# HUA YI SECONDARY SCHOOL
# PRELIMINARY EXAM 2024

**4-G3 / 5-G2**

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|---|---|---|---|---|
| NAME | | | | |
| CLASS | | | INDEX NUMBER | |

**ADDITIONAL MATHEMATICS** **4049/02**

**PAPER 2**

**26 August 2024**

**2 hour 15 minutes**

Candidates answer on the Question Paper

No Additional Materials is required.

# ANSWER SCHEME

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## 1. ALGEBRA

*Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

*Binomial expansion*

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where $n$ is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

## 2. TRIGONOMETRY

*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

*Formulae for* $\Delta ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

**1** Show that $x = \frac{1}{2}$ is a solution of the equation $2x^3 + x^2 - 3x + 1 = 0$ and hence solve the equation completely. [5]

$f(x) = 2x^3 + x^2 - 3x + 1$
$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - 3\left(\frac{1}{2}\right) + 1$ ---------------------M1
$= 0$
$f(x) = (2x - 1)(x^2 + bx - 1)$ ------------------------M1

Compare coeff. of $x^2$, $1 = 2b - 1$
b= 1 ---------------------------M1

$f(x) = (2x - 1)(x^2 + x - 1)$-------------------------M1
$x = \frac{1}{2}, \frac{-1+\sqrt{5}}{2}, \frac{-1-\sqrt{5}}{2}$ --------------------------------------A1

Alternative Method : Using long division to find $(x^2 + x - 1)$.

**2** **(a)** By considering the general term in the binomial expansion of $\left(px + \frac{1}{x^3}\right)^9$, where $p$ is a constant, explain why there are no even powers of $x$ in this expansion. [3]

General Term $= \binom{9}{r}(px)^{9-r}\left(\frac{1}{x^3}\right)^r$------------------M1

$= \binom{9}{r}p^{9-r}x^{9-4r}$ --------------------M1

Since $9 - 4r = 1 + 4(2 - r)$, one added to any even number will give an odd value, hence there are no even powers of *x* in this expansion. --------A1

**(b)** Given that the coefficient of $x^8$ is equal to the coefficient of $x$ in the expansion of $(2x^3+1)\left(px+\frac{1}{x^3}\right)^9$, find the value of *p*. [4]

$(2x^3+1)\left(\ldots\ldots\binom{9}{1}(px)^8\left(\frac{1}{x^3}\right)^1+\binom{9}{2}(px)^7\left(\frac{1}{x^3}\right)^2\right)^9$ ---------------------M1

$=(2x^3+1)\,(\ldots+9p^8x^5+36p^7x+\cdots)$ ----------------------------------M1

$2(9p^8)=36p^7$ -------------------------------------------------M1
$p=2$ ------------------------------------------------------------A1

**(c)** Using the value of *p* in **(b)**, find the term independent of *x* in the expansion of $(2x^3+1)\left(px+\frac{1}{x^3}\right)^9$. [2]

Term independent of $x=(2)\binom{9}{3}(2^6)$ ----------------------M1
=10752 ---------------------------A1

**3** **(a)** Given that $y = \frac{2x}{(3x+1)^{\frac{1}{2}}}$, show that $\frac{dy}{dx} = \frac{3x+2}{(3x+1)^{\frac{3}{2}}}$. [4]

$\frac{dy}{dx} = \frac{2(3x+1)^{\frac{1}{2}} - 2x\left(\frac{1}{2}\right)(3)(3x+1)^{-\frac{1}{2}}}{(3x+1)^1}$ --------------------M2 (numerator and denominator)

$= \frac{2(3x+1)-3x}{(3x+1)^{\frac{3}{2}}}$ ----------------------------------M1

$= \frac{3x+2}{(3x+1)^{\frac{3}{2}}}$ ------------------------------------------A1

**(b)** Hence find the value of $\int_0^2 \frac{x}{(3x+1)^{\frac{3}{2}}}\, dx$. [5]

$From\ \frac{dy}{dx} = \frac{3x+2}{(3x+1)^{\frac{3}{2}}}$,

$$\frac{dy}{dx} = \frac{3x}{(3x+1)^{\frac{3}{2}}} + \frac{2}{(3x+1)^{\frac{3}{2}}}$$

$$\frac{3x}{(3x+1)^{\frac{3}{2}}} = \frac{dy}{dx} - \frac{2}{(3x+1)^{\frac{3}{2}}}$$

