 6$ -------(1)

$6y - 7x = 16$ -------(2)

sub (1) into (2)

$6\left(-\frac{1}{2}x + 6\right) - 7x = 16$

$-3x + 36 - 7x = 16$

$-10x = -20$

$x = 2$

$y = 5$

$\therefore L(2,5)$

Answer L (………….… , …………..…) [3]

(d) State the ratio $JL : JK$.

$JL = \sqrt{(-2-2)^2 + (7-5)^2} = \sqrt{20}$

$JK = \sqrt{180}$

$\therefore JL : JK = \sqrt{20} : \sqrt{180}$

or $JL_{run} : JK_{run} = 4:12$ or $JL_{rise} : JK_{rise} = 2:6$

$\therefore JL : JK = 1:3$

]

4 Hello Panda is a popular Japanese snack. Its cream-filled biscuits are shaped like panda faces. The biscuits are commonly sold in a hexagonal prism box modelled below.

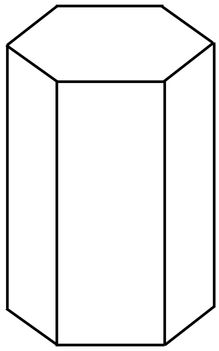


Diagram 1 shows the cross section area of the normal box.
It consists of a hexagon formed by two identical trapeziums.
$PS = 7.5$ cm, $QR = 4.5$ cm and $PQ = RS = 2.5$ cm.

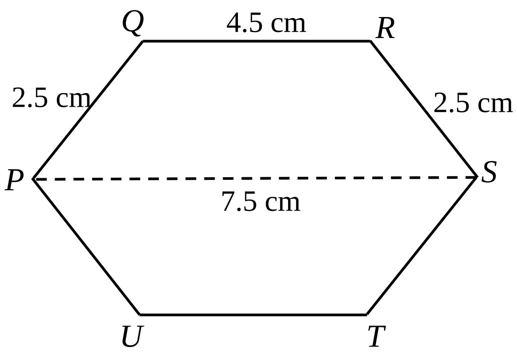
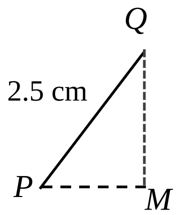


Diagram 1

(a) Show that the area of trapezium *PQRS* is 12 cm^2.

Answer

$$PM = \frac{7.5-4.5}{2} = 1.5$$
$$QM = \sqrt{2.5^2 - 1.5^2} = 2$$
$$\text{Area} = \frac{1}{2}\times[4.5+7.5]\times 2$$
$$= 12 \text{ cm}^2 \text{ (shown)}$$



[2]

(b) Given that the height of the box is 14 cm, calculate the total surface area of the box.

$$\text{Total SA} = (2\times12\times2) + 4(2.5\times14) + 2(4.5\times14)$$
$$= 314 \text{ cm}^2$$

Answer ……………………….…….. cm^2 [3]

Hello Panda has a family box, containing ten individual packets. Given that the two boxes are geometrically similar and the height of the family box is 3 times the height of the normal box.

(c) Calculate the total surface area of the family box, correct to 2 significant figures.

$$\frac{\text{Area}}{314} = \left(\frac{3}{1}\right)^2$$
$$\text{Area} = 9 \times 314$$
$$= 2826$$
$$\approx 2800 \text{ cm}^2$$

Answer …………………………..…….. cm^2 [2]

Jack went to the supermarket to purchase some Hello Panda biscuits.
They were sold as Normal boxes and Family boxes.

Size	**Normal Box**	**Family Box**
Weight	Contains 1 packet weighing 50 grams	Contains 10 packets, each weighing 26 grams
Price	Buy 5 boxes for $5.80	Buy 1 box for $6.61

(d) Which packaging is more value for money? Justify your answer with calculations.

Answer

Normal Family

$\frac{5.80}{5 \times 50} = 0.0232$ \$/g $\frac{6.61}{10 \times 26} = 0.0254$ \$/g

or

$\frac{5 \times 50}{5.80} = 43.1$ g/\$ $\frac{10 \times 26}{6.61} = 39.3$ g/\$

Either show 0.0232 < 0.0254 or 43.1 > 39.3

Buying the normal pack (at 5 for $5.80) is more value for money.

