Calculator Model: ________________

NAME:	CLASS:	INDEX NO:

QUEENSWAY SECONDARY SCHOOL

PRELIMINARY EXAMINATION 2021

SECONDARY 4 EXPRESS

Parent's Signature:

ADDITIONAL MATHEMATICS

Paper 1

4049/01

13 September 2021

2 hour 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction tape.

Answer **all** questions.

If working is needed for any question, it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in bracket [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots + \binom{n}{r} a^{n-r} b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \dfrac{n!}{(n-r)!\,r!} = \dfrac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = cos^2 A - sin^2 A = 2cos^2 A - 1 = 1 - 2sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \tfrac{1}{2} bc \sin A$$

1. (a) A beaker of solution is heated until it reaches a temperature of P ℃. It is then placed in a room to cool. Its temperature θ ℃, when cooling for t minutes, is given by $\theta = 38 + 50e^{-0.7t}$.

(i) Find the value of P. [1]

(ii) Find the value of t when $\theta = \frac{1}{2}P$. [2]

(iii) Explain, with working, if the water will cool to a temperature of 30 ℃ . [2]

2. (a) Find the coefficient of x in the expansion of $\left(\frac{2}{x} - \frac{x^2}{4}\right)^8$. [3]

(b) It is given that $(1 + kx)^n = 1 + 10x + \frac{175}{4}x^2 + px^3 + \ldots\ldots\ldots$ where $n > 0$.

Find the values of n, k and p. [5]

3. (a) $2x^2 - x - 3$ is a factor of $2x^4 + x^3 + px^2 + 3x + q$.

(i) Show that $p = -16$ and $q = 18$. [3]

(ii) Hence, solve the equation $2x^4 + x^3 - 16x^2 + 3x + 18 = 0$. [3]

(b) The equation $(x+a)(x+b)=c^2$ is given such that a, b and c are real values.

(i) Show that the roots of the equation are always real. [3]

(ii) State a set of possible values for a, b and c when the roots are real and equal. [2]

4. (a) Solve the equation $log_{16}(4x-5) = log_4 2x - log_4\sqrt{5}$. [4]

(b) Show that $2^x + \frac{1}{2}(2^{x+4}) - 2^{x+2}$, where x is positive, is exactly divisible by 5. [2]

5. (i) Prove that $\frac{\tan x}{1+\sec x} + \frac{1+\sec x}{\tan x} = 2\ cosec\ x$. [3]

(ii) Hence, solve $\frac{\tan x}{1+\sec x} + \frac{1+\sec x}{\tan x} = 3 \sec x$, where $0° \leq x \leq 540°$. [4]

6. A circle is defined by the equation $x^2 + y^2 - 8x + 12y = 48$.

(i) Determine the coordinates of the centre of the circle and its radius. [3]

(ii) Justify whether point A (7, 3) lies on, inside or outside the circle. [2]

(iii) The tangent to the circle at B is parallel to the line $4y = 3x - 6$. Determine two possible coordinates of B. [6]

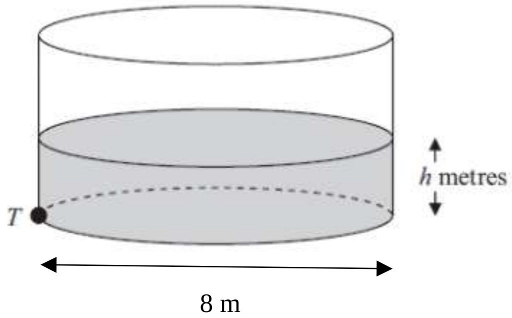
7. The equation of a curve is $y = ax^4 + bx^2 - 16x$. The coordinates of a stationary point on the curve is given as $(-2, 12)$. Find the coordinates of all the stationary points on the curve and determine their nature. [8]

8. (i) Differentiate $(3x+5)\ln(2x-1)$ with respect to x. [2]

(ii) Express $\frac{6x+10}{2x-1}$ in the form $A+\frac{B}{2x-1}$. [1]

(iii) Hence, find $\int \ln(2x-1)\,dx$. [3]