$\int_0^2 \frac{3x}{(3x+1)^{\frac{3}{2}}} dx = \frac{2x}{(3x+1)^{\frac{1}{2}}} - \int_0^2 \frac{2}{(3x+1)^{\frac{3}{2}}}\, dx$ -----------------------------------M1

$$\int_0^2 \frac{x}{(3x+1)^{\frac{3}{2}}} dx = \frac{1}{3}\left[\frac{2x}{(3x+1)^{\frac{1}{2}}}\right]_0^2 - \frac{1}{3}\int_0^2 \frac{2}{(3x+1)^{\frac{3}{2}}}\, dx$$

$= \frac{1}{3}\left[\frac{2x}{(3x+1)^{\frac{1}{2}}}\right]_0^2 - \frac{1}{3}\left[\frac{2(3x+1)^{-\frac{1}{2}}}{\left(-\frac{1}{2}\right)(3)}\right]_0^2$ -----------------------M1, M1

$= \frac{1}{3}\left(\frac{4}{\sqrt{7}} - 0\right) + \left(\frac{4}{9}\right)\left(\frac{1}{\sqrt{7}} - 1\right)$ ------------------------------M1

$= 0.227$ ------------------------------------A1

**4** Show that the equation $5e^x = \frac{1}{e^x} - 4$ has only one solution and find its value correct to 2 decimal places. [4]

$5e^x = \dfrac{1}{e^x} - 4$

$5e^{2x} = 1 - 4e^x$
$5e^{2x} + 4e^x - 1 = 0$ ------------------------M1
$5(e^x)^2 + 4e^x - 1 = 0$
$(5e^x - 1)(e^x + 1) = 0$ ---------------------M1
$e^x = \frac{1}{5}$ or $e^x = -1$
$x = ln\left(\frac{1}{5}\right)$ (no solution, reject)----------A1
$= -1.61$ (2 dp) ------A1

**5** The equation of a curve is $y = -2x^2 + 3x + 5$.

**(a)** Find the set of values for $x$ for which the curve lies below the line $y = 3$ and represent this set of values on a number line. [4]

$-2x^2 + 3x + 5 < 3$ -----------------------M1

$2x^2 - 3x - 2 > 0$

$(2x + 1)(x - 2) > 0$ ----------------------M1

$x < -\frac{1}{2}$ or $x > 2$ ------------------------A1


A1

The line $y = x + k$ is a tangent to the curve at the point $Z$.

**(b)** Find the value of the constant $k$. [3]

$-2x^2 + 3x + 5 = x + k$
$2x^2 - 2x + k - 5 = 0$ --------------------------------M1
Line is tangent to curve => $b^2 - 4ac = 0$

$(-2)^2 - 4(2)(k - 5) = 0$ -----------------------------M1
$4 - 8k + 40 = 0$
$k = \frac{11}{2}$ or $5\frac{1}{2}$ --------------------------------------------A1

**(c)** Find the coordinates of $Z$. [2]

$y = x + \frac{11}{2}$--------------eqn 1
Sub eqn 1 into eqn of curve,
$-2x^2 + 3x + 5 = x + \frac{11}{2}$ ----------------M1
$-4x^2 + 6x + 10 = 2x + 11$
$-4x^2 + 4x - 1 = 0$
$4x^2 - 4x + 1 = 0$
$(2x - 1)^2 = 0$
$x = \frac{1}{2}, y = 6$
$Z = \left(\frac{1}{2},\ 6\right)$ --------------------------------A1

**6** **(a)** The speed $V$ m/s of a vehicle, $t$ s after passing a fixed point $O$, is given for $t \geq 0, V = 1 + pe^{qt}$, where $p$ and $q$ are constants.
Explain how a straight line can be drawn to represent the formula, and state how the value of $p$ and $q$ can be obtained from the line. [4]

$V - 1 = pe^{qt}$
$\ln(V - 1) = \ln(pe^{qt})$
$\ln(V - 1) = \ln p + qt$ --------------------M1
Draw $\ln(V - 1)$ against $t$ ---------------M1

y- intercept = $\ln p$ --------------------A1
gradient = $q$ -------------------------- A1

**(b)(i)** Data of the speeds of the vehicle was collected. The table below shows the corresponding values of $V$ and $t$.