……………………………………………………………………………………………………

…………………………………………………………………………………………….. [3]

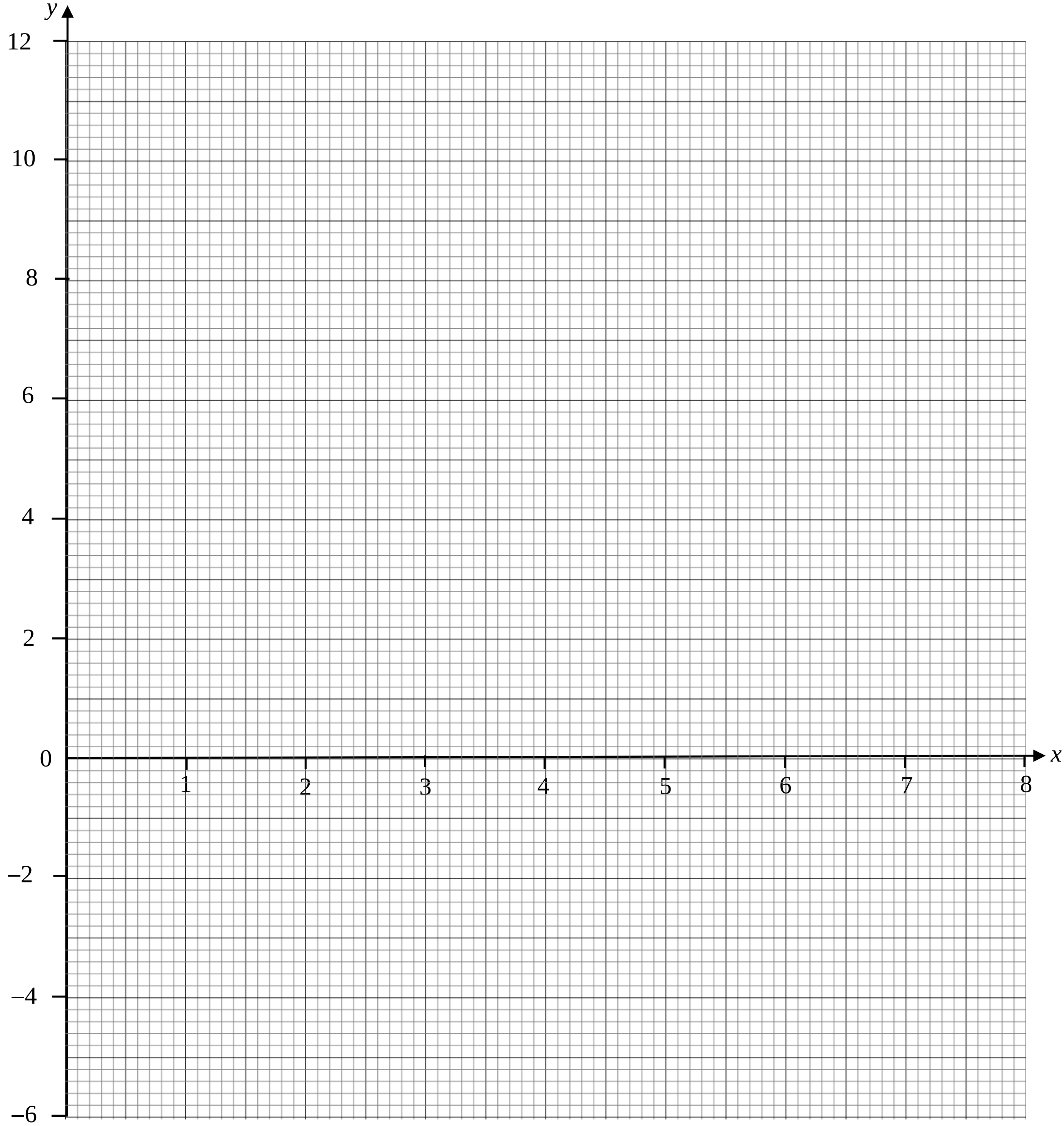
5 **(a)** Complete the table of values for $y = \frac{x^2}{6} + \frac{8}{x} - 5$.

x	0.5	1	2	3	4	5	6	7	8
y		3.2	–0.3	–0.8	–0.3	0.8	2.3	4.3	6.7

[1]

Sub x = 0.5, y =11.0

(b) On the grid, draw the graph of $y = \frac{x^2}{6} + \frac{8}{x} - 5$ for $0 < x \le 8$.



