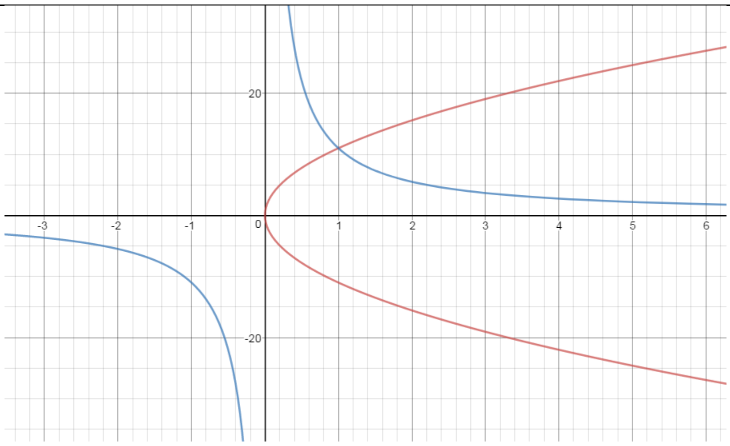
Regent Secondary School
Additional Mathematics
Sec 4 Express Preliminary Examination 2020
Paper 1
(Setter: Ms Su RY)
Marking Scheme

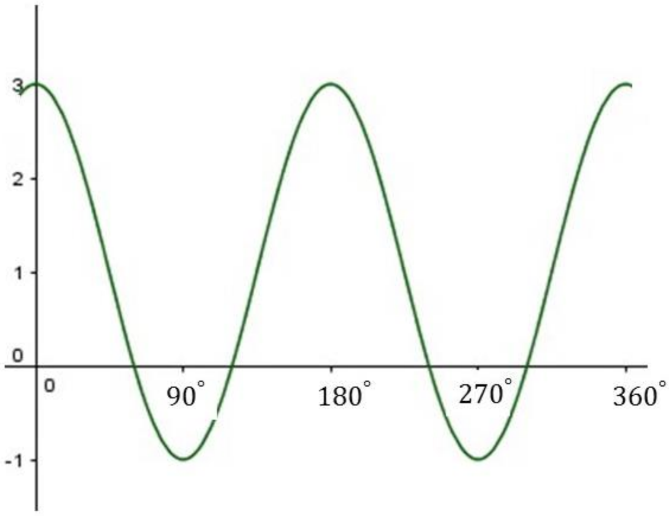
Question	Solution	Marks	Total	Marker's Report
1i	$\cos A$ $= -\frac{12}{13}$	B2	2	
1ii	$\sin(A+B)$ $= \sin A\cos B + \cos A\sin B$ $= \left(-\frac{5}{13}\right)\left(-\frac{4}{5}\right) + \left(-\frac{12}{13}\right)\left(-\frac{3}{5}\right)$ $= \frac{56}{65}$	M1 A1	2	
2	Let $\frac{2x-5}{x^2-2x-3}$ $= \frac{2x-5}{(x-3)(x+1)}$ $= \frac{A}{x-3} + \frac{B}{x+1}$ $2x - 5 = A(x+1) + B(x-3)$ Let $x = 3$, $1 = 4A + 0$ $A = \frac{1}{4}$ Let $x = -1$, $-7 = 0 - 4B$ $B = \frac{7}{4}$ Hence $\frac{2x-5}{x^2-2x-3} = \frac{1}{4(x-3)} + \frac{7}{4(x+1)}$	M1 M1 M1 M1 A1	5	
3i		B1 graph, B1 mark each	2	

3ii	$(\frac{11}{x})^2 = 121x$ $\frac{121}{x^2} = 121x$ $x^3 = 1$ $x = 1$ $y = 11$ $(1, 11)$	M1 M1 A1	3	
4	Length $= \frac{8+7\sqrt{2}}{5-2\sqrt{2}}$ $= \frac{8+7\sqrt{2}}{5-2\sqrt{2}} \times \frac{5+2\sqrt{2}}{5+2\sqrt{2}}$ $= = \frac{40+16\sqrt{2}+35\sqrt{2}+28}{25-8}$ $= \frac{68+51\sqrt{2}}{17}$ $4+3\sqrt{2}$	M1 M1 M1 A1	4	
5i	$5x^2+7x-8 = x$ $(5x-4)(x+2) = 0$ $x = \frac{4}{5}$ or $x = -2$ (rejected)	M1 M1 A1, A1	4	
5ii	$\log_3 a - \frac{\log_3 b}{\log_3 3^2} = 2$ $2\log_3 a - \log_3 b = 4$ $\log_3 \frac{a^2}{b} = 4$ $b = \frac{a^2}{81}$	M1 M1 A1	3	
6i	$8^x - 2^{x+2} = 15$ $2^{3x} - 2^{x+2} = 15$ $(2^x)^3 - 2^x.2^2 = 15$ Let $u = 2^x$ $u^3 - 4u = 15$ $u^3 - 4u - 15 = 0$	M1 M1 A1	3	
6ii	Let f(u) = $u^3 - 4u - 15 = 0$ $f(3) = 0$	M1 M1		

