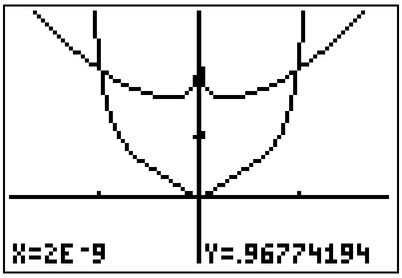
Qn	Paper 2 Solution
1 (i)	$\begin{pmatrix} 2.5k+\lambda \\ k+3\lambda \\ 2k+\lambda \end{pmatrix} = \begin{pmatrix} 3+4\mu \\ 0.5k+7\mu \\ 0.5k+5\mu \end{pmatrix}$ *Equating* **i**, **j**, **k** *components*, $2.5k+\lambda-4\mu=3$ **---(1)** $0.5k+3\lambda-7\mu=0$ **---(2)** $1.5k+\lambda-5\mu=0$ **---(3)** *Solving Equations* **(1)**, **(2), (3)**: $k=\mathbf{2}.$ $(\mu=1, \lambda=2.)$
(ii)	*When* $k=2, \lambda=2,$ $\mathbf{r} = \begin{pmatrix} (2.5)(2)+2 \\ 2+(3)(2) \\ (2)(2)+2 \end{pmatrix} = \begin{pmatrix} 7 \\ 8 \\ 6 \end{pmatrix}$ **Intersection point is** $(7,8,6)$
2	Let $y=f(x)$ $y=e^{\sin^{-1}2x}$ $\ln y=\sin^{-1}2x$ diff. w.r.t. x $\frac{1}{y}\frac{dy}{dx}=\frac{2}{\sqrt{1-4x^2}}$ $\sqrt{1-4x^2}\frac{dy}{dx}=2y$ diff. w.r.t. x $\frac{-4x}{\sqrt{1-4x^2}}\frac{dy}{dx}+\sqrt{1-4x^2}\frac{d^2y}{dx^2}=2\frac{dy}{dx}$ $-4x\frac{dy}{dx}+(1-4x^2)\frac{d^2y}{dx^2}=2\sqrt{1-4x^2}\frac{dy}{dx}$ $(1-4x^2)\frac{d^2y}{dx^2}=4x\frac{dy}{dx}+2(2y)$ $(1-4x^2)\frac{d^2y}{dx^2}=4x\frac{dy}{dx}+4y$ (shown) diff. w.r.t. x $(1-4x^2)\frac{d^3y}{dx^3}+\frac{d^2y}{dx^2}(-8x)=4x\frac{d^2y}{dx^2}+\frac{dy}{dx}(4)+4\frac{dy}{dx}$ $(1-4x^2)\frac{d^3y}{dx^3}=12x\frac{d^2y}{dx^2}+8\frac{dy}{dx}$

	$f(0)=1$ $f'(0)=2$ $f''(0)=4$ $f'''(0)=16$ $f(x)=1+2x+2x^2+\frac{8}{3}x^3+\ldots$ $e^{\sin^{-1}2x}=e^{\frac{\pi}{6}}$ $\sin^{-1}2x=\frac{\pi}{6}$ $2x=\frac{1}{2}$ $x=\frac{1}{4}$ $e^{\frac{\pi}{6}}=1+2\left(\frac{1}{4}\right)+2\left(\frac{1}{4}\right)^2+\frac{8}{3}\left(\frac{1}{4}\right)^3$ $=1\frac{2}{3}$
3i	$x=1+\sin u$ diff. w.r.t. x $\frac{dx}{du}=\cos u$ $\int\sqrt{2x-x^2}\,dx$ $=\int\sqrt{2(1+\sin u)-(1+\sin u)^2}\cos u\ du$ $=\int\sqrt{2+2\sin u-1-2\sin u-\sin u^2}\cos u\ du$ $=\int\sqrt{1-\sin^2 u}\cos u\ du$ $=\int\cos^2 u\ du$ $=\int\frac{\cos 2u+1}{2}\ du$ $=\frac{\sin 2u}{4}+\frac{1}{2}u+c$ $=\frac{2\sin u\cos u}{4}+\frac{1}{2}u+c$ $=\frac{(x-1)}{2}\sqrt{2x-x^2}+\frac{1}{2}\sin^{-1}(x-1)+c$

ii a)



Area $R = 2\left[\int_0^1 |x| + 2 - \sqrt{2|x| - x^2}\,dx - \int_0^1 |x|^9 + |x|\,dx\right]$

$= 2\left[\int_0^1 x + 2 - \sqrt{2x - x^2}\,dx - \int_0^1 x^9 + x\,dx\right]$

$= 2\left[\frac{x^2}{2} + 2x - \frac{x-1}{2}\sqrt{2x - x^2} + \frac{1}{2}\sin^{-1}(x-1) - \frac{x^{10}}{10} - \frac{x^2}{2}\right]_0^1$