[2]

(c) The equation $\dfrac{x^2}{6}+\dfrac{8}{x}=8$ has two solutions.

Use your graph to solve the equation.

$$\frac{x^2}{6}+\frac{8}{x}-5=8-5$$
$$\frac{x^2}{6}+\frac{8}{x}-5=3$$
draw $y=3$

from graph, $x=1\pm0.1$, 6.4 ± 0.1

Answer $x=$…………… or …………….. [2]

(d) **(i)** On the grid in part **(b)**, draw the $4x$ for $0\le x\le 8$.

[1]

(ii) Write down the x-coordinates of points where this line intersects the line y = 9 –

curve.

From graph,
$x=0.7\pm0.1, 2.4\pm0.1$

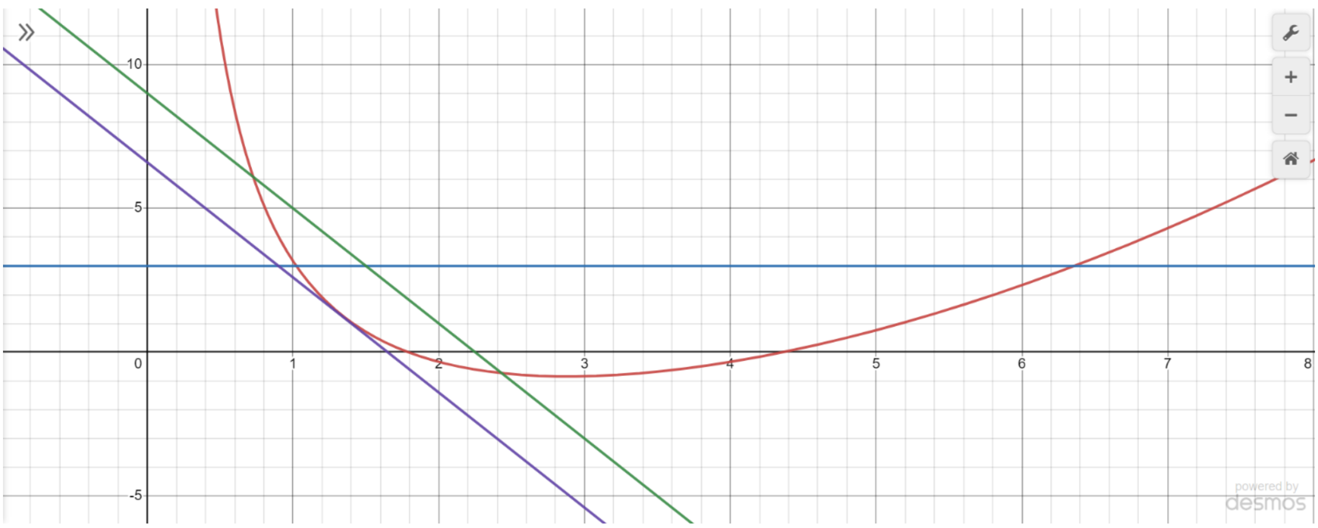
Answer $x=$…………… and …………….. [2]

(iii) Another line $y = c - 4x$, where c is a constant, intersects the curve at only one point.

Use your graph to find the value of c.

Draw tangent
From graph,
$y=6.6\pm0.2$

Answer $c=$……………………………….. [2]



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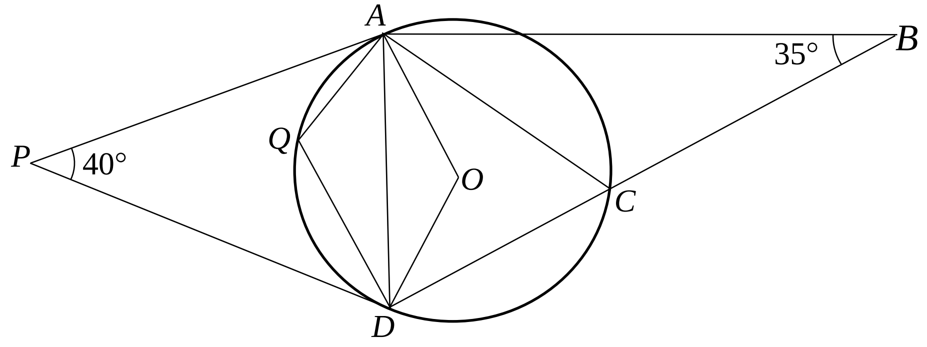


The diagram shows a circle *ACDQ*, centre *O*.
PA and *PD* are tangents to the circle.
$AC = DC$ and DCB is a straight line.
Angle $APD = 40°$ and angle $ABD = 35°$.