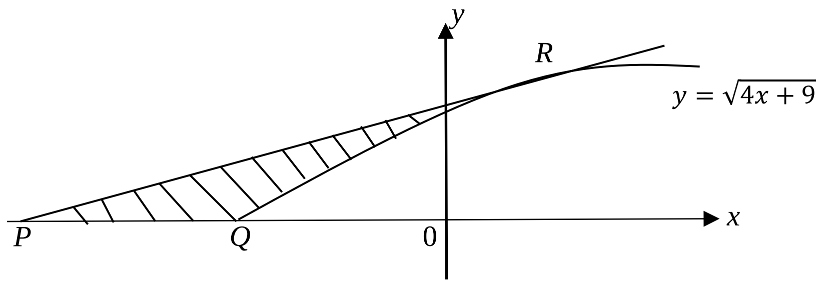
9. The diagram shows a cylindrical tank used to store water. The diameter of the circular cross-sectional area is 8 m. Water is pumped into the tank at a constant rate of 0.45π m^3/min. After t minutes, the depth of the water in the tank is h m. A tap, T, at the bottom of the tank will release the water at a rate of $0.6\pi h$ m^3/min when it is opened.



(a) Show that the rate of change of the depth of water, t minutes after the tap is opened, is $\frac{dh}{dt} = \frac{9-12h}{320}$. [4]

(b) The water from the tank is mixed with chemicals and poured onto a flat surface, forming a circular patch with negligible depth. The patch expands at a constant rate of $6\pi\ cm^2/s$. Find the rate of change of the radius of the circular patch 10 seconds after the liquid is poured out. [4]

10. (a) The figure shows the curve $y = \sqrt{4x+9}$. The line PR is a tangent to the curve at R with gradient $\frac{2}{5}$.

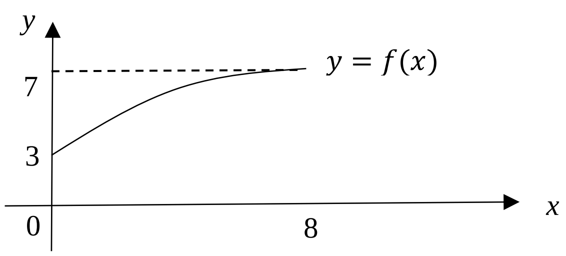


(i) Find the coordinates of P, Q and R. [5]

(ii) Find the area of the shaded region bounded by the line PR, curve QR and the x-axis. [5]

(b) The diagram shows part of the curve $y = f(x)$. Show that [2]

$40 < \int_0^8 f(x)\,dx < 56$.



11. A particle, P, travelling in a straight line passes a fixed point O with a velocity of 24 m/s. Its acceleration, $a\ m/s^2$, is given by the equation $a = 6t - 18$, where t is the time in seconds after passing O.

(i) Find the velocity of P when $t = 3$. [3]

(ii) Find the expression for s, the displacement of P from O. [2]

(iii) Calculate the total distance travelled from $t = 0$ to $t = 6$. [3]

