TOPIC 1: MEASUREMENT TUTORIAL
Suggested Answers

1 (a)

Derived quantities	**Definition**	**Defining Equation**	**Derived units (base)**	**Alternative unit(s)**
Area	The extent of a two-D figure or shape in a plane.	$A_{\text{rectangle}} = l \times w$ $A_{\text{circle}} = \pi r^2$	m^2	-
Moment	Moment of a force about a point is the product of the magnitude of the force and the perpendicular distance of the force from the point.	moment of a force $= F \times d_{\perp}$	kg m^2 s^{-2}	N m
Pressure	Force per unit area.	$p = \dfrac{F}{A}$	kg m^{-1} s^{-2}	N m^{-2} , Pa
Work done	Force × Displacement in direction of force	$W = Fd\cos\theta$	kg m^2 s^{-2}	J
Power	Work done per unit time.	$P = \dfrac{W}{t}$	kg m^2 s^{-3}	J s^{-1} , W
Resistance	The ratio of potential difference across the conductor to the current flowing in it.	$R = \dfrac{V}{I}$	kg m^2 s^{-3} A^{-2}	V A^{-1}, Ω

$$1\text{ V} = \frac{1\text{ J}}{1\text{ C}} = \frac{1\text{ Nm}}{1\text{ C}} = \frac{1\left(\text{kg m s}^{-2}\right)\text{m}}{1\text{ C}} = 1\text{ kg m}^2\text{ s}^{-2}\text{ C}^{-1}$$
$$= 1\text{ kg m}^2\text{ s}^{-2}\ (\text{A s})^{-1}$$
$$= 1\text{ kg m}^2\text{ s}^{-3}\text{ A}^{-1}$$

1(b)

$$E = hf$$
$$\Rightarrow h = \frac{E}{f}$$
$$\text{Units of } h = \frac{\text{Units of } E}{\text{Units of } f} = \frac{\text{J}}{\text{s}^{-1}} = \text{J s}$$
$$\text{Base units of } h = \left(\text{kg m}^2\text{ s}^{-2}\right)\text{s} = \text{kg m}^2\text{ s}^{-1}$$

Common mistake: Students often incorrectly equate **physical quantity** with **units** in the working, for example, by writing: h = kg m^2 s^{-1}, which is an incorrect equation.

2.

$$\text{base units of } C_D = \frac{\text{base units of } F}{\text{base units of } \frac{1}{2}\rho A v^2}$$
$$= \frac{\text{kg m s}^{-2}}{(1)(\text{kg m}^{-3})(\text{m}^2)(\text{m}^2\ \text{s}^{-2})}$$
$$= \frac{\text{kg m s}^{-2}}{\text{kg m s}^{-2}} = 1$$

Hence, C_D has no units. It is dimensionless.

3. A homogeneous equation is one in which every term has the same units. To determine whether an equation is homogeneous, find the units of each term and compare. If the units are the same, the equation is homogeneous.

Unit of v = m s^{-1}.

(A)
$v = \sqrt{g\lambda}$

Units of $\sqrt{g\lambda} = \sqrt{(\text{m s}^{-2})(\text{m})} = \sqrt{\text{m}^2\ \text{s}^{-2}} = \text{m s}^{-1}$

Comparing units of v and $\sqrt{g\lambda}$, we conclude that the equation $v = \sqrt{g\lambda}$ is homogeneous.

(B)
$v = \sqrt{\frac{g}{h}}$

Units of $\sqrt{\frac{g}{h}} = \sqrt{\frac{\text{m s}^{-2}}{m}} = \sqrt{\text{s}^{-2}} = \text{s}^{-1}$

Comparing units of v and $\sqrt{\frac{g}{h}}$, we conclude that the equation $v = \sqrt{\frac{g}{h}}$ is not homogeneous.

(C)
$v = \sqrt{\rho g h}$

Units of $\sqrt{\rho g h} = \sqrt{(\text{kg m}^{-3})(\text{m s}^{-2})(\text{m})} = \sqrt{\text{kg m}^{-1}\text{s}^{-2}} = \text{kg}^{\frac{1}{2}}\ \text{m}^{-\frac{1}{2}}\text{s}^{-1}$

Comparing units of v and $\sqrt{\rho g h}$, we conclude that the equation $v = \sqrt{\rho g h}$ is not homogeneous.