(a) **(i)** Find angle AOD.
Give a reason for each step of your working.

$\angle PAO = \angle PDO = 90°$ (**tangent $\perp$ radius**)

$\angle AOD = 360° - 40° - 90° - 90°$ ($\angle$sum of quad)

$= 140°$

Answer Angle *AOD* =……………………. [2]

(ii) Find angle AQD.
Give a reason for each step of your working.

Reflex $\angle AOD = 360° - 140°$ ($\angle$s at a pt)
$= 220°$

$\angle AQD = \dfrac{220°}{2}$ (**$\angle$at centre = 2$\angle$ at circumf**)
$= 110°$

Answer Angle *AQD* =……………………. [2]

(iii) Find angle ACD
Give a reason for each step of your working.

$\angle ACD = \dfrac{140°}{2} = 70^o$ (**$\angle$at centre = 2$\angle$ at circumf**)

OR $\angle ACD$ =180° - 110° **($\angle$s in opp segment)**
$= 70°$

Answer Angle *ACD* =……………………. [1]

(b) Explain why a semicircle with DB as a diameter, passes through *A*.

Answer

Since *AC*=*DC* (given), base $\angle$s of isos Δ

$$\angle ADC = \angle DAC = \frac{180^\circ - 70^\circ}{2} = 55^\circ$$

$\angle CAB = 70^\circ - 35^\circ = 35^\circ$ (ext $\angle$of Δ)

$\angle DAB = 55^\circ + 35^\circ = 90^\circ$

Hence by **right angle in semicircle** property, a semicircle with *DB* as diameter will pass through *A*.

……………………………………………………………………………………………………

……………………………………………………………………………………………….. [2]

(c) The radius of circle $ACDQ$ is 10 cm.
Find the area of the region bounded by PA, PD and the minor arc AQD.