$= 2\left[2 - \frac{1}{10} + \frac{\pi}{4}\right]$

$= \frac{19}{5} - \frac{\pi}{2}$

b) Vol. of revolution formed when R is rotated completely about x-axis

$= 2\pi\left[\int_0^1 \left(x + 2 - \sqrt{2x - x^2}\right)^2 dx - \int_0^1 \left(x^9 + x\right)^2 dx\right]$

$= 14.995$

4 (i) Method 1:

$line\ l : \mathbf{r} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}$

Since (1,1,0) lies on p_2,

$\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ a \\ 1 \end{pmatrix} = 1$

$-1 + a = 1$

$a = 2$

Method 2:

Since direction vector of line l is perpendicular to normal of p_2,

4(ii)

$$\begin{pmatrix}1\\2\\-3\end{pmatrix}\cdot\begin{pmatrix}-1\\a\\1\end{pmatrix}=0$$

$$-1+2a-3=0$$

$$a=2$$

Method 3:

$$\begin{pmatrix}-1\\a\\1\end{pmatrix}\times\begin{pmatrix}1\\1\\1\end{pmatrix}=\begin{pmatrix}a-1\\2\\-1-a\end{pmatrix}$$

$$a-1=1$$

$$a=2$$

Method 1: Normal method

$$p_1:\mathbf{r}\cdot\begin{pmatrix}1\\1\\1\end{pmatrix}=2$$

$$line\ AN:\mathbf{r}=\begin{pmatrix}0\\0\\1\end{pmatrix}+\mu\begin{pmatrix}1\\1\\1\end{pmatrix}$$

$$\begin{pmatrix}0+\mu\\0+\mu\\1+\mu\end{pmatrix}\cdot\begin{pmatrix}1\\1\\1\end{pmatrix}=2$$

$$\mu+\mu+1+\mu=1$$

$$\mu=\frac{1}{3}$$

$$\overrightarrow{ON}=\begin{pmatrix}0\\0\\1\end{pmatrix}+\left(\frac{1}{3}\right)\begin{pmatrix}1\\1\\1\end{pmatrix}=\left(\frac{1}{3}\right)\begin{pmatrix}1\\1\\4\end{pmatrix}$$

Method 2: Projection Method

$$\overrightarrow{AN}=\left(\overrightarrow{AB}.\frac{1}{\sqrt{3}}\begin{pmatrix}1\\1\\1\end{pmatrix}\right)\left(\frac{1}{\sqrt{3}}\begin{pmatrix}1\\1\\1\end{pmatrix}\right)$$

$$=\left(\begin{pmatrix}1\\1\\-1\end{pmatrix}\cdot\frac{1}{\sqrt{3}}\begin{pmatrix}1\\1\\1\end{pmatrix}\right)\left(\frac{1}{\sqrt{3}}\begin{pmatrix}1\\1\\1\end{pmatrix}\right)$$

$$=\left(\frac{1+1-1}{3}\right)\begin{pmatrix}1\\1\\1\end{pmatrix}=\frac{1}{3}\begin{pmatrix}1\\1\\1\end{pmatrix}$$

<table>
<tr><td></td><td>

$\overrightarrow{AN} = \overrightarrow{ON} - \overrightarrow{OA}$

$\overrightarrow{ON} = \overrightarrow{AN} + \overrightarrow{OA}$

$= \frac{1}{3}\begin{pmatrix}1\\1\\1\end{pmatrix} + \begin{pmatrix}0\\0\\1\end{pmatrix} = \frac{1}{3}\begin{pmatrix}1\\1\\4\end{pmatrix}$

</td></tr>
<tr><td>4 (iii)</td><td>

Let the acute angle between planes p_1 and p_2 be θ.

$\sin\theta = \left|\frac{\begin{pmatrix}1\\1\\1\end{pmatrix}\cdot\begin{pmatrix}-1\\2\\1\end{pmatrix}}{\left|\begin{pmatrix}1\\1\\1\end{pmatrix}\right|\left|\begin{pmatrix}-1\\2\\1\end{pmatrix}\right|}\right|$

$= \left|\frac{-1+2+1}{\sqrt{3}\sqrt{6}}\right| = \frac{2}{\sqrt{18}}$

$\theta = 28.1^\circ$

</td></tr>
<tr><td>4 (iv)</td><td>

Distance from Origin to $p_1 = \frac{|2|}{\left|\begin{pmatrix}1\\1\\1\end{pmatrix}\right|} = \frac{2}{\sqrt{3}}$

Distance from Origin to $p_3 = \frac{|b|}{\left|\begin{pmatrix}2\\2\\2\end{pmatrix}\right|} = \frac{b}{\sqrt{12}}$ $(b > 0)$

$\frac{b}{\sqrt{12}} - \frac{2}{\sqrt{3}} = \sqrt{3}$

$\frac{b}{\sqrt{12}} = \frac{2}{\sqrt{3}} + \sqrt{3}$

$b = 4 + 6 = 10$

</td></tr>
<tr><td>4 (v)</td><td>

p_1, p_2 and p_4 meet in a line *l*.

