Anglo - Chinese School

(Independent)

FINAL EXAMINATION 2018

YEAR THREE EXPRESS

ADDITIONAL MATHEMATICS

PAPER 1

Friday **5 October 2018** **1 hour 30 minutes**

Additional Materials: Answer Paper (6 Sheets)

READ THESE INSTRUCTIONS FIRST

Write your index number on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 60.

This question paper consists of 5 printed pages.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

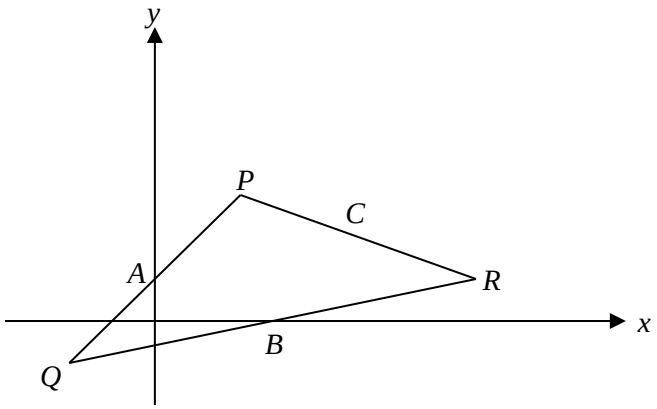
$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

Answer all the questions.

1 **Solutions to this question by accurate drawing will not be accepted.**

In the diagram below, $A(0, 2)$, $B(6, 0)$ and $C(h, k)$ are the midpoints of the sides of the triangle PQR, and Q is the point $(-2, -1)$. Find the coordinates of C. [4]



2 **(a)** Simplify $2\sqrt{180} + \sqrt{245} - 3\sqrt{125}$, leaving your answer in surd form [2]

(b) Solve the equation $\frac{x}{\sqrt{1-3x}} = \frac{1}{2}$. [3]

3 When the graph of $xy + x$ against y is drawn, a straight line is obtained. Given that the line passes through the points $(5, -2)$ and $(1, 4)$, find

(i) y in terms of x, and [4]

(ii) the value of x when $y = 2$. [2]

4 A rectangle has a length of 2α cm and a width of 2β cm, where α and β are the roots of a quadratic equation.

(i) Given that the equation $x^2 - 53x + 196 = 0$ has roots α^2 and β^2, find the quadratic equation in x whose roots are α and β. [5]

(ii) Hence, find the dimensions of the rectangle. [2]

5 The circle $C_1 : x^2 + y^2 - 2x + 8y = 23$ passes through the point $P(-5, -2)$.

(i) Find the equation of the line *PQ* which is a tangent to the circle at *P*. [3]

(ii) Determine if the point $R(-3,1)$ lies within C_1. [3]

(iii) Find the equation of circle C_2 which is a reflection of C_1 about the line $x = -3$. [2]

6 **Answer the whole of this question on a single sheet of graph paper.**

An experiment was done to estimate the population of bacteria, *P*, present in a culture at time *T* days after the start of the experiment. The results are shown in the table below.

Time, *T* (days)	3	5	10	20	40
Population of Bacteria, *P*	900	1884	10000	16000	47315

It is known that *P* and *T* are related by the equation $P = aT^b$, where a and b are constants.

(i) Using 4 cm to represent 1 unit on the vertical axis and 2 cm to represent 0.2 units on the horizontal axis, plot the graph of *lg P* against *lg T*. [2]

(ii) Use your graph to

(a) estimate the value of *a* and of *b*, [3]

(b) identify the abnormal reading and estimate its correct value, [2]

(c) estimate the population of bacteria after 4 days. [2]

7 **(a)** **(i)** Solve the equation $|x-2| = 3$. [2]

(ii) Sketch the graph of $y = -|x-2| + 3$, labelling clearly the intercepts and the vertex.

[2]

(iii) Find the equation of the straight line which is to be inserted in the diagram in **(ii)** in order to solve the equation $5 + 4x = |x-2|$. [2]

(b) Sketch the graph of $y = \cos 2x$ for $0 \le x \le \pi$. By adding a suitable straight line, state the number of solutions to the equation $\pi - 2x = 2\pi \cos 2x$. [4]

8 **(a)** Simplify $\dfrac{\log_n m \times \log_m n^2}{\log_{m^2} m}$. [2]

(b) It is given that $\log_2 x = r$ and $\log_4 y = s$. If $x^2 y = 64$ and $xy^2 = 128$, determine the value of *r* and of *s*. [6]

(c) Solve $\dfrac{1}{\log_{(5x-e)} e} = 1 + \ln x$, leaving your answer in terms of *e*. [3]

END OF PAPER ONE

Answer Key

1 (8, 3)

2 (a) $4\sqrt{5}$ (b) $x = \dfrac{1}{4}$

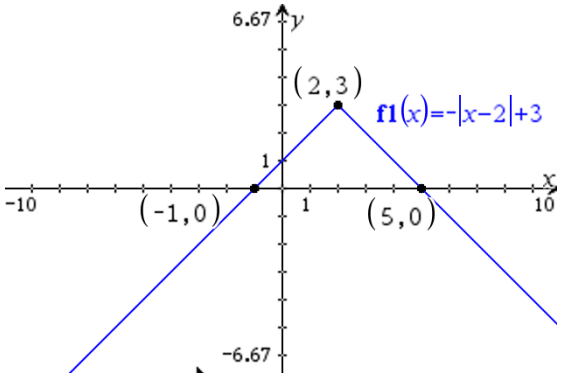
3 (i) $y = \dfrac{11-2x}{2x+3}$ (ii) $x = \dfrac{5}{6}$

4 (i) $x^2 - 9x + 14 = 0$ (ii) 4 cm by 14 cm

5 (i) $y = 3x + 13$ (ii) R lies outside of C_1 (iii) $x^2 + y^2 + 14x + 8y + 25 = 0$

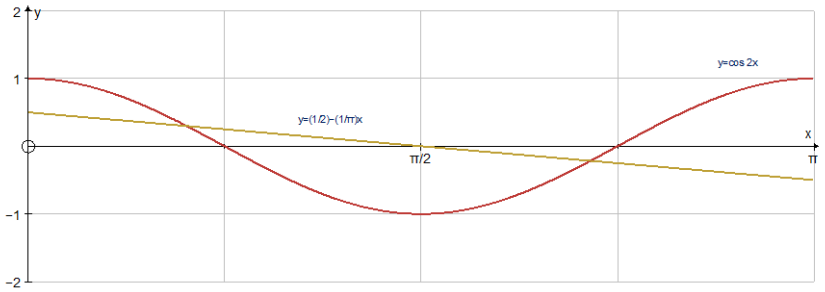
6 (ii) (a) a = 158.5, b = 1.52 (b) 5623 (c) 1413

7 (a)(i) $x = 5$ or $x = -1$ (ii)



(iii) $y = -2 - 4x$

7 (b) 2 solutions



8 (a) 4 (b) $s = \dfrac{4}{3}$; $r = \dfrac{5}{3}$ (c) $x = \dfrac{e}{5-e}$