$\angle APO = \dfrac{40^\circ}{2} = 20^\circ$ **(tangents from external pt)**

$\tan \angle APO = \dfrac{10}{AP}$

$AP = \dfrac{10}{\tan 20^\circ}$ or $10 \tan 70^o$

$= 27.47477$

Area $AODP = 2\left[\dfrac{1}{2}(27.47477)(10)\right]$

≈ 274.7477

Area minor sector $ODQA = \dfrac{140^\circ}{360^\circ} \times \pi \times 10^2$

Area of sector $= 122.17304$ cm^2

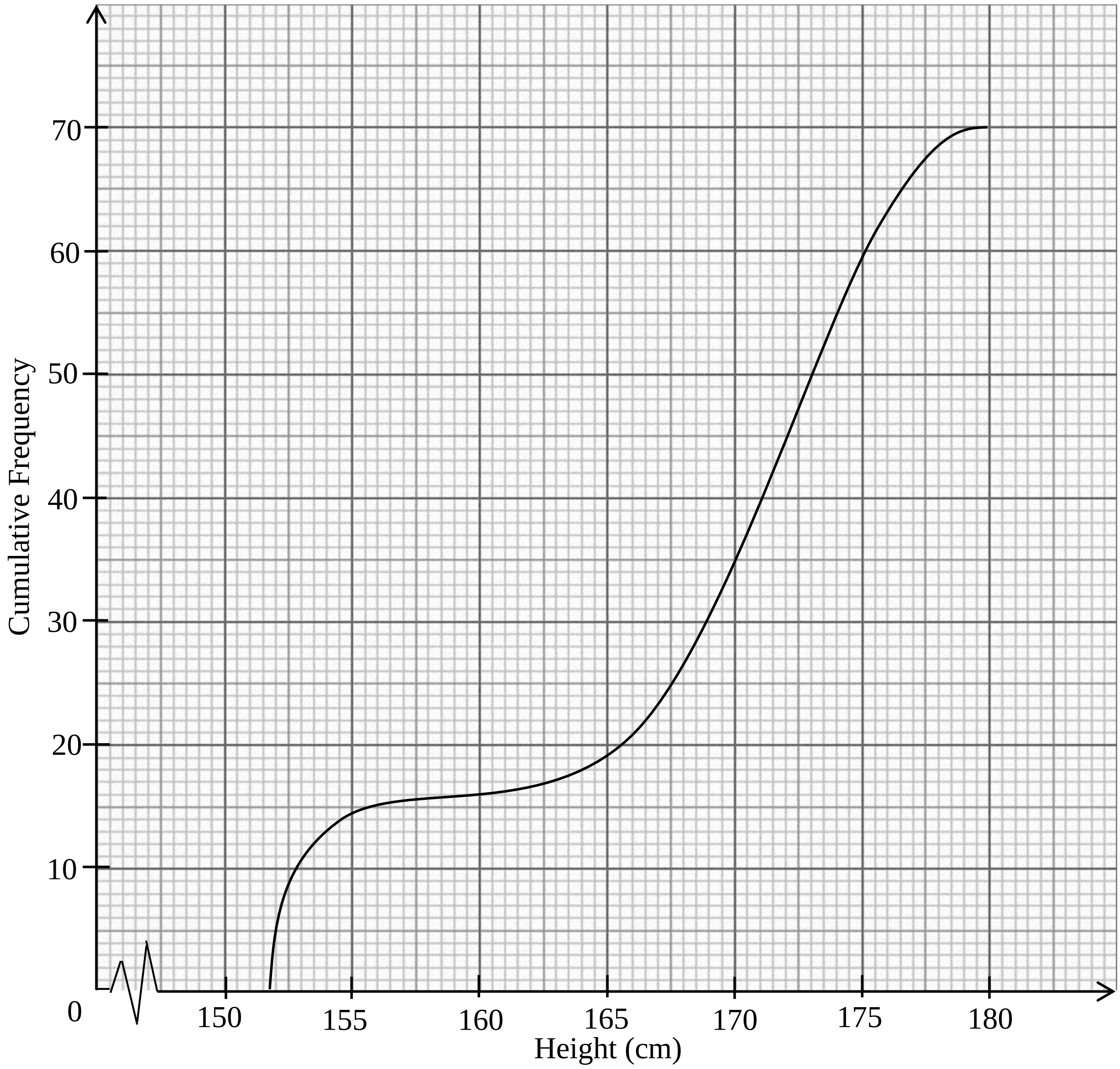
Required area $= 274.7477 - 122.1730$

$= 153$ cm^2 (3sf)

Answer ………………………….…….. cm^2 [4]

7 The heights, in cm, of 70 adults are measured.

The cumulative frequency curve shows the distribution of their heights.



(a) Use the graph to estimate

(i) the median height,

Median height $= 170$ cm

Answer ……….……….……… cm [1]

(ii) the interquartile range,

$Q_1 = 163.5 \quad (17.5\text{th}) \qquad Q_3 = 173.5 \quad (52.5\text{th})$

$\text{ITQ} = 173.5 - 163.5$

$= 10$

Answer ……….……….……… cm [2]

(iii) the percentage of adults who are taller than 174.5 cm.

$$\text{Percentage} = \frac{70-57}{70} \times 100\%$$
$$\approx 18.6\% \ \ \text{(3sf)}$$

Answer ……….………….……. % [2]

(b) One adult is selected at random from the 70 adults. If the probability that this adult is shorter than h cm is $\frac{2}{5}$, find the value of h.

$\frac{2}{5}$ represents 28 adults

$h = 168.5$

Answer h =……………….………… [2]

(c) Two adults were selected at random. Find the probability that one of the adults is shorter 154 cm and the other is taller than 176 cm.

$$\text{Probability} = \frac{13}{70} \times \frac{7}{69} \times 2$$
$$= \frac{13}{345}$$

Answer ……………….………….…… [2]

(d) A mistake has been discovered when measuring the heights of the adults.
An additional 2 cm needs to be added to all the heights.

Explain how the median height and the interquartile range have been affected by this mistake.