| $t$ | 2 | 4 | 6 | 8 | 10 |
|---|---|---|---|---|---|
| $V$ | 10.35 | 8.40 | 6.70 | 5.42 | 5.95 |

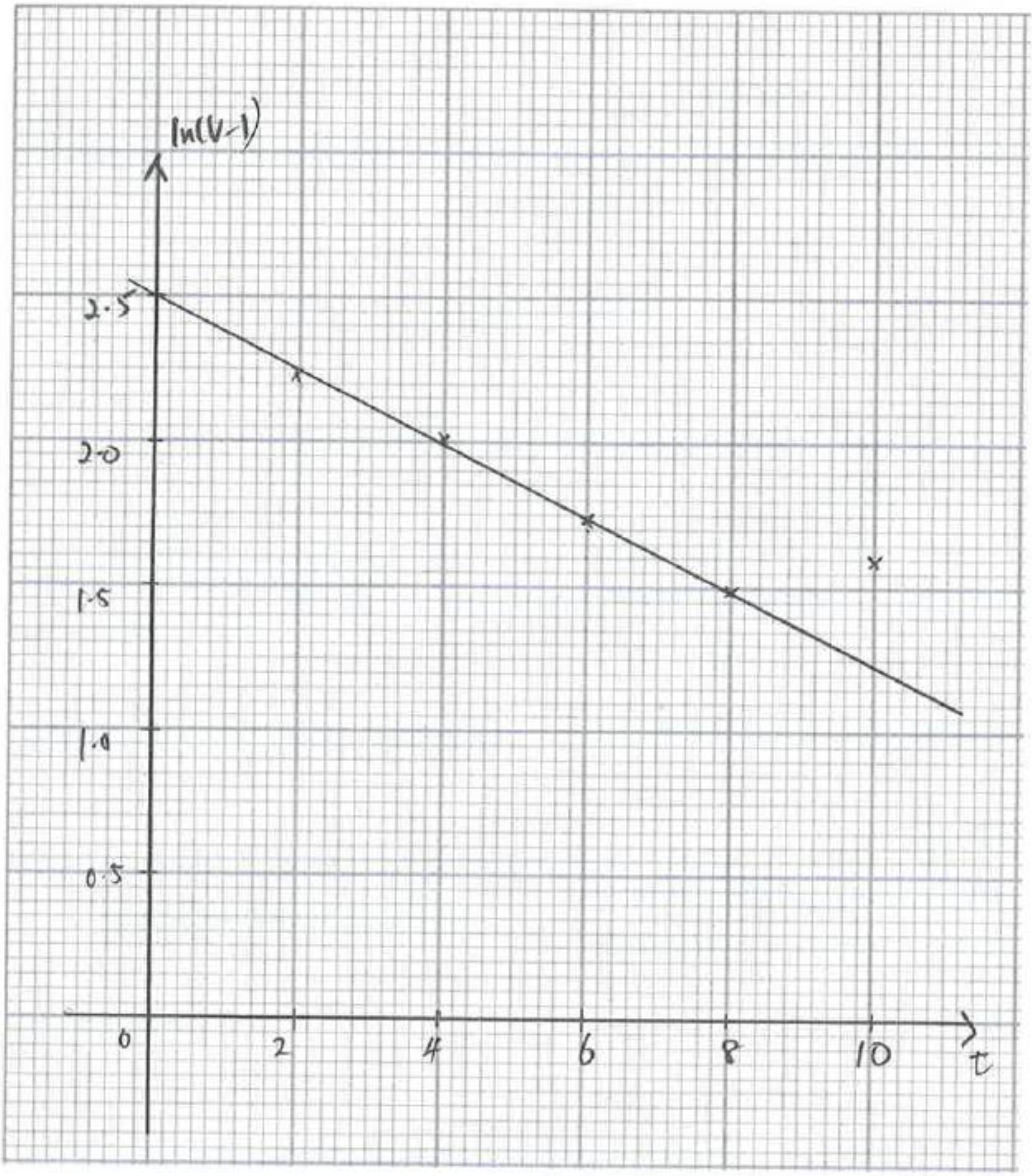
Using **(a),** draw the straight line graph on the next page. [3]

| $t$ | 2 | 4 | 6 | 8 | 10 |
|---|---|---|---|---|---|
| ln($V$-1) | 2.24 | 2.00 | 1.74 | 1.49 | 1.60 |