(D)
$v = \sqrt{\frac{g}{\rho}}$

Units of $\sqrt{\frac{g}{\rho}} = \sqrt{\frac{\text{m s}^{-2}}{\text{kg m}^{-3}}} = \sqrt{\text{kg}^{-1}\ \text{m}^4\ \text{s}^{-2}} = \text{kg}^{-\frac{1}{2}}\ \text{m}^2\ \text{s}^{-1}$

Comparing units of v and $\sqrt{\frac{g}{\rho}}$, we conclude that the equation $v = \sqrt{\frac{g}{\rho}}$ is not homogeneous.

4. (i) Pressure is defined as $P = \dfrac{\text{force}}{\text{area}}$.

Base unit of pressure P = Base unit of $\dfrac{\text{force}}{\text{area}} = \dfrac{\text{kg m s}^{-2}}{\text{m}^2} = \text{kg m}^{-1}\ \text{s}^{-2}$

(ii) Units of $c = \sqrt{\dfrac{\gamma P}{\rho}} = \sqrt{\dfrac{\text{kg m}^{-1}\text{s}^{-2}}{\text{kg m}^{-3}}} = \sqrt{\text{m}^2\text{s}^{-2}} = \text{m s}^{-1}$

(iii) c could be velocity or speed.

5. $p + h\rho g + \dfrac{1}{2}\rho v^2 = k$

(i)

Base units of $p = \left(\text{kg m s}^{-2}\right)\text{m}^{-2} = \text{kg m}^{-1}\ \text{s}^{-2}$

Base units of $h\rho g = (\text{m})\left(\text{kg m}^{-3}\right)\left(\text{m s}^{-2}\right) = \text{kg m}^{-1}\ \text{s}^{-2}$

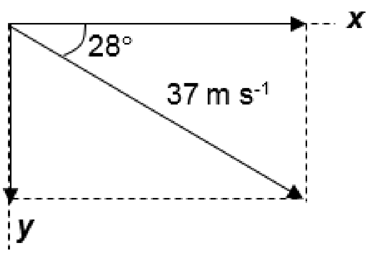
Base units of $\dfrac{1}{2}\rho v^2 = \left(\text{kg m}^{-3}\right)\left(\text{m s}^{-1}\right)^2 = \text{kg m}^{-1}\ \text{s}^{-2}$

Since all terms have the same base units, the equation is dimensionally consistent.
Note: Homogenous equation may not be physically correct.

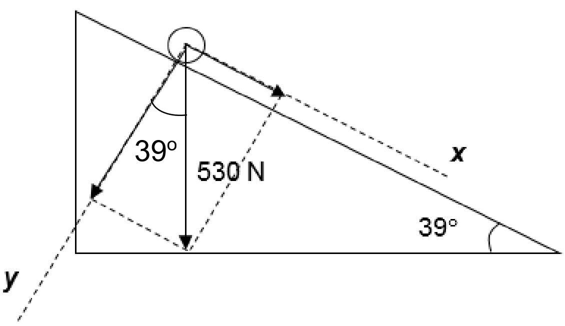
(ii) The base units for k should also be $\text{kg m}^{-1}\ \text{s}^{-2}$.

6. (a)

v_x = 37 cos 28° = 33 m s^{-1}, horizontally to the right
v_y = 37 sin 28° = 17 m s^{-1}, vertically downwards



(b) F_x = 530 cos51° = 530 sin 39 = 334 N, downwards parallel to the incline plane
F_y = 530 sin51° = 530 cos 39 = 412 N, perpendicularly towards the incline plane



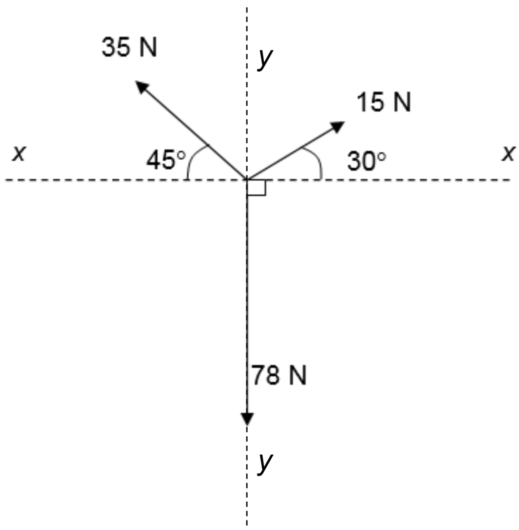
7. (i)