New Median height $= 170 + 2$ $= 172$ cm New Interquartile Range = 10 cm

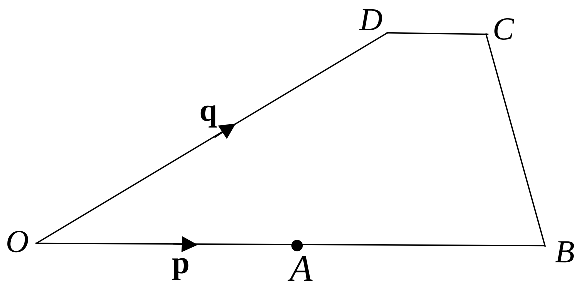
With the mistake, the median height has to add 2 cm more, but the interquartile range which is the difference between the upper and lower quartiles remain unchanged.

……………………………………………………………………………………………

…………………………………………………………………………………………….. [2]

8 **(a)** $OBCD$ is a trapezium where DC is parallel to OB.

$\overrightarrow{OA} = \mathbf{p}$ and $\overrightarrow{OD} = \mathbf{q}$. $OA = \frac{1}{2}OB$ and $DC : OB = 1 : 6$.



(i) Show that $\overrightarrow{BC} = \mathbf{q} - \frac{5}{3}\mathbf{p}$.

Answer

$$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$$
$$= \underline{q} + \frac{1}{6}(2\underline{p}) - 2\underline{p}$$
$$= \underline{q} - \frac{5}{3}\underline{p}$$

[2]

(ii) X is a point on OB such that XD is parallel to BC.

Find $\overrightarrow{XC}$, as simply as possible, in terms of $\mathbf{p}$ and $\mathbf{q}$.

$$\overrightarrow{XC} = \overrightarrow{XB} + \overrightarrow{BC}$$
$$= \overrightarrow{DC} + \overrightarrow{BC}$$
$$= \frac{1}{6}(2\underline{p}) + \underline{q} - \frac{5}{3}\underline{p}$$
$$= \frac{1}{3}\underline{p} + \underline{q} - \frac{5}{3}\underline{p}$$
$$= \underline{q} - \frac{4}{3}\underline{p}$$

Answer $\overrightarrow{XC}$ = ……………..…..……. [2]

(iii) Find the ratio area *XBCD* **:** area *OBCD*.

area *XBCD* : area *OBCD*

$$= XB \times h : \frac{1}{2}(DC + OB)h$$

$$= \frac{1}{3} : \frac{1}{2}(2 + \frac{1}{3}) \qquad = XB : \frac{1}{2}(DC + OB)$$

$$= 2 : 3(\frac{7}{3}) \qquad \text{or} \qquad = 1 : \frac{1}{2}(1 + 6)$$

$$= 2 : 7 \qquad = 1 : \frac{7}{2}$$

$$= 2 : 7$$

Answer …………..… **:** ………………. [1]

(iv) Y is a point on BC produced such that $\overrightarrow{OY} = h\,\mathbf{q}$.

By using $\overrightarrow{BY} = k\overrightarrow{BC}$, find the value of h and the value of k.

$$\overrightarrow{OY} = \overrightarrow{OB} + \overrightarrow{BY}$$

$$h\underline{q} = 2\underline{p} + \mathrm{k}\underline{q} - \frac{5\mathrm{k}}{3}\underline{p}$$

$$h\underline{q} = (2 - \frac{5k}{3})\underline{p} + \mathrm{k}\underline{q}$$

$$h = k, \quad 2 - \frac{5k}{3} = 0$$

$$h = \frac{6}{5} = 1\frac{1}{5}, \quad k = \frac{6}{5} = 1\frac{1}{5}$$

Answer h = ………..…………………

k = ………..………………[3]

(b) T is the point (–2, 6) and $\overrightarrow{ST} = \begin{pmatrix} -7 \\ 9 \end{pmatrix}$.

(i) Find the position vector of S.

$\overrightarrow{OT} = \begin{pmatrix} -2 \\ 6 \end{pmatrix}$

$\overrightarrow{OS} = \overrightarrow{OT} + \overrightarrow{TS}$

$= \begin{pmatrix} -2 \\ 6 \end{pmatrix} - \begin{pmatrix} -7 \\ 9 \end{pmatrix}$

$= \begin{pmatrix} 5 \\ -3 \end{pmatrix}$

$A = (5, -3)$

Answer $\begin{pmatrix} \quad \\ \quad \end{pmatrix}$ [1]

(ii) Find the magnitude of $\begin{pmatrix} -7 \\ 9 \end{pmatrix}$.