	$f(u)=(u-3)(u^2+3u+5)$ consider u^2+3u+5 b^2-4ac $=3^2-4(1)(5)$ $=-11$ Since $b^2-4ac<0$, the quadratic factor has no real roots. Hence $u = 3$ is the only real solution of this equation (shown)	M1 A1	 4	
6iii	$f(u) = 0$ $u = 0$ $2^x = 3$ Taking lg on both sides $\lg 2^x = \lg 3$ $x\lg 2 = \lg 3$ $x = 1.58496...$ $x = 1.58$	 M1 M1 A1	 3	
7i	$t = 20$ $A = 48 \times 12$ $= 576\ cm^2$ $A = \pi r^2$ $r = \sqrt{\frac{576}{\pi}}$ $= \frac{24}{\sqrt{\pi}}$ (shown)	 M1 M1 A1	 3	
7ii	$\frac{dA}{dt} = 48$ $\frac{dA}{dr} = 2\pi r$ $\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$ $48 = 2\pi r \times \frac{dr}{dt}$ $\frac{dr}{dt} = \frac{48}{2\pi r}$ $= \frac{48}{2\pi\left(\frac{24}{\sqrt{\pi}}\right)}$ $= \frac{1}{\sqrt{\pi}}\ cm/s$	 M1 M1 A1	√ based on 7i 3	
8i	$\frac{1}{2}\times\frac{4}{3}\pi r^3+\pi r^2 h=3200$ $\pi r^2 h = 3200-\frac{2}{3}\pi r^3$ $h=\frac{3200}{\pi r^2}-\frac{2}{3}r$	M1 A1	 2	

8ii	$A=\frac{1}{2}\times 4\pi r^2+2\pi rh$ $=2\pi r^2+2\pi r\left(\frac{3200}{\pi r^2}-\frac{2}{3}r\right)$ $=2\pi r^2+\frac{6400}{r}-\frac{4}{3}\pi r^2$ $=\frac{6400}{r}+\frac{2}{3}\pi r^2$ (shown)	M1 A1	√ based on 8i 2	
8iii	When A has a stationery value, $\frac{dA}{dr}=0$ $-\frac{6400}{r^2}+\frac{4}{3}\pi r=0$ $\frac{1}{3}\pi r^3-1600=0$ $\pi r^3=4800$ $r=11.51764..$ $r=11.5cm$ $\frac{d^2A}{dr^2}=\frac{12800}{r^3}+\frac{4}{3}\pi$ When $r=11.5176...$, $\frac{d^2A}{dr^2}=12.5665$ (>0) This value of A is a minimum	M1 M1 M1 M1 A1	5	
9i	$k=-1$ as 2 is the mid-value of k and 5.	B1	1	
9ii	$x+1=0$ or $x-5=0$ $a[(x+1)(x-5)]=0$ $a[x^2-4x-5]=0$ $\Rightarrow y=a(x^2-4x-5)$ $sub(2,-18)$ $-18=a(4-8-5)$ $-18=a(-9)$ $a=2$ when $a=2$ $y=2x^2-8x-10$ $\Rightarrow b=-8$ $c=-10$	M1 A1 A1 A1	4	
9iii	3	B1	1	

10i	Gradient of $PR = \frac{9-6}{14-5}$ $= \frac{1}{3}$ Gradient of $QS = -3$ Midpoint of $PR = \left(\frac{5+14}{2}, \frac{6+9}{2}\right)$ $= \left(\frac{19}{2}, \frac{15}{2}\right)$ Let the coordinates of *S* be $(x_S, 0)$ $\frac{\frac{15}{2}-0}{\frac{19}{2}-x_s} = -3$ $\frac{15}{2} = 3x_s - \frac{57}{2}$ $3x_s = 36$ $x_s = 12$ *S* = (12, 0)	M1 M1 M1 M1 A1	5	
10ii	$\frac{k-0}{8-12} = -3$ $k = 12$	M1 A1	2	
10iii	Area of *PQRS* = $\frac{1}{2}\begin{vmatrix} 12 & 14 & 8 & 5 & 12 \\ 0 & 9 & 12 & 6 & 0 \end{vmatrix}$ $= \frac{1}{2}[108+168+48-72-60-72]$ = 60 units^2	M1 A1	2	
11i	$3 - 4\sin^2 x$ $= 3 - 4\left(\frac{1-\cos 2x}{2}\right)$ $= 3 - 2(1-\cos 2x)$ $= 2\cos 2x + 1$ $a = 2, b = 1$	M1 A1, A1	3	
11ii	Greatest value of f(*x*) = 3 Least value of f(*x*) = -1	B1 B1	2	
11iii	Period of f(*x*) = 180° (accept π) Amplitude of f(*x*) = 2	B1 B1	2	

11iv		Shape C1 Correct max and min values C1 $1\frac{1}{2}$ cycles C1	3	
12i	$\frac{dy}{dx}$ $= \frac{(2x-1)^{\frac{1}{2}}(3)-(3x+4)\frac{1}{2}(2x-1)^{-\frac{1}{2}}(2)}{(2x-1)}$ $= \frac{6x-3-3x-4}{(2x-1)^{\frac{3}{2}}}$ $= \frac{3x-7}{\sqrt{(2x-1)^3}}$	M1 M1 A1	3	
12ii	y is increasing $\frac{3x-7}{\sqrt{(2x-1)^3}} > 0$ $3x-7>0$ $x > \frac{7}{3}$	M1 A1	2	