Resultant vector in the x-x direction

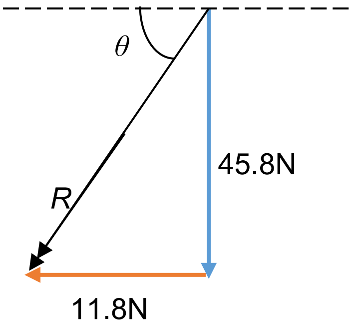
R_x = -35 cos45° + 15 cos30°+ 0= −11.8 N

Resultant vector in the y-y direction

R_y = 35 sin45° + 15 sin30° - 78 = −45.8 N



Vector diagram to show resultant:



Magnitude of resultant force $R = \sqrt{45.8^2 + 11.8^2} = 47.3$ N

$\theta = \tan^{-1}\left(\dfrac{45.8}{11.8}\right) = 76°$

Resultant force R = 47.3 N in the direction 76° anticlockwise from the negative x-axis.

(ii) Smallest force that must be applied such that the resultant is along x-x direction is 45.8 N in the direction 90° anticlockwise from the positive x-axis.

8. Force component along the N-S direction

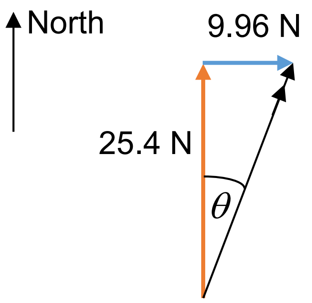
= 27.56 + 7.89 cos34° + 10 sin20° – 12 sin 60° – 10 sin10°

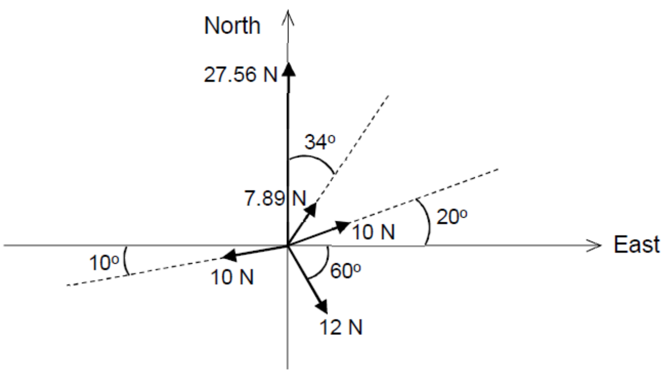
= 25.4 N

Force component along the E-W direction

= 0 + 7.89 sin34° +10 cos20° + 12 cos60° -10 cos10°

= 9.96 N



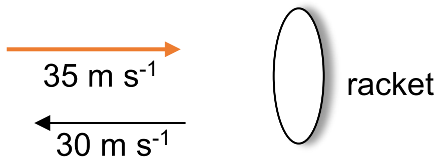


Magnitude of resultant force = $\sqrt{9.96^2 + 25.4^2} = 27.3$ N

$\theta = \tan^{-1}\left(\dfrac{9.96}{25.4}\right) = 21.4°$

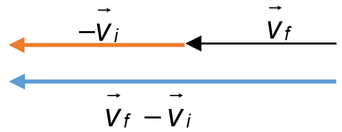
The resultant force is 27.3 N, 21.4° clockwise from the North direction.

9a



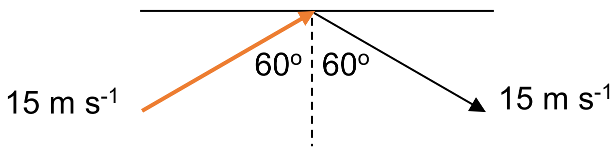
Taking leftward direction as positive,
Change in velocity $\Delta\vec{v}$ = final velocity – initial velocity = $\vec{v}_f - \vec{v}_i = \vec{v}_f + \left(-\vec{v}_i\right)$

Vector diagram:

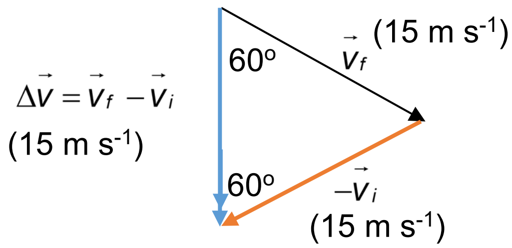


Thus the change in velocity is 65 m s^{-1} away from racket.