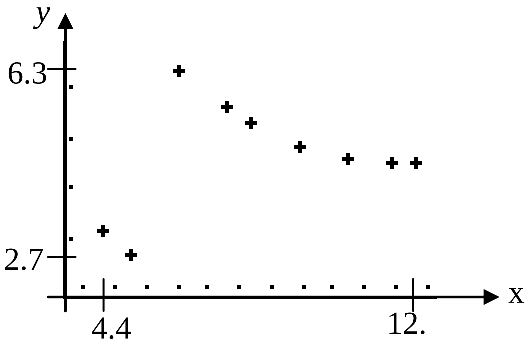
Hence (1,1,0) lies on p_4.

$\begin{pmatrix}1\\1\\0\end{pmatrix}\cdot\begin{pmatrix}c\\2\\3\end{pmatrix} = d$

$c + 2 = d$

Normal of p_4 is perpendicular to line *l*.

</td></tr>
</table>

(vi)	$\begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} c \\ 2 \\ 3 \end{pmatrix} = 0$ $c + 4 - 9 = 0$ $c = 5$ $\therefore d = 7$ $c = 5\ ,\ \ d \neq 7$
5	**Randomly** choose **10 programmers**, **1 secretary** and **1 section head** for the sample. Stratified sampling guarantees a **representative sample** of each group (i.e. programmers, secretaries and section heads) in the population. CANNOT accept any of the following answers: ♦ "… allows the opinions of different strata to be considered separately." ♦ "… accurate …" ♦ "… unbiased …"
6 (a) (i)	4 boys and 3 girls _ B _ B _ B _ B _ $4! \times {}^5P_3 = 24 \times 60$ $= 1440$ Or $4! \times ({}^5C_3 \times 3!) = 24 \times 60 = 1440$
(ii)	G BBBB GG type so $4! \times 4! = 576$
(b)	Case (i): 3 boys and 3 girls G B B G G B (arranged in a circle) ${}^4C_3 \times (3-1)! \times 3!$ $= 4 \times 2 \times 6 = 48$ Case (ii): 4 boys and 2 girls G B B B G B (arranged in a circle) $(4-1)! \times {}^3C_2 \times {}^4P_2$ $= 6 \times 3 \times 12$ $= 216$ Or $(4-1)! \times {}^3C_2 \times {}^4C_2 \times 2! = 216$ Total number of arrangements = 48 + 216 = 264
7	 Plot of y against x. Highest y = 6.3; Lowest y = 2.7 Highest x = 12.3; Lowest x = 4.4 Correlation coefficient = r = 0.263 A linear model is <u>not</u> appropriate as the **scatter diagram** shows that the **points are not close to a straight line** and the **value of *r* is quite close to 0**.

7	For $y = ax^2 + b$, value of $r = -0.913$ For $y = a \ln x + b$, value of $r = -0.971$ Therefore, (b) $y = a \ln x + b$ is a better model as the value of r in (b) is **closer to −1** than for (a). Line of regression is $y = 11.042\,948\,39 - 2.687\,231\,256 \ln x$ When $y = 6.1$, $\ln x = 1.839\,420\,548$ $\Rightarrow x = 6.29$ (3 sf) Accept $x = 6.3$ (1 dec place) This estimate is <u>valid</u> since the value of ***r* is close to −1** and the **value of *y* used in within the range of experimental data** ($4.5 \le y \le 6.3$). Or, may say that x comes from **interpolation** instead of "within the range of experimental data".
8(a)	Let X be the number of cars arriving at the jetty in 30 mins $X \sim Po(6)$ (i) $P(X=2)=0.0446$ (ii) $P(X \le 2)=0.0620$
8(b)	Let Y be the number of cars arriving at the jetty in 20 mins $Y \sim Po(4)$ $P(Y>k)<0.1$ $P(Y \le k)>0.9$ $Least\, k=7$ | k | $P(Y \le k)$ | | 6 | 0.88933 | | 7 | 0.94887 | | 8 | 0.97864 |
9 (a)	Let X be the time of journey from Town A to Town B. Let $X \sim N(\mu, \sigma^2)$ $P(X>60)=\frac{1}{4}$ $P(X>70)=\frac{1}{20}$ $P\left(Z>\frac{60-\mu}{\sigma}\right)=\frac{1}{4}$ $P\left(Z>\frac{70-\mu}{\sigma}\right)=\frac{1}{20}$ $\frac{60-\mu}{\sigma}=0.67449$ $\frac{70-\mu}{\sigma}=1.\,64485$ Solve: $\mu=53.0491 \approx 53$ $\sigma=10.3054 \approx 10$ (nearest minute)
(b)	Let Y be the time of journey from Town B to Town C. $Y \sim N(80, 9)$ $P(Y>k) \le 0.02$ $k \ge 86.1612$ Min $k=87$ mins Last time of departure 10 33h