------ End of Paper ----

4E AM Prelim P2 Solutions

1.	(i)	$t = 0, P = 38 + 50^{-0.7(0)} = 88°C$	A1
	(ii)	$\frac{1}{2}P = 44$ $38 + 50e^{-0.7t} = 44$ $50e^{-0.7t} = 6$ $e^{-0.7t} = 0.12$ $t = \frac{ln0.12}{-0.7}$ $t = 3.03\ min$	M1 A1
	(iii)	When t becomes very large, $50e^{-0.7t} \to 0$ $\theta \to 38 + 0 = 38°C$ and will never reach 30°C	M1 A1
2.	(a)	$T_{r+1} = \binom{8}{r}\left(\frac{2}{x}\right)^{8-r}\left(-\frac{x^2}{4}\right)^r$ $= \binom{8}{r}(2)^{8-r}\left(-\frac{1}{4}\right)^r x^{-8+r}(x^2)^r$ $x^{-8+3r} = x^1$ $3r = 9$ $r = 3$ Coefficient of $x = \binom{8}{3}(2)^{8-3}\left(-\frac{1}{4}\right)^3 = -28$	M1 M1 A1
	(b)	$(1 + kx)^n = 1^n + \binom{n}{1}(kx) + \binom{n}{2}(kx)^2$ $= 1 + nkx + \frac{n(n-1)}{2}k^2x^2$ $nk = 10$ $k = \frac{10}{n}$ $\frac{n(n-1)}{2}k^2 = \frac{175}{4}$ $\frac{n(n-1)}{2}\left(\frac{10}{n}\right)^2 = \frac{175}{4}$ $\frac{n(n-1)100}{2n^2} = \frac{175}{4}$ $200n(n-1) = 175n^2$ $25n^2 - 200n = 0$ $25n(n-8) = 0$ $n = 0\ or\ n = 8$ (rejected) $k = \frac{10}{8} = 1\frac{1}{4}\ or\ 1.25$ $px^3 = \binom{n}{3}(kx)^3 = \binom{8}{3}\left(\frac{5}{4}\right)^3 x^3$ $p = \frac{875}{5}\ or\ 109\frac{3}{8}$	M1 M1 A1 A1 A1

3.	(a)(i)	$2x^2 - x - 3 = (2x-3)(x+1)$ $f(x) = 2x^4 + x^3 + px^2 + 3x + q$ $f(-1) = 0$ $p + q = 2$ $f\left(\frac{3}{2}\right) = 0$ $\frac{9}{4}p + q = -18$ Using elimination, $\frac{5}{4}p = -20$ $p = -16$ $q = -18\ (shown)$	M1 M1 A1
	(ii)	$f(x) = 2x^4 + x^3 - 16x^2 + 3x + 18$ $f(x) = (2x^2 - x - 3)(ax^2 + bx + c)$ Long division or compare coefficients $f(x) = (2x^2 - x - 3)(x^2 + x - 6)$ $f(x) = 0$ $x = -3, -1,\ 1\frac{1}{2}\ or\ 2$	M1 A2
	(b)(i)	$x^2 + ax + bx + ab - c^2 = 0$ $b^2 - 4ac = (a+b)^2 - 4(ab - c^2)$ $= a^2 + 2ab + b^2 - 4ab + 4c^2$ $= a^2 - 2ab + b^2 + 4c^2$ $= (a-b)^2 + 4c^2$ $(a-b)^2 \geq 0\ and\ 4c^2 \geq 0$ for all values of $a, b\ and\ c$ $b^2 - 4ac \geq 0$ and thus roots are always real	M1 M1 A1
	(ii)	$Accept\ any\ values\ of\ a = b\ but\ c = 0$	A2
4.	(a)	$\frac{log_4(4x-5)}{log_4 16} = log_4\frac{2x}{\sqrt{5}}$ $\frac{log_4(4x-5)}{2} = log_4\frac{2x}{\sqrt{5}}$ $log_4(4x-5) = 2log_4\frac{2x}{\sqrt{5}}$ $log_4(4x-5) = log_4\left(\frac{2x}{\sqrt{5}}\right)^2$ $4x - 5 = \frac{4x^2}{5}$ $4x^2 - 20x + 25 = 0$	M1 M1 M1