9b

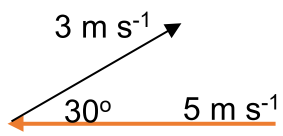


Vector diagram:

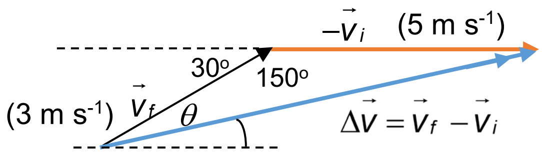


It is an equilateral triangle!
Thus the change in velocity is 15 m s^{-1}, vertically downwards.

9c



Vector diagram:



By Cosine Rule, $\Delta v = \sqrt{3^2 + 5^2 - 2\times 3\times 5\times \cos\left(150^\circ\right)} = 7.7 \text{ m s}^{-1}$

By Sine Rule, $\dfrac{\sin\theta}{5} = \dfrac{\sin 150^\circ}{7.7} \Rightarrow \theta = \sin^{-1}\left(5\times\dfrac{\sin 150^\circ}{7.7}\right) = 19^\circ$

The change in velocity is 7.7 m s^{-1}, (30° - 19°) = 11° anticlockwise from the horizontal.

10. **D**

$$\text{Average} = \frac{1.02\times 4+1.01}{5} = \frac{5.09}{5} = 1.02$$

$$\text{Corrected diameter} = \text{average diameter} - \text{zero reading}$$
$$= 1.02-(-0.02) = 1.04 \text{ mm}$$

11. **C**

Random errors - where repeating the measurement gives an unpredictably different result.

- cannot possibly be eliminated (P2)
- has varying sign and magnitude (Q2)
- can be reduced by averaging repeated measurements (R1)
  The random errors tend to cancel each other out, and the residual error is divided by the number of readings, so it gets shared out among many readings. Increasing the number of measurements generally reduces the uncertainty in the mean value.

12. **C**

Experimental technique that reduces the systematic error of the quantity being investigated: Adjusting an ammeter to remove its zero error before measuring a current

13. An object of mass 1.000kg is placed on four different balances. For each balance the reading is taken five times. The table shows the values obtained together with the means. Which balance has the smallest systematic error but is not very precise?
(N2002/I/2)

Balance	Reading/ kg					mean/kg	|Mean – 1.000 kg|
	1	**2**	**3**	**4**	**5**		
A	1.000	1.000	1.002	1.001	1.002	1.001	0.001
B	1.011	0.999	1.001	0.989	0.995	0.999	0.001
C	1.012	1.013	1.012	1.014	1.014	1.013	0.013
D	0.993	0.987	1.002	1.000	0.983	0.993	0.007

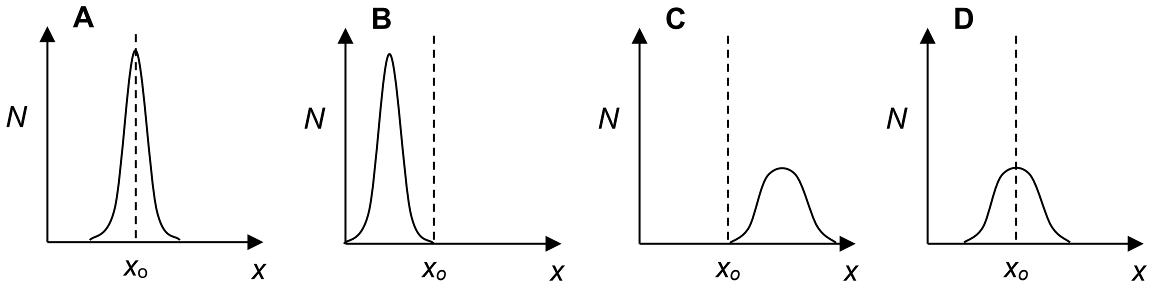
Smallest systematic error gives better accuracy, i.e. the 'mean' of the readings is closer to the true value of 1.000 kg. (Either balance A or B whose mean readings are 1.001 and 0.999 respectively)

'Not precise' means that the measurements has large random errors, i.e. the readings are 'not close' to each other. This can be 'quantified' using the 'spread' of the readings given by (largest reading – smallest reading).
Range or 'Spread' for balance A = 1.002 - 1.000 = 0.002
Range or 'Spread' for balance B = 1.011 - 0.989 = 0.022
Readings for Balance B are less precise than those for Balance A.

