A1: QUADRATIC FUNCTIONS

- Finding the maximum or minimum value of a quadratic function using the method of completing the square
- Conditions for $y = ax^2 + bx + c$ to be always positive (or always negative)
- Using quadratic functions as models

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1. (a) Show that $f(x) = 3x^2 - 9x + 7$ can be written as $f(x) = a(x - b)^2 + c$, where a, b and c are constants to be found.	[4]
1. (b) Hence, explain why the function $f(x)$ is always positive.	[1]
2. (a) Express $-2x^2 - 4x + 3$ in the form $a(x - h)^2 + k$, where a, h and k are constants.	[3]
2. (b) Hence, state the maximum value of the curve $y =- 2x^2 - 4x + 3$.	[1]
3. (a) Express $6x^2 - 12x + 7$ in the form $p(x - q)^2 + r$, where p, q and r are constants to be found.	[3]
3. (b) Hence find the greatest value $\left(6x^2 - 12x + 7\right)^{-1}$ and state the value of x at which this occurs.	[2]
4. (a) Express $x^2 - 8x + 5$ in the form $(x + a)^2 + b$ where a and b are constants.	[2]
4. (b) Hence state the line of symmetry and the coordinates of the vertex of the curve $y = x^2 - 8x + 5$.	[2]
5. (a) Show that $g(x) =- x^2 + 6x - 14$ can be written as $g(x) = a(x - b)^2 + c$ where a, b and c are constants.	[3]
5. (b) Explain why the function $g(x)$ is always negative.	[2]

A1: QUADRATIC FUNCTIONS (MARKING SCHEME)

1. (a) Show that $f(x) = 3x^2 - 9x + 7$ can be written as $f(x) = a(x - b)^2 + c$, where $a,\ b$ and c are constants to be found. [4]

$f(x) = 3x^2 - 9x + 7$ ★ factorise so that coefficient of x^2 is 1

$f(x) = 3\left(x^2 - 3x\right) + 7$ ★ you can choose to keep 7 untouched, less error

$f(x) = 3\left[x^2 - 2\left(\frac{3}{2}x\right) + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2\right] + 7$

$f(x) = 3\left[\left(x - \frac{3}{2}\right)^2 - \frac{9}{4}\right] + 7$

$f(x) = 3\left(x - \frac{3}{2}\right)^2 - \frac{27}{4} + 7$

$f(x) = 3\left(x - \frac{3}{2}\right)^2 + \frac{1}{4}$ (shown)

1. (b) Hence, explain why the function $f(x)$ is always positive. [1]

$\therefore$ Since $3\left(x - \frac{3}{2}\right)^2 \geq 0, 3\left(x - \frac{3}{2}\right)^2 + \frac{1}{4} \geq \frac{1}{4}$ which is more than 0.

2. (a) Express $-\ 2x^2 - 4x + 3$ in the form $a(x - h)^2 + k$, where a, h and k are constants. [3]

$-\ 2x^2 - 4x + 3$ ★ factorise so that coefficient of x^2 is 1

$=-\ 2(x^2 + 2x) + 3$ ★ you can choose to keep 3 untouched, less error

$=-\ 2\left[x^2 + 2(x) + 1^2 - 1^2\right] + 3$

$=-\ 2\left[(x + 1)^2 - 1\right] + 3$

$=-\ 2(x + 2)^2 + 2 + 3$

$=-\ 2(x + 1)^2 + 5$

$\therefore -\ 2(x + 1)^2 + 5$

2. (b) Hence, state the maximum value of the curve $y =-\ 2x^2 - 4x + 3$. [1]

$\therefore$ Maximum value is 5.

3. (a) Express $6x^2 - 12x + 7$ in the form $p(x - q)^2 + r$, where p, q and r are constants to be found. $6x^2 - 12x + 7$ ★ factorise so that coefficient of x^2 is 1 $= 6[(x^2 - 2x)] + 7$ you can choose to keep 7 untouched, less error $= 6[x^2 - 2(x) + 1^2 - 1^2] + 7$ $= 6[(x - 1)^2 - 1] + 7$ $= 6(x - 1)^2 - 6 + 7$ $= 6(x - 1)^2 + 1$ $\therefore 6(x - 1)^2 + 1$	[3]
3. (b) Hence find the greatest value $(6x^2 - 12x + 7)^{-1}$ and state the value of x at which this occurs. Greatest value $= \frac{1}{6x^2-12x+7} = \frac{1}{1} = 1$ $x = 1$ $\therefore x = 1$	[2]
4. (a) Express $x^2 - 8x + 5$ in the form $(x + a)^2 + b$ where a and b are constants. $x^2 - 8x + 5$ $= [x^2 - 2(4x) + 4^2 - 4^2] + 5$ $= (x - 4)^2 - 16 + 5$ $= (x - 4)^2 - 11$ $\therefore (x - 4)^2 - 11$	[2]
4. (b) Hence state the line of symmetry and the coordinates of the vertex of the curve $y = x^2 - 8x + 5$. Line of symmetry, $x = 4$ Coordinates of the vertex of the curve, $(4, -11)$	[2]

5. (a) Show that $g(x) =- x^2 + 6x - 14$ can be written as $g(x) = a(x - b)^2 + c$ where $a,\ b$ and c are constants. $g(x) =- x^2 + 6x - 14$ ★ factorise so that coefficient of x^2 is 1 $g(x) =- 1\left[\left(x^2 - 6x\right)\right] - 14$ ★ you can choose to keep 14 untouched, less error $g(x) =- 1\left[x^2 - 2(3x) + 3^2 - 3^2\right] - 14$ $g(x) =- 1\left[(x - 3)^2 - 9\right] - 14$ $g(x) =- (x - 3)^2 + 9 - 14$ $g(x) =- (x - 3)^2 - 5$ (shown)	[3]
5. (b) Explain why the function $g(x)$ is always negative. $(x - 3)^2 \geq 0$ $- (x - 3)^2 \leq 0$ $- (x - 3)^2 - 5 \ \leq- 5$ Hence, maximum value of $g(x) = 5$, $g(x)$ is always negative.	[2]