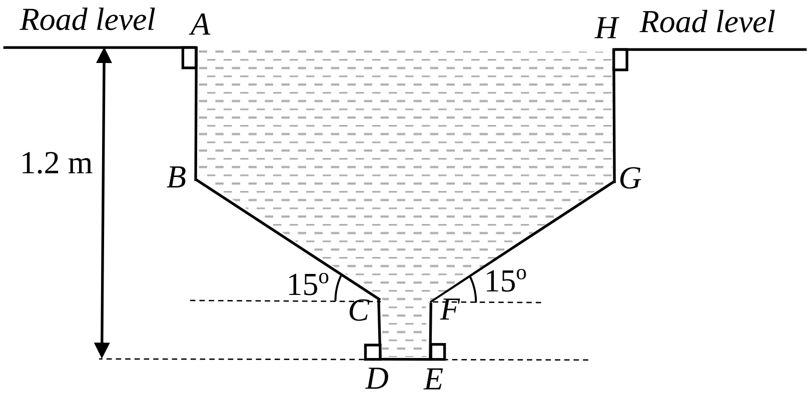
Magnitude $= \sqrt{(-7)^2 + 9^2}$

$= \sqrt{130} \approx 11.401 = 11.4$ units (3 s.f.)

Answer ……………………………………. [1]

9 A project manager is constructing a drain running along a straight boundary of a piece of land. The drain runs along a slope with a bed gradient of $\frac{1}{200}$ which means for every 200 m length there is a drop of 1 m.

The diagram below, not drawn to scale, shows the cross section of the drain.



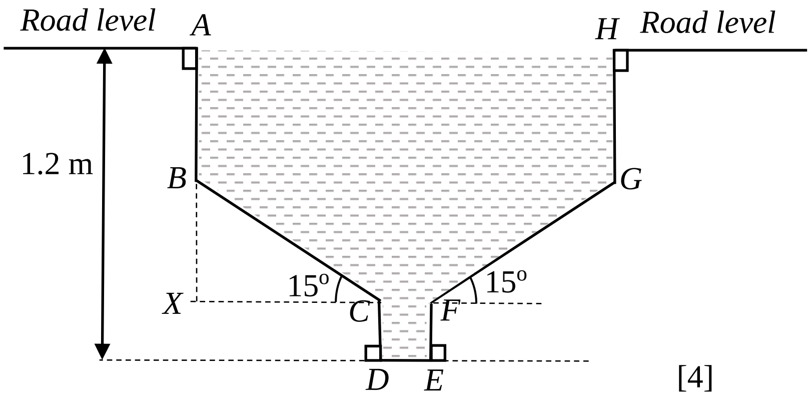
The depth of the drain is 1.2 metre from the road level, with *AB* = *HG*.

BC = *FG* = 27 cm, *CD* = *EF* = 10 cm and *DE* = 4 cm.

The wetted perimeter of the planned drain is the sum of the length of *AB*, *BC*, *CD*, *DE*, *EF*, *FG* and *GH*.

(a) Show that the cross-sectional area *ABCDEFGH* of the planned drain is 0.6035 m^2, correct to 4 decimal places.

Answer



[4]

	Lengths	Area (method 1)		Area (method 2)	
1	CD =10 DE = 4	40		480	
2	CX = 27cos15°=26.0799 BX = 27sin15°=6.9881	$\frac{1}{2}(CX)(BX)2+4BX$ =210.2013		$\frac{1}{2}(CX)(BX)2$ $=182.2489$	
3	AB = 120 – (27sin15°+10) =103.0118	$(AB)(CX)2+4AB$ =5373.0748+412.0472 =5785.122		$(AB)(CX)2$ =5373.0748	
	in cm^2	6035.3233		6035.3237	
	in m^2 (see divide 10000)	0.6035		0.6035	

The velocity of water flow is computed from the Manning's Formula:

$$v = \frac{1}{n} R^{\frac{2}{3}} S^{\frac{1}{2}},$$

where v = velocity of water flow in the drain (m/s)
n = roughness coefficient
A = flow area* (m^2)
P = wetted perimeter (m)
$R = \frac{A}{P}$ = hydraulic radius (m)
S = bed gradient
**Flow area is the cross-sectional area of the drain.*

The value of the roughness coefficient (n) depends on the drain's flow surface and is given below.