Answer: Balance **B** has 'small systematic error' but 'not very precise'.
(So Balance A has 'small systematic error' as well as 'small random error'.
Also,
Range for Balance C = 1.014 - 1.012 = 0.002
Balance C has 'large systematic error' but 'small random error'.

Range for Balance D = 1.002 - 0.987 = 0.015
Balance D has 'large systematic error' as well as 'large random error'.

14. Which graph best represents precise measurements with poor accuracy?



B

Interpretation of the graph of *N* against *x*:

N is the number of times a particular value of *x* is obtained, i.e. it is the frequency of *x*. The graph of *N* against *x* is a plot of frequency against the measured value *x*.

The graphs are more or less 'bell-shaped', i.e. the values of *x* are 'normally distributed'. The 'mean' (average) of the distribution coincides with the 'median' (largest number of *N*), which is the 'peak' of the curve.

'Precise' measurements *x* are close to each other, that is, the measurements has smaller 'spread' when plotted on the *x* axis. The 'bell-shape' curve will be thinner (smaller spread), with taller and sharper peak. Measurements with 'poor precision' have a larger 'spread', the bell-shape curve appears broader (larger spread) and flatter.

Poor 'accuracy' means the 'average' of the measurements is 'not close' to the true value x_o, that is, the 'peak' of the curve is not close to x_o. Good accuracy means the 'peak' of the curve is close to x_o. Poor 'accuracy' means the 'average' of the measurements is 'not close' to the true value x_o.

15. (a) The omission of 'zero' reading introduces a systematic error in all the readings, which results in all the readings consistently larger or consistently smaller than the actual reading.

(b) The readings are not accurate because the 'mean' of these readings will not be close to the true reading since the readings have not been corrected by subtracting the 'zero' reading. The readings may be 'precise' because the values may be close to each other due to the ability of the micrometer screw gauge to differentiate minute distances, about 0.01 mm.

16. **C**

Given $p \pm \Delta p,\ q \pm \Delta q$

Let $r = \dfrac{p}{q}$

$$\frac{\Delta r}{r} = \frac{\Delta p}{p} + \frac{\Delta q}{q}$$

17.

$$P = \frac{V^2}{R}$$

$$\text{percentage uncertainty in } P = \left(\frac{\Delta P}{P}\right) \times 100\%$$

$$= \left(2\frac{\Delta V}{V} + \frac{\Delta R}{R}\right) \times 100\%$$

$$= 2\left(\frac{\Delta V}{V} \times 100\%\right) + \left(\frac{\Delta R}{R} \times 100\%\right)$$

$$= 2(3\%) + (2\%)$$

$$= 8\%$$

18. Given **X ± ΔX** = (1.0 $\pm$ 0.1) cm
Y ± ΔY = (4.0 $\pm$ 0.1) cm

$$D = \frac{\mathbf{Y} - \mathbf{X}}{4} = \frac{4.0 - 1.0}{4} = \frac{3.0}{4} = 0.75 \text{ cm}$$

$$D = \frac{\mathbf{Y}}{4} - \frac{\mathbf{X}}{4}$$

$$\Delta D = \frac{1}{4}\Delta\mathbf{Y} + \frac{1}{4}\Delta\mathbf{X} = \frac{1}{4}(0.1) + \frac{1}{4}(0.1) = 0.05 \text{ cm}$$

$$\therefore D = (0.75 \pm 0.05) \text{ cm}$$

19. **D**

$$d = \frac{d_1 + d_2 + d_3 + d_4 + d_5}{5}$$

$$d = \frac{1.52 + 1.48 + 1.49 + 1.51 + 1.49}{5} = 1.498 \text{ mm}$$

$$\Delta d = \frac{1}{5}(\Delta d_1 + \Delta d_2 + \Delta d_3 + \Delta d_4 + \Delta d_5) = \frac{1}{5}(5 \times 0.01) = 0.01 \text{ mm}$$

$$\therefore d = (1.50 \pm 0.01) \text{ mm}$$

The *least count* of the instrument scale is the smallest division on the measurement scale. This is the unit of the smallest reading that can be made without estimating.
The least count is taken to be the measurement error.
The least count of micrometer screw gauge is 0.01 mm, so $\Delta d_1 = \Delta d_2 = \Delta d_3 = \Delta d_4 = \Delta d_5 = 0.01$ mm.

