1. C

2. B

3. (a) When the boat is in equilibrium,
By Principle of Floatation:
$U = m_s g$
$= 1.2 \times 10^7 (9.81)$
$= 1.18 \times 10^8$ N

(b) By Archimedes's Principle:
$U = \rho_w V_{displaced} g = 1.18 \times 10^8$
$(1000) V_{displaced} (9.81) = 1.18 \times 10^8$
$V_{displaced} = 1.20 \times 10^4$ m^3
Volume of boat below the water line = $V_{displaced} = 1.20 \times 10^4$ m^3

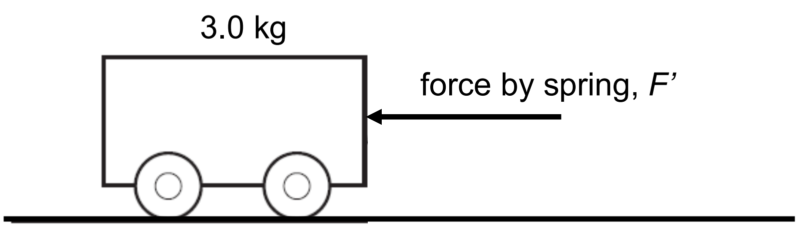
4. (a) $k_A = 6.0$ N m^{-1}
$k_B = 3.0$ N m^{-1}
Tension in A and B = 0.6 N

extension in spring A = (0.6 / 6) = 0.10 m
extension in spring B = (0.6 / 3) = 0.20 m

(b) Total extension = 0.20 + 0.10 = 0.30 m
Hence, the effective spring constant of A and B = (0.60 / 0.3) = 2.0 N m^{-1}

5. Acceleration of both trolleys = $\frac{20}{5.0} = 4.0$ m s^{-2}

Consider the 3.0 kg trolley:



$F' = 3.0 \times 4.0 = 12$ N

Compression of spring = $\frac{12}{4.0} = 3.0$ cm Ans: B

6. <u>Method 1</u>

Let the spring constant of *each* spring be *k*.
In the first case, the downward force on *each* spring is *W*/3.
So, the spring constant is given by:

$$\frac{W}{3} = kx \qquad \Rightarrow \qquad k = \frac{W}{3x}$$

In the second case, the force on *each* spring is (2*W*)/2 = *W*
Extension *e* is given by:

$$W = ke \qquad \Rightarrow \qquad e = \frac{W}{k} = \frac{W}{W/(3x)} = 3x$$

<u>Method 2</u>

Let the spring constant of each spring be *k*.
Using the formula for parallel springs,

In the first case, $k_{eff} = k_1 + k_2 + k_3 = k + k + k = 3k$

Using Hooke's Law, "$F_1 = k_1 x_1$"

$W = 3k\,x$ ---------------- (1)

In the second case, $k_{eff} = k_1 + k_2 = k + k = 2k$

Using Hooke's Law, "$F_2 = k_2 x_2$"

$2W = 2k\,x_2$ ---------------- (2)

Sub (1) into (2)

$2(3k\,x) = 2k\,x_2$

$x_2 = 3x$

7. Method 1

Let k be the spring constant of the spring.

By Hooke's Law, $k = \frac{F}{x} = \frac{20}{2.0 \times 10^{-2}} = 1000 \text{ N m}^{-1}$

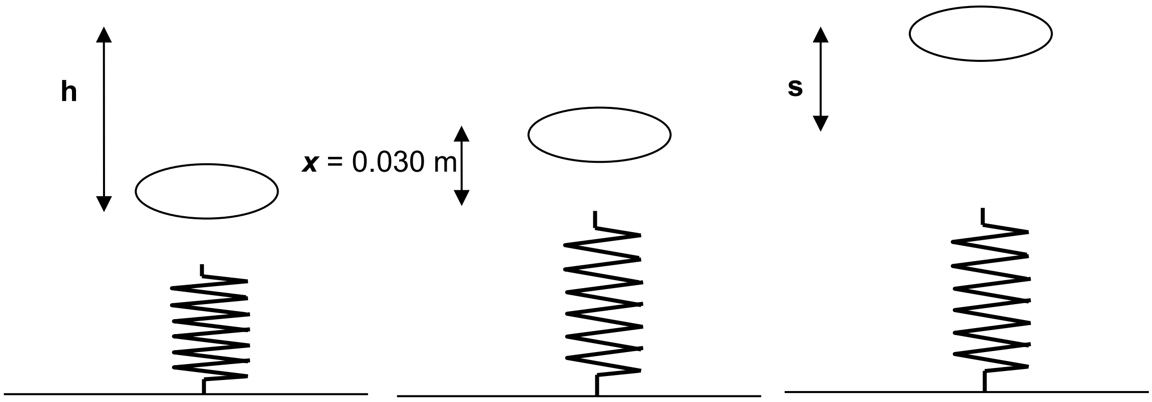
Elastic potential energy (PE) stored in the spring when it is compressed by 3.0 cm:

$U = \frac{1}{2}kx^2 = \frac{1}{2}(1000)(0.030)^2 = 0.45$ J

When the toy reaches its maximum height h above its point of release,

Gain in gravitational PE = Loss in elastic PE

mgh = 0.45 J $\Rightarrow$ h = 0.917 m



Position 1 **Position 2** **Position 3**

(At max compression) (when toy *just* leaves spring) (At max height)

Method 2

Energy conversion from Position 1 to Position 2

Gain in GPE + Gain in KE = Loss in EPE

mgx + ½ mu^2 = ½ kx^2

(0.050) (9.81) (0.030) + ½ (0.050)u^2 = ½ (1000)$(0.030)^2$

u = 4.17 m s^{-1} (upwards) where u is the velocity of the toy which *just* leaves the spring

Using equation of motion between Position 2 and Position 3,

Taking upwards as positive,

$v^2 = u^2 + 2as$

$0^2 = (4.17)^2 + 2(-9.81)s$

s = 0.886 m

h = x + s = 0.030 + 0.886 = 0.916 m

8. (a) When the block is submerged in water, there is a pressure difference between the top and bottom surfaces of the block, with the water pressure being greater at the bottom surface of the block.

Since pressure is force per unit area, the force the water exerts on the bottom surface of the block is greater than the force it exerts on the top surface of the block. This results in a net upward force the water exerts on the block which is the force of upthrust.

8. (bi) By Archimedes' Principle,

$$U = \rho_w V_{block} g$$
$$= 1.0 \times 10^3 (27.8 \times 10^{-6})(9.81)$$
$$= 0.272718 \text{ N}$$
$$\approx 0.27 \text{ N}$$

8. (bii) From Fig. 8.1:

Since the system is in equilibrium, taking moments about the pivot
Sum of clockwise moments = Sum of anticlockwise moments

$$0.18F = 0.083W$$

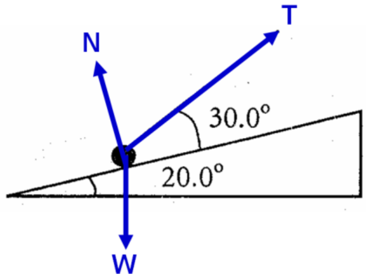
$W(0.083) = F(0.180)$ ----- (1)

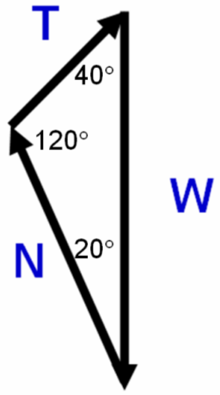
From Fig. 8.2:

Since the system is in equilibrium,
Sum of clockwise moments = Sum of anticlockwise moments
$W(0.078) = (F - U)(0.19)$ ----- (2)

Sub (1) into (2): $F = 2.49\ N$

9. Method1:





Using sine rule:

$$\frac{T}{\sin 20^\circ} = \frac{(2.00)(9.81)}{\sin 120^\circ}$$
$$T = 7.75\ \text{N}$$

$$\frac{N}{\sin 40^\circ} = \frac{(2.00)(9.81)}{\sin 120^\circ}$$
$$N = 14.6\ \text{N}$$

Method 2:

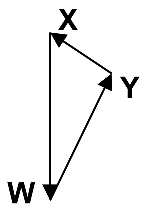
Let *x* represent the direction along the slope; ↗ as positive.
Let *y* represent the direction perpendicular to the slope; ↖ as positive.

