RAFFLES INSTITUTION
H2 Mathematics (9758)
2024 Year 5

Term 4 RTT2: Post-Promo Revision Lesson Worksheet
Session 3: C4A to C4C: Vectors

1 Referred to the origin O, the points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively. The vectors $\mathbf{a}$ and $\mathbf{b}$ are given by

$$\mathbf{a} = 2p\mathbf{i} - 6p\mathbf{j} + 3p\mathbf{k} \quad \text{and} \quad \mathbf{b} = \mathbf{i} + \mathbf{j} - 2\mathbf{k},$$

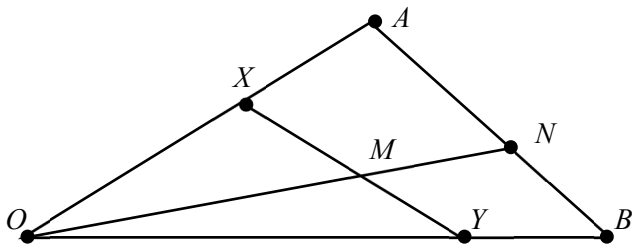
where p is a constant.

Find $\dfrac{|\mathbf{a}\cdot\mathbf{b}|}{|\mathbf{a}|}$ and give a geometrical interpretation of $\dfrac{|\mathbf{a}\cdot\mathbf{b}|}{|\mathbf{a}|}$. **[2]**

Find $\mathbf{a}\times\mathbf{b}$ and give a geometrical interpretation of $|\mathbf{a}\times\mathbf{b}|$. **[2]**

Given that $\mathbf{a}$ is a unit vector, find the possible value(s) of p. **[2]**

2 DHS Prelim 9758/2020/01/Q6b modified



With reference to the origin O, the points A and B are such that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

It is given that $\overrightarrow{OX} = \dfrac{2}{3}\mathbf{a}$, $\overrightarrow{OY} = \dfrac{3}{4}\mathbf{b}$ and the line ON bisects the line XY at the point M.

(i) By considering the ratio $XM : MY$, find the vector $\overrightarrow{OM}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. [1]

(ii) Given that $AN : NB = \lambda : 1-\lambda$ and $ON : OM = k : 1$ where λ and k are real constants, find the ratio $AN : NB$. [4]

3 **(a)** The non-zero vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are such that $\mathbf{a}\times\mathbf{b} = \mathbf{c}\times\mathbf{a}$. Given that $\mathbf{b} \neq -\mathbf{c}$, find a linear relationship between $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$. [3]

(b) The variable vector $\mathbf{v} = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ satisfies the equation $\mathbf{v}\times(\mathbf{i} - 3\mathbf{k}) = \mathbf{j}$.
Find the set of vectors $\mathbf{v}$ and describe this set geometrically. [3]

4 ACJC Promo 9758/2020/Q8

The lines l and m are defined by the equations

$$l: \mathbf{r} = \mathbf{i} - \mathbf{k} + \lambda(2\mathbf{i} - 6\mathbf{j} + 3\mathbf{k}),$$
$$m: \frac{x-1}{4} = \frac{a-y}{a} = \frac{z+3}{4}.$$

(i) Given that the lines intersect, show that $a = 6$. [2]

(ii) Find the position vector of N, the foot of perpendicular from the point $A(5, 0, 1)$ to the line l. [3]

(iii) Find the position vector of the two points on l that are 5 units from A. [3]

5 9740/2015/02/Q2

The line L has equation $\mathbf{r} = \mathbf{i} - 2\mathbf{j} - 4\mathbf{k} + \lambda(2\mathbf{i} + 3\mathbf{j} - 6\mathbf{k})$.

(i) Find the acute angle between L and the x-axis. **[2]**

The point P has position vector $2\mathbf{i} + 5\mathbf{j} - 6\mathbf{k}$.

(ii) Find the points on L which are a distance of $\sqrt{(33)}$ from P. Hence or otherwise find the point on L which is closest to P. **[5]**

(iii) Find a cartesian equation of the plane that includes the line L and the point P. **[3]**

6 JPJC Promo 9758/2020/Q9 modified

The plane p_1 has equation $\mathbf{r} = \lambda\begin{pmatrix}1\\2\\0\end{pmatrix} + \mu\begin{pmatrix}0\\1\\1\end{pmatrix}$, where λ and μ are real parameters. .

(i) Find an equation of p_1 in the form $\mathbf{r}.\mathbf{n} = d$. [3]

The plane p_2 has equation $2x - y + z = 12$.

(ii) State the relationship between p_1 and p_2. [1]

The line l has equation $\mathbf{r} = t\begin{pmatrix}3\\2\\2\end{pmatrix}$, where t is a real parameter.