Material used to construct the drain	**Roughness coefficient (n)**
Unplasticised Polyvinyl Chloride (PVC)	0.0125
Concrete	0.0150
Brick	0.0170
Earth	0.0270
Mud	0.0350
Gravel	0.0300

The safety requirements of a drain are listed below.

- The minimum velocity of water flow in a drain must exceed 3.6 km/h.
- The maximum velocity of water flow in a mud drain must be no more than 5.4 km/h.
- The maximum velocity of water flow in any other drain must be no more than 10.8 km/h.

(b) Concrete is used as a material for the construction of the drain.
Suggest whether the safety requirements will be met.
Justify your decision and show your calculations clearly.

Answer

$AB = HG = 120 - \left(27\sin 15^{o}\right) - 10$

$AB = HG = 103.011$ cm

wetted perimeter

$= 2\left(103.011 + 27 + 10\right) + 4$

$= 284.022$ cm

$= 2.84022$ m

$R = \frac{A}{P} = \frac{6035.323}{284.022}$ or $\frac{0.6035323}{2.84022}$

$\therefore R = 21.249$ cm or 0.21249 m

$S = \frac{1}{200}$

$n = 0.0150$

$v = \left(\frac{1}{0.0150}\right)\left(0.21249\right)^{\frac{2}{3}}\left(\frac{1}{200}\right)^{\frac{1}{2}}$

$v = 1.678$ m/s

$v = \frac{1.678 \times 3600}{1000}$ km/h

$v = 6.0408 = 6.04$ km/h (3sf)

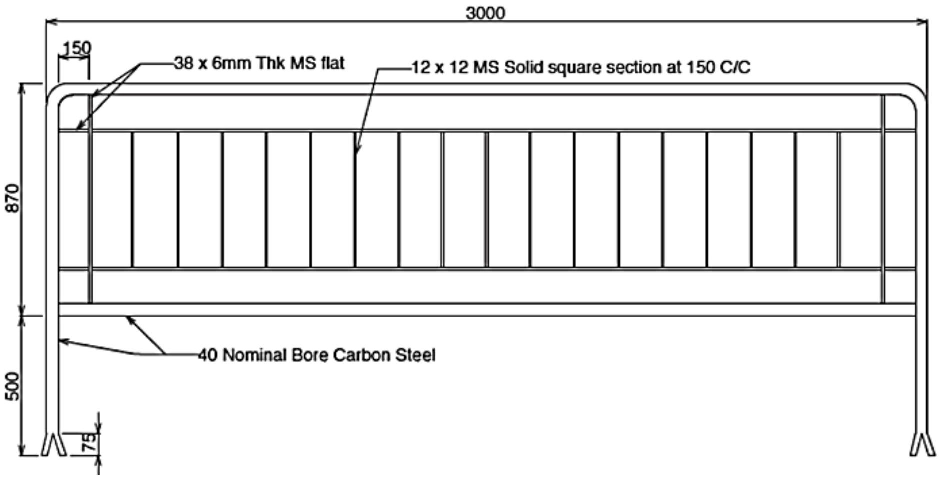
$\therefore 3.6$ km/h < 6.04 km/h < 10.8 km/h

or 1 m/s < 1.678 m/s < 3 m/s

Yes, safety requirement is met.

[6]

(c)



This diagram shows one standard safety railing taken from https://kusgrp.com/metal-works/product/mild-steel-railing/

The open drain runs for the length of 47 metres.
For open drains of more than 1.0 m deep, standard safety railings are placed along the side of the drain.

A standard safety railing measures 3000 mm by 1370 mm, with a 50 mm space in between each railing.

Assuming the first railing is placed at the start of the drain, the project manager claims that there is a need to modify the last railing to sufficiently line the open drain safely.

Calculate the length, in mm, of the last railing.

estimate on number of railings needed

$=\frac{47}{3}=15\frac{2}{3}$

$\therefore 15$ complete railings needed

Length covered by railing including the 50mm space

$=15\times 3+15\times 0.05$

$=45.75$ m

Remaining length not covered by railing

$=47-45.75$

$=1.25$ m

=1250 mm

Answer ……………..…..………..… mm [2]