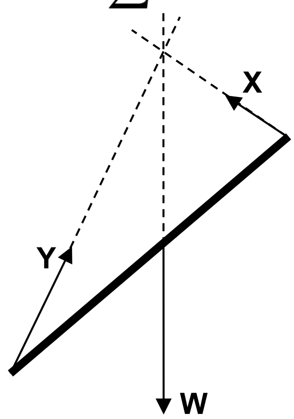
For translational equilibrium, $\sum F_x = 0$ and $\sum F_y = 0$.

$\sum F_x = 0$ $T\cos 30^\circ - W\sin 20^\circ = 0$ $T = \dfrac{W\sin 20^\circ}{\cos 30^\circ} = 7.75\ \text{N}$	$\sum F_y = 0$ $N + T\sin 30^\circ - W\cos 20^\circ = 0$ $N = 14.6\ \text{N}$

10. (a) X: Resultant of normal contact force and frictional force by wall on ladder. Friction is upwards as ladder tends to slide down.
Y: Resultant of normal contact force and frictional force by ground on ladder. Friction is to the right as ladder sliding down and to the left.
W: Gravitational force of Earth on ladder, called the weight.

(b) $\sum \vec{F} = 0$ $\sum \vec{\tau} = 0$



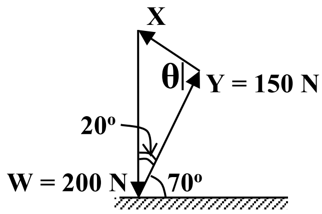


(c) Using the cosine rule,

$X^2 = W^2 + Y^2 - 2WY\cos 20^0$

$X^2 = 200^2 + 150^2 - 2(200)(150)\cos 20^0$

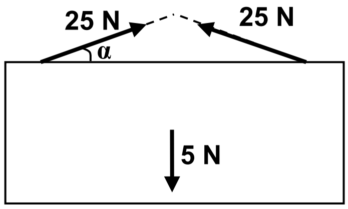
$X = 78.2$ N



11. (a) Considering the free body of the picture:
Considering forces acting along the vertical axis:
Since in equilibrium,

$2(25 \sin \alpha) = 5$

$\alpha = 5.74^\circ$



(b) $2 T \sin \alpha$ = weight of picture = 5 N
So when the tension T = breaking tension which is the maximum tension, sin α and hence α will be a minimum.
{This is a <u>minimum</u> value as when *α* gets smaller, the tension will be greater than 25 N which will break the cord.}

<u>Mathematical Proof</u>

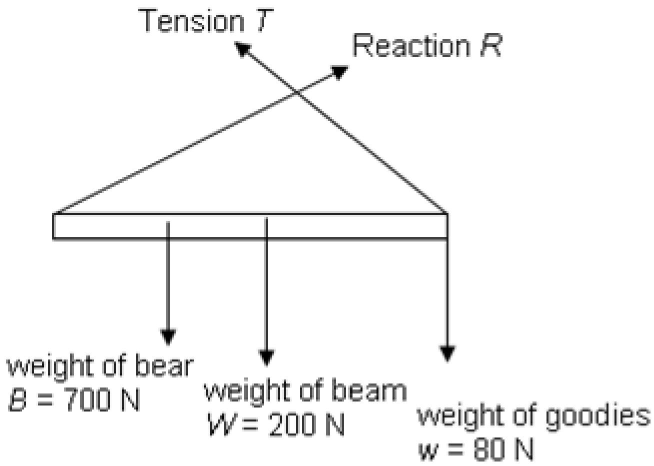
$2 T \sin \alpha = 5$

$T = \dfrac{5}{2\sin\alpha} \leq 25$

$\sin \alpha \geq 0.10$

$\alpha \geq 5.739$

12. (a)



(b) Take moments about the hinge,

$700(1) + 200(3) + 80(6) = T(6 \sin 60^0)$
$T = 343$ N

At the hinge,
$R_x = T \cos 60^0 = 172$ N
$R_y + T \sin 60^0 = 700 + 200 + 80$
$R_y = 683$ N

(c) Let the maximum distance be y
Take moments about the hinge,
$700(y) + 200(3) + 80(6) \leq 900(6 \sin 60^0)$
$y \leq 5.14$ m

13.

(a) What reason does the paragraph give for the construction with many cables?