Note : Uncertainty should be rounded to one significant figure, and the diameter rounded to the same number of decimal places as the uncertainty.

20.

Given $T = 2\pi\sqrt{\frac{l}{g}}$

$$g = \frac{4\pi^2}{T^2} l = \frac{4\pi^2}{2.16^2}(1.150) = 9.7308 \text{ m s}^{-2}$$

$$\frac{\Delta g}{g} = 2\frac{\Delta T}{T} + \frac{\Delta l}{l} = 2\left(\frac{0.01}{2.16}\right) + \left(\frac{0.005}{1.150}\right)$$

$$\frac{\Delta g}{9.7308} = 2\left(\frac{0.01}{2.16}\right) + \left(\frac{0.005}{1.150}\right)$$

$$\Delta g = 0.132 \text{ m s}^{-2}$$

$$= 0.1 \text{ m s}^{-2} \text{ (to 1 sig. fig.)}$$

$$g \pm \Delta g = (9.7 \pm 0.1) \text{ m s}^{-2}$$

21. (i)

$$F = 6\pi\eta r v = 6\pi(0.13)(0.01)(2.7) = 0.06617 \text{ N}$$

$$\frac{\Delta F}{F} = \frac{\Delta \eta}{\eta} + \frac{\Delta r}{r} + \frac{\Delta v}{v}$$

$$\frac{\Delta F}{0.06617} = \frac{0.02}{0.13} + \frac{0.1}{2.0} + 0.05$$

$$\Delta F = 0.02 \text{ N (to 1 s.f.)}$$

$$F = (0.07 \pm 0.02) \text{ N}$$

(ii) Measurement of the timing might not be accurate as it will be difficult to determine when the sphere has passed the length. Better to use light gates instead to measure the time of fall.

22.

$$\frac{V}{t}=\frac{\pi r^4(p_1-p_2)}{8\eta L}$$

$$\eta=\frac{\pi r^4(p_1-p_2)t}{8LV}$$

$$=\frac{\pi\left(0.43\times10^{-3}\right)^4(1.150-1.000)\times10^5\times(4.0)}{8\left(5.5\times10^{-2}\right)\left(10.0\times10^{-6}\right)}\quad\frac{\left(\mathrm{m}^4\right)\left(\mathrm{kg\ m\ s^{-2}}\times\mathrm{m}^{-2}\right)(\mathrm{s})}{(\mathrm{m})\left(\mathrm{m}^3\right)}$$

$$=1.4646\times10^{-3}\ \mathrm{kg\ m^{-1}\ s^{-1}}$$

Let $P=p_1-p_2$

$$\therefore\eta=\frac{\pi r^4Pt}{8LV}$$

$$\Rightarrow\frac{\Delta\eta}{\eta}=4\frac{\Delta r}{r}+\frac{\Delta P}{P}+\frac{\Delta t}{t}+\frac{\Delta L}{L}+\frac{\Delta V}{V}$$

Since $P=p_1-p_2$, $\Rightarrow\Delta P=\Delta p_1+\Delta p_2\Rightarrow\dfrac{\Delta P}{P}=\dfrac{\Delta p_1+\Delta p_2}{(p_1-p_2)}$

$$\frac{\Delta\eta}{\eta}=4\frac{\Delta r}{r}+\frac{\Delta p_1+\Delta p_2}{(p_1-p_2)}+\frac{\Delta t}{t}+\frac{\Delta L}{L}+\frac{\Delta V}{V}$$

$$\frac{\Delta\eta}{1.4646\times10^{-3}}=4\frac{0.01}{0.43}+\frac{0.010}{0.150}+\frac{0.1}{4.0}+\frac{0.1}{5.5}+\frac{0.1}{10.0}$$

$$\Delta\eta=0.213\times1.4646\times10^{-3}$$

$$=0.3\times10^{-3}\ \mathrm{kg\ m^{-1}\ s^{-1}}\ \text{(to 1 sig. fig.)}$$

$$\eta\pm\Delta\eta=(1.5\pm0.3)\times10^{-3}\ \mathrm{kg\ m^{-1}\ s^{-1}}\quad\text{OR}\quad\mathrm{N\ s\ m^{-2}}$$

Note that factor 4 for $\dfrac{\Delta r}{r}$ when computing $\dfrac{\Delta\eta}{\eta}$, so the measurement of r, in this experiment, has more effect on the uncertainty in η than all other readings.