	$\bar{Y} \sim N\left(80, \frac{9}{20}\right)$ $P\left(\left\|\bar{Y}-80\right\|<2\right) = P\left(78<\bar{Y}<82\right)=0.997$
10	Let L be the length and B be the breadth of a tile $L \sim N\left(18.9, 0.3^2\right) \qquad B \sim N\left(8.9, 0.1^2\right)$ $Let\ P=2L+2B$ $E\left(P\right)=2(18.9)+2(8.9)=55.6$ $Var\left(P\right)=4(0.3^2)+4(0.1^2)=0.4$ $E\left(P_1+P_2+P_3+\ldots\ldots\ldots\ldots+P_{10}\right)=10(55.6)=556$ $Var\left(P_1+P_2+P_3+\ldots\ldots\ldots\ldots+P_{10}\right)=10(0.4)=4$
	Let W be the number of tiles with a red tint. $W \sim B\left(500, 0.6\right)$ $P\left(W \geq k\right) \geq 0.95$ $np=300>5\ and\ nq=200>5, n$ is large Use Normal approximation: $W \sim N\left(300, 120\right)\ approximately$ $P\left(W \geq k\right) \geq 0.95$ $P\left(W<k\right) \leq 0.05$ $P\left(W<k-\frac{1}{2}\right) \leq 0.05$ $k-\frac{1}{2}<281.98$ $k<282.48$ Greatest $k=282$
11	To test: $H_0 : \mu = 27$ $H_1 : \mu > 27$ at 2% level Under $H_0 : T = \frac{\bar{X}-27}{s/\sqrt{27}} \sim t\left(26\right)$ Test statistic = 1.22474 p-value = 0.1158 >0.02 Do not reject H_0 There is **insufficient evidence at the 2% level of significance** to conclude **the mean foot length** of an 18 year old man of high intelligence is **more than 27cm** Assumption: Assume the population of foot length of 18 year old man of high intelligence is normally distributed

To test: $\mathrm{H}_0 : \mu = k$

$\mathrm{H}_1 : \mu \neq k$ at 4% level

Under H_0 by Central limit theorem, since n is large

$$\bar{X} \sim N\left(k, \frac{s^2}{60}\right) approx, where\ s^2 = \frac{1}{59}(123.20) = 2.088136$$

To reject H_0 :

$$2P\left(Z < \frac{\bar{x}-k}{s/\sqrt{60}}\right) < 0.04$$

$$P\left(Z < \frac{\bar{x}-k}{s/\sqrt{60}}\right) < 0.02$$

$$\frac{\bar{x}-k}{s/\sqrt{60}} < -2.05375 \quad or \quad \frac{\bar{x}-k}{s/\sqrt{60}} > 2.05375$$

$$\frac{26.6-k}{\sqrt{2.088136}/\sqrt{60}} < -2.05375 \quad or \quad \frac{26.6-k}{\sqrt{2.088136}/\sqrt{60}} > 2.05375$$

$k > 26.9831$ $\quad or\ k < 26.21687$

$k > 27.0 \quad or\ k < 26.2$

12 (i)

Let X be the r.v. "number of shots, out of 20, that hit the target". X ~ B(20, 0.25)

X	Y1	
0	.00317	
1	.02114	
2	.06695	
3	.1339	
4	.18969	
5	.20233	
6	.16861	

Press + for ΔTbl

Most likely number of shots is 5.

12 (ii)

P($X \geq 5 \mid X \leq 10$)

$$= \frac{\mathrm{P}(5 \leq X \leq 10)}{\mathrm{P}(X \leq 10)}$$

$$= \frac{\mathrm{P}(X \leq 10) - \mathrm{P}(X \leq 4)}{\mathrm{P}(X \leq 10)}$$

$$= \frac{0.5812163575}{0.9960578583} = 0.584 \text{ (3 sf)}$$

12 (iii)	P(same number of shots each to hit target) $= P(1, 1) + P(2, 2) + P(3, 3) + \ldots$ $= 0.25^2 + (0.75 \times 0.25)^2 + (0.75 \times 0.75 \times 0.25)^2 + \ldots$ $= 0.25^2\ (1 + 0.75^2 + 0.75^4 + 0.75^6 + \ldots)$ $= 0.0625 \times \dfrac{1}{1-0.75^2}$ $= 0.0625 \times \dfrac{1}{0.4375} = \dfrac{1}{7}$ or 0.143 (3 sf)