The bridge will be better as the **load** of the structure and traffic will be **distributed** amongst the cables instead of being concentrated on a cable. [2]

(b) When a civil engineer designs the tower, he needs to consider the maximum total mass which the tower may need to support. Calculate that maximum total mass.

Maximum total mass $= 3.5 \times 10^5 + 2.9 \times 10^5 + 6.8 \times 10^4$ [1]
$= \mathbf{7.08 \times 10^5}$ kg [1]

(c) Calculate the mass of 10 m of the roadway and the maximum mass of traffic which the 10 m of roadway may have to support.

mass of 10 m of roadway $= \dfrac{10}{100} \times 3.5 \times 10^5$ kg
$= \mathbf{3.5 \times 10^4}$ kg [1]

mass of traffic on 10 m of roadway $= \dfrac{10}{100} \times 2.9 \times 10^5$ kg
$= \mathbf{2.9 \times 10^4}$ kg [1]

(d) Calculate the angle, θ, (in degree) between a cable and the horizontal.

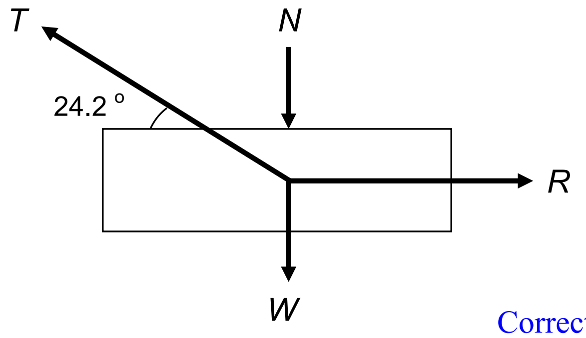
$\tan \theta = \dfrac{4.5}{10}$ [1]

$\theta = \tan^{-1}\left(\dfrac{4.5}{10}\right)$

$= \mathbf{24.2^o}$ [1]

(e) Draw a force diagram of a fully laden 10 m section of road at segment **A** given that the following forces act at the segment.

W : Weight of 10 m of roadway
N : Force exerted by traffic on 10 m of roadway
T : Tension in cable
R : Net force exerted by the rest of the roadway



Correct directions of the forces [4]

(f) By considering the conditions necessary for translational equilibrium, state the equations relating W, N, T and R.

$T \sin \theta = N + W$ [1]

$T \cos \theta = R$ [1]

(g) Hence, or otherwise, calculate the tension in a cable when the bridge is fully laden. (The tension in all cables is assumed to be the same.)

$$T = \frac{N+W}{\sin \theta} = \frac{2.9 \times 10^4 \times 9.81 + 3.5 \times 10^4 \times 9.81}{\sin 24.2}$$ [1]

$= 1.53 \times 10^6$ N [1]

(h) When the civil engineer designs the bridge, he needs to consider the possibility of vibration of the bridge.

Under what condition will the bridge vibrate with maximum amplitude? Suggest two possible external sources that will give rise to this condition.

The bridge will vibrate with maximum amplitude when the driving frequency matches the natural frequency of the bridge. [1]

This phenomenon is known as resonance. [1]

The possible external sources which can provide the driving frequency are:

(i) soldiers matching across the bridge,
(ii) vehicles moving across the bridge,
(iii) wind blowing on the bridge,
(iv) earth movement caused by earthquake in Sumatra or Java.

(i) Calculate the deflection of the bridge.

Given that the deflection, d, of the bridge is governed by

$$d = \frac{5PL^3}{384EI}$$

where P is the loading which is equal to 4.7×10^6 N.
L is the length of the bridge.

E is the Modulus of Elasticity which is equal to 8.0×10^{11} N m^{-2}

I is the Moment of Inertia which is equal to 15 m^4.

$$d = \frac{5PL^3}{384EI}$$

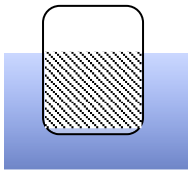
$$= \frac{5(4.7\times10^6)(100)^3}{384(8\times10^{11})(15)} \quad [1]$$

$= 0.005$ m

$= 5.0$ mm [1]

__Challenging Questions__

14. Since the ice is floating, by the principle of floatation:



$$U_{on\ ice} = W_{ice}$$