23.

Given $l \pm \Delta l = (40 \pm 1)$ mm

$D \pm \Delta D = (12.0 \pm 0.2)$ mm

$d \pm \Delta d = (10.0 \pm 0.2)$ mm

(a) $\frac{\Delta l}{l} \times 100\% = \frac{1}{40} \times 100\% = 2.5\%$

$\frac{\Delta D}{D} \times 100\% = \frac{0.2}{12.0} \times 100\% = 1.7\%$

$\frac{\Delta d}{d} \times 100\% = \frac{0.2}{10.0} \times 100\% = 2.0\%$

(b) $V = V_D - V_d = \frac{1}{4}\pi l D^2 - \frac{1}{4}\pi l d^2 = \frac{1}{4}\pi l\left(D^2 - d^2\right)$

Let $V = \frac{1}{4}\pi l P$, where $P = \left(D^2 - d^2\right)$

$\frac{\Delta V}{V} = \frac{\Delta l}{l} + \frac{\Delta P}{P}$

$= \frac{1}{40} + \frac{9}{44}$

$\therefore$ percentage uncertainty in V

$= \frac{\Delta V}{V} \times 100\% = \left(\frac{1}{40} + \frac{9}{44}\right) \times 100\%$

$= 23\%$ (2 sig. fig.)

Let $P = D^2 - d^2$
Let $Q = D^2 = 12.0^2 = 144$ mm^2
Let $R = d^2 = 10.0^2 = 100$ mm^2

Then $P = Q - R$
$= 144 - 100$
$= 44$ mm^2

$\frac{\Delta Q}{Q} = 2\frac{\Delta D}{D} = 2\left(\frac{0.2}{12.0}\right) = \frac{1}{30} \Rightarrow \Delta Q = \frac{1}{30}(144) = 5 \text{ mm}^2$

$\frac{\Delta R}{R} = 2\frac{\Delta d}{d} = 2\left(\frac{0.2}{10.0}\right) = \frac{1}{25} \Rightarrow \Delta R = \frac{1}{25}(100) = 4 \text{ mm}^2$

Since $P = Q - R$
Then $\Delta P = \Delta Q + \Delta R$
$= (5 + 4)$ mm^2
$= 9$ mm^2

(c)

$V = \frac{1}{4}\pi l\left(D^2 - d^2\right) = 1.382301 \times 10^3 \text{ mm}^3$

% uncertainty in V	ΔV / mm^3	$(V \pm \Delta V)$ / mm^3
23%	0.3×10^3 (to 1 sig. fig.)	$(1.4 \pm 0.3) \times 10^3$

Alternatively, using first principles:

$V_{max} = \frac{1}{4}\pi l_{max}\left(D_{max}{}^2 - d_{min}{}^2\right) = \frac{1}{4}\pi(41)\left(12.2^2 - 9.8^2\right) = 1700.23$

$V_{min} = \frac{1}{4}\pi l_{min}\left(D_{min}{}^2 - d_{max}{}^2\right) = \frac{1}{4}\pi(39)\left(11.8^2 - 10.2^2\right) = 1078.19$

$\Delta V = \frac{V_{max} - V_{min}}{2} = 311$

$\frac{\Delta V}{V} = \frac{311}{1382.301} = 23\%$

24. **D**

25. **A**

For this question, you may solve it using the first principle numerical method.

R_1 = (250 ± 30) kΩ

R_2 = (1000 ± 50) kΩ

$$\frac{1}{R_{\text{effective}}} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow R_{\text{effective}} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$

$$\text{maximum } R_{\text{effective}} = \frac{1}{\frac{1}{(R_1)_{\text{max}}} + \frac{1}{(R_2)_{\text{max}}}} = \frac{1}{\frac{1}{280} + \frac{1}{1050}} = 221.05 \text{ k}\Omega$$

$$\text{minimum } R_{\text{effective}} = \frac{1}{\frac{1}{(R_1)_{\text{min}}} + \frac{1}{(R_2)_{\text{min}}}} = \frac{1}{\frac{1}{220} + \frac{1}{950}} = 178.63 \text{ k}\Omega$$

$$\Delta R_{\text{effective}} = \frac{\text{maximum } R_{\text{effective}} - \text{minimum } R_{\text{effective}}}{2} = \frac{221.05 - 178.63}{2} = 21 \text{ k}\Omega$$