		$(2x-5)^2 = 0$ $x = 2.5$		A1
	(b)	$2^x + 2^{-1}2^{x+4} - 2^x2^2$ $= 2^x + 2^x2^3 - 2^x2^2$ $= 2^x(1+2^3-2^2)$ $= 2^x(5)$ Thus the term is always divisible by 5		M1 A1
5	(i)	$LHS = \frac{tanx}{1+secx} + \frac{1+secx}{tanx}$ $= \frac{tan^2x + (1+secx)^2}{tanx(1+secx)}$ $= \frac{tan^2x + 1 + 2secx + sec^2x}{tanx(1+secx)}$ $= \frac{2sec^2x + 2secx}{tanx(1+secx)} = \frac{2secx(secx+1)}{tanx(1+secx)}$ $= \frac{2secx}{tanx} = \frac{2}{cosx} \div \frac{sinx}{cosx}$ $= \frac{2}{sinx} = 2cosecx = RHS(shown)$		M1 M1 A1
	(ii)	$2cosecx = 3secx$ $\frac{2}{sinx} = \frac{3}{cosx}$ $\frac{sinx}{cosx} = \frac{2}{3}$ $tanx = \frac{2}{3}$ $basic\ angle = 33.7$ $x = 33.7°, 213.7°, 393.7°$		M1 A3
6	(i)	$x^2 + y^2 - 8x + 12y = 48$ $(x-4)^2 - 16 + (y+6)^2 - 36$ $= 48$ $(x-4)^2 + (y+6)^2 = 100$ Centre of circle = $(4, -6)$ Radius = 10 units	Alternative $2g = -8, 2f = 12$ $g = -4, f = 6$	M1 A1 A1

	(ii)	Length = $\sqrt{(7 = 4)^2 + \left(3 - (-6)\right)^2} = \sqrt{90}$ units $\sqrt{90} < 10\ units$, thus point A lies inside the circle	M1 A1
	(iii)	$$gradient\ of\ tangent = \frac{3}{4}$$ $$gradient\ of\ perpendicular = -\frac{4}{3}$$ Equation of diameter $y + 6 = -\frac{4}{3}(x - 4)$ $$y = -\frac{4}{3}x - \frac{2}{3}$$ Solve simultaneous equations $$(x-4)^2 + \left(-\frac{4}{3}x - \frac{2}{3} + 6\right)^2 = 100$$ $$x^2 - 8x + 16 + \frac{16}{9}x^2 - \frac{128}{9}x + \frac{256}{9} = 100$$ $$25x^2 - 200x - 500 = 0$$ $$x^2 - 8x - 20 = 0$$ $$(x-10)(x+2) = 0$$ $$x = 10\ or - 2$$ $$y = -14\ or\ 2$$ Coordinates = $(10, -14) or\ (-2, 2)$	M1 M1 M1 M1 A2
7		$$\frac{dy}{dx} = 4ax^3 + 2bx - 16$$ $$x = -2, \quad \frac{dy}{dx} = 0$$ $$8a + b = -4$$ $(-2, 12)$ $\quad 12 = 16a + 4b + 32$ $$4a + b = -5$$ $$a = \frac{1}{4}, \quad b = -6$$ $$y = \frac{1}{4}x^4 - 6x^2 - 16x$$ $$\frac{dy}{dx} = x^3 - 12x - 16$$ $$(x+2)(ax^2 + bx + c) = 0$$ Long Division or compare coefficient, $$(x+2)(x^2 - 2x - 8) = 0$$ $$(x+2)^2(x-4) = 0$$ $$x = -2\ or\ 4$$ $$y = 12\ or\ -96$$ Stationary points $(-2,12)$	M2 M1 M1 M1 A1

		$\frac{d^2y}{dx^2} = 3x^2 - 12 = 0$, point of inflexion $\begin{array}{cccc} x & -2.1 & -2 & -1.9 \\ \frac{dy}{dx} & - & 0 & - \end{array}$ Stationary point $(4, -96)$ $\frac{d^2y}{dx^2} = 3x^2 - 12 = 36 > 0$, minimum point	A1 A1
8	(i)	$\frac{d}{dx}(3x+5)\ln(2x-1)$ $= \frac{2(3x+5)}{2x-1} + 3\ln(2x-1)$ $= \frac{6x+10}{2x-1} + 3\ln(2x-1)$	A2
	(ii)	$\frac{6x+10}{2x-1} = 3 + \frac{13}{2x-1}$ Using long division	A1
	(iii)	$\int \frac{6x+10}{2x-1} + 3\ln(2x-1)\,dx = [(3x+5)\ln(2x-1)] + c$ $\int 3\ln(2x-1)dx = [(3x+5)\ln(2x-1)] - \left(\int 3 + \frac{13}{2x-1}dx)\right) + c$ $\int 3\ln(2x-1)\,dx = [(3x+5)\ln(2x-1)] - 3x - \frac{13}{2}\ln(2x-1) + c$ $\int \ln(2x-1)dx = \frac{1}{3}\{[(3x+5)\ln(2x-1)] - 3x - \frac{13}{2}\ln(2x-1) + c$ $\int \ln(2x-1)\,dx = \frac{1}{3}[(3x+5)\ln(2x-1)] - x - \frac{13}{6}\ln(2x-1) + c$	 M1 M1 A1
9	(a)	$\frac{dv}{dt} = 0.45\pi - 0.6\pi h$ $V = \pi r^2 h = \pi(4^2)h = 16\pi h$ $\frac{dV}{dh} = 16\pi$ $\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$ $\frac{dh}{dt} = \frac{0.45\pi - 0.6\pi h}{16\pi} = \frac{0.45 - 0.6h}{16}$ $= \frac{45 - 60h}{1600} = \frac{9 - 12h}{320}$ $(shown)$	M1 M1 M1 A1