(iii) Find the acute angle between l and p_1. [2]

(iv) Find the foot of perpendicular from the origin to p_2. Hence, or otherwise, find the exact distance between p_1 and p_2. [4]

More Practice Questions

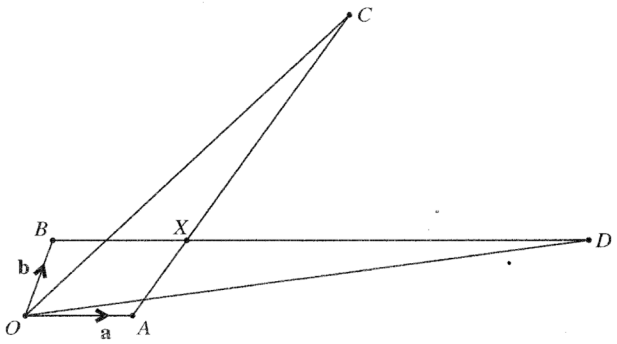
7 MI PU3 Mid-Year CT 9758/2018/01/Q3

Relative to the origin O, two points A and B have position vectors a and b respectively. It is given that $\mathbf{b}$ is a unit vector, $|\mathbf{a}| = \sqrt{3}$, and $|4\mathbf{a} - 3\mathbf{b}| = \sqrt{41}$. θ is defined as the acute angle between $\mathbf{a}$ and $\mathbf{b}$.

(i) By considering the scalar product $(4\mathbf{a} - 3\mathbf{b})\cdot(4\mathbf{a} - 3\mathbf{b})$, find θ. [4]

(ii) Give the geometrical meaning of $|(\mathbf{a} - \mathbf{b}) \times \mathbf{b}|$ and find its exact value. [2]

8 9758/2019/02/Q5



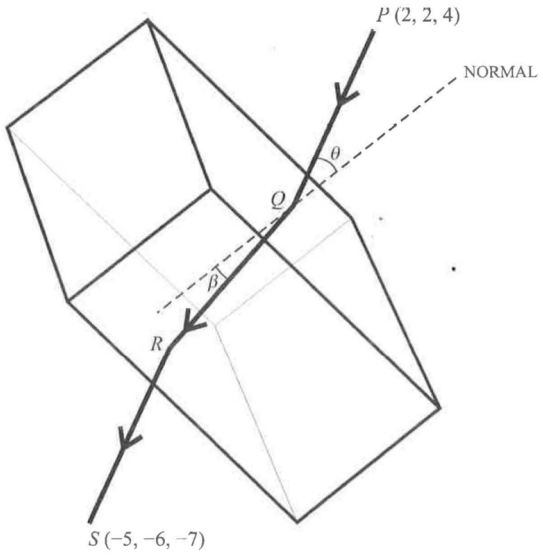
With reference to the origin O, the points A, B, C and D are such that $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$, $\overrightarrow{OC} = 2\mathbf{a} + 4\mathbf{b}$ and $\overrightarrow{OD} = \mathbf{b} + 5\mathbf{a}$. The lines BD and AC cross at X (see diagram).

(i) Express $\overrightarrow{OX}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. [4]

The point Y lies on CD and is such that the points O, X and Y are collinear.

(ii) Express $\overrightarrow{OY}$ in terms of $\mathbf{a}$ and $\mathbf{b}$ and find the ratio $OX : OY$. [6]

9 9758/2019/01/Q12



A ray of light passes from air into a material made into a rectangular prism. The ray of light is sent in direction $\begin{pmatrix}-2\\-3\\-6\end{pmatrix}$ from a light source at the point P with coordinates (2,2,4). The prism is placed so that the ray of light passes through the prism, entering at the point Q and emerging at the point R and is picked up by a sensor at point S with coordinates $(-5,-6,-7)$. The acute angle between PQ and the normal to the top of the prism at Q is θ and the acute angle between QR and the same normal is β (see diagram).

It is given that the top of the prism is a part of the plane $x+y+z=1$, and that the base of the prism is a part of the plane $x+y+z=-9$. It is also given that the ray of light along PQ is parallel to the ray of light along RS so that P, Q, R and S lie in the same plane.

(i) Find the exact coordinates of Q and R. [5]

(ii) Find the values of $\cos\theta$ and $\cos\beta$. [3]

(iii) Find the thickness of the prism measured in the direction of the normal at Q. [3]

Snell's law states that $\sin\theta = k\sin\beta$, where k is a constant called the refractive index.

(iv) Find k for the material of this prism. [1]

(v) What can be said about the value of k for a material for which $\beta > \theta$? [1]