M1 - Calculate ln $(v - 1)$, M1 – correct points plotted, M1 – Best fit line

**(ii)** Estimate the values of $p$ and $q$. [3]

$q = \frac{2.5-1.5}{0-8}$ --------------------M1
$= -0.125$ ---------------------A1
$lnp = 2.5$
$p = 12.18$ ----------------A1




**(iii)** Estimate a value of $V$ to replace one incorrect recording of $V$ found in the straight line graph. [2]

Incorrect value of $V$ occurs at $x = 10$.
$\ln(V-1) = 1.25$ ------------------------------M1
$V = 4.49$ -----------------------------------A1

**7** A motorcyclist, travelling along a straight road, passes a lamp post *X*, with speed of *h* km/h. A while later, the motorcyclist passes a second lamp post *Y*, with a speed of *60* km/h.
Between the two lamp posts, the speed is given by $V = 20e^{50t} + 10$ km/h where *t*, the time after passing lamp post *X* is measured in hours.

**(a)** Find the value of *h*. [2]

$t = 0,\ h = 20e^0 + 10$ ----------------------------M1
$h = 30\ km/h$ ---------------------------------------A1

**(b)** Calculate to the nearest second, the time taken to travel from *X* to *Y*. [3]

$60 = 20e^{50t} + 10$ --------------M1
$ln(2.5) = 50t$
$t = 0.0183258\ h$ -----------------M1
$= 66$ seconds ----------------A1

**(c)** Find the acceleration of the motorcyclist as he passes *Y*. [3]

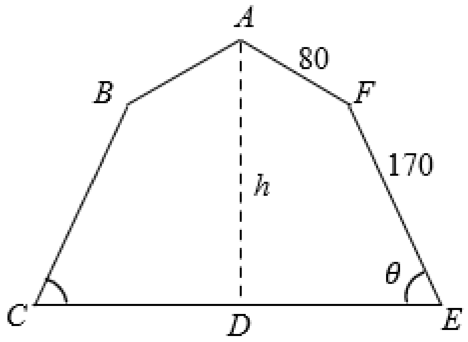
Acceleration $= \frac{dv}{dt}$
$= 20(50)e^{50t}$ -----------------M1
$= 20\ (50)e^{50(0.0183258)}$------------M1
$= 2500\ km/h^2$ ------------------A1

**(d)** Find the distance *XY*. [5]

Distance *XY*
$= \int_0^{\frac{ln2.5}{50}} vdt$
$= \int_0^{\frac{ln2.5}{50}} 20e^{50t} + 10dt$ ----------------------M1
$= \left[\frac{20}{50}e^{50t} + 10t\right]_0^{\frac{ln2.5}{50}}$ -----------------------M1, M1

$= \left(\frac{2}{5}e^{ln2.5} + 10\left(\frac{ln2.5}{50}\right)\right) - \frac{2}{5}$ -------------------M1
$= 0.783km$ ---------------------------------A1

**8** The diagram shows the side view *ABCDEF* of an ornament. The ornament rests with *CE* on horizontal ground and is symmetrical about the vertical *AD*, where *D* is the midpoint of *CE*.
Angle *DEF* = Angle *DAF*= $\theta$ radians and the lengths of *AF* and *FE* are 80 cm and 170 cm respectively. The vertical height of the ornament is *h* cm.




**(a)** Explain clearly why $h = 80cos\theta + 170sin\theta$ [2]

$cos\theta = \frac{AM}{80}$ $sin\theta = \frac{FN}{170}$ ------------------M1

$h = AM + MD$
$= 80cos\theta + 170sin\theta$-------------------------A1

**(b)** Express *h* in the form $Rsin(\theta + \alpha)$, where *R* > 0 and $\alpha$ is an acute angle. [3]
$h = 170sin\theta + 80cos\theta$
$R = \sqrt{80^2 + 170^2} = \sqrt{35300}$ or 187.88 -------------------------M1
$\alpha = tan^{-1}\left(\frac{80}{170}\right) = 0.44\ rad$---------------------------------------M1
$\sqrt{35300}\ sin(\theta + 0.44)$ ------------------------------------------A1