	(b)	$A = 10 \times 6\pi = 60\pi cm^2$ $\pi r^2 = 60\pi$ $r = \sqrt{60} cm$ $\frac{dA}{dr} = 2\pi r = 2\pi(\sqrt{60})$ $\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$ $\frac{dr}{dt} = \frac{6\pi}{2\pi\sqrt{60}} = 0.387\ cm/s$	M1 M1 M1 A1
10	(a) (i)	$y = 0, \sqrt{4x+9} = 0$ $x = -2.25 \quad Q(-2.25, 0)$ $\frac{dy}{dx} = \frac{1}{2}(4x+9)^{\left(-\frac{1}{2}\right)}(4) = \frac{2}{\sqrt{4x+9}}$ $\frac{2}{\sqrt{4x+9}} = \frac{2}{5}$ $4x + 9 = 25$ $x = 4, y = 5 \quad R(4,5)$ $y - 5 = \frac{2}{5}(x-4)$ $y = \frac{2}{5}x + \frac{17}{5}$ $y = 0, x = -8.5 \quad P(-8.5, 0)$	A1 M1 A1 M1 A1
	(ii)	$Area = \frac{1}{2}(12.5)(5) — \int_{-2.25}^{4} \sqrt{4x+9}\, dx$ $= 31.25 - \left[\frac{(4x+9)^{\frac{3}{2}}}{(4)\left(\frac{3}{2}\right)}\right]_{-2.25}^{4}$ $= 31.25 - \left[\frac{1}{6}(4x+9)^{\frac{3}{2}}\right]_{-2.25}^{4}$ $= 31.25 - \left[\frac{125}{6} - 0\right] = 10\frac{5}{12}\ units^2$	M2 M1 A2
	(b)	$Area\ of\ trapezium < \int_0^8 f(x)\, dx < Area\ of\ rectangle.$ $\frac{1}{2} \times 8 \times (3 + 7) < \int_0^8 f(x)\, dx < 7 \times 8.$ $40 < \int_0^8 f(x)\, dx < 56$ (shown)	A2

11	(i)	$$a = 6t - 18$$ $$v = \int 6t - 18\,dx = \frac{6}{2}t^2 - 18t + c$$ $$t = 0, v = 24$$ $$v = 3t^2 - 18t + 24$$ $$t = 3, v = -3m/s$$	M1 M1 A1
	(ii)	$$s = \int 3t^2 - 18t + 24\,dx$$ $$s = \frac{3}{3}t^3 - \frac{18}{2}t^2 + 24t + c$$ $$t = 0, s = 0$$ $$s = t^3 - 9t^2 + 24t$$	M1 A1
	(iii)	$$v = 0,\ \ 3t^2 - 18t + 24 = 0$$ $$t^2 - 6t + 8 = 0$$ $$(t-2)(t-4) = 0 \qquad t = 2\ or\ 4$$ $$t = 0, s = 0 \qquad t = 2, s = 20 \qquad t = 4, s = 16 \qquad t = 6, s = 36$$ Distance travelled = 20 + 4 + 20 = 44 m	M1 M1 A1