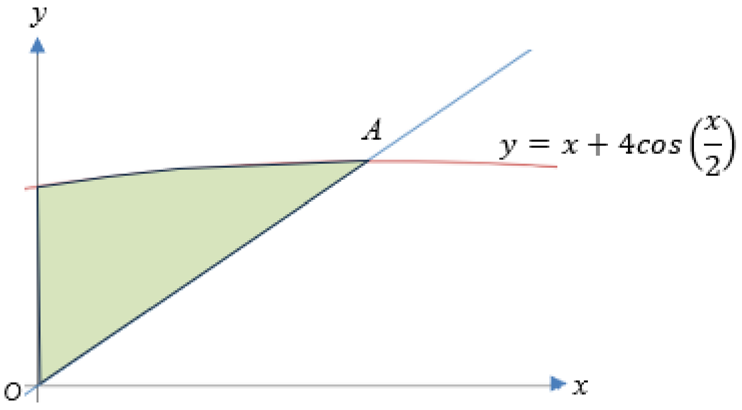
**(c)** Find the greatest possible value of *h* and the value of $\theta$ at which this occurs. [3]

Greatest value of *h* =187.88 ----------------A1
$sin(\theta + 0.44) = 1$ ---------------------------M1
$\theta + 0.44 = \frac{\pi}{2}$
$\theta = 1.13\ rad$ --------------------------------A1

**(d)** Find the value of $\theta$ when *h* = 180 cm. [2]

$\sqrt{35300}sin(\theta + 0.44) = 180$
$\theta + 0.44 = 1.280$ , 1.862 ---------------------------------M1
$\theta = 0.84$ rad , 1.42 rad--------------------------------------A1

**9**




The diagram shows the curve $y = x + 4cos\left(\frac{x}{2}\right)$ for $0 \leq x \leq \pi$ radians. The point $A$ is the stationary point of the curve and $OA$ is a straight line.

**(a)** Find the coordinates of $A$. [5]

$\frac{dy}{dx} = 1 - 2sin\left(\frac{x}{2}\right)$ ----------------------M1, M1

At the stationary point $A$, $\frac{dy}{dx} = 0$

$1 - 2sin\left(\frac{x}{2}\right) = 0$ -----------M1

$sin\left(\frac{x}{2}\right) = 0.5$

$x = \frac{\pi}{3}$ ----------------------M1

$y = \frac{\pi}{3} + 2\sqrt{3}$

$A = \left(\frac{\pi}{3}, \frac{\pi}{3} + 2\sqrt{3}\right)$ --------------------------A1

**(b)** Show that the area of the shaded region is $4 - \frac{\sqrt{3}}{3}\pi$ units$^2$. [5]

Shaded area

$= \int_0^{\frac{\pi}{3}} x + 4cos\left(\frac{x}{2}\right) dx - \frac{1}{2}\left(\frac{\pi}{3}\right)\left(\frac{\pi}{3} + 2\sqrt{3}\right)$ ------------------ M1 (Area of Δ)

$= \left[\frac{x^2}{2} + \frac{4sin\left(\frac{x}{2}\right)}{\frac{1}{2}}\right]_0^{\frac{\pi}{3}} - \frac{\pi^2}{18} - \frac{\pi\sqrt{3}}{3}$ -------------M2 (Integration of the two terms)

$= \frac{\pi^2}{18} + 8sin\left(\frac{\pi}{2}\right) - 0 - \frac{\pi^2}{18} - \frac{\pi\sqrt{3}}{3}$ --------------------M1

$= 4 - \frac{\sqrt{3}}{3}\pi$ --------------------------A1

**10** A tangent to a circle at the point (3,2) cuts the y-axis at 5. The line with the equation $3y = 2x + 5$ is normal to the circle.

**(a)** Show all your workings, find the equation of the circle. [7]

Gradient of tangent = $\frac{5-2}{0-3} = -1$ ---------------------M1
Gradient of normal =1 -------------------------------M1
Eqn of normal : $y = x - 1$ --------------------------M1

Solve simultaneous eqns of the two normals to get the centre of circle.
----M1
$y = x - 1$ -------------eqn 1
$3y = 2x + 5$ -----------eqn 2
Sub eqn (1) into eqn 2, $3(x - 1) = 2x + 5$
Centre : $x = 8, y = 7$ ------------------M1
Radius of circle = $\sqrt{(8-3)^2 + (7-2)^2}$=$\sqrt{50}$ -----------M1
Equation of Circle : $(x - 8)^2 + (y - 7)^2 = 50$ ----------A1

**(b)** Find the tangents to the circle that are parallel to the x-axis. [2]

$y = 7 + \sqrt{50}$ and $y = 7 - \sqrt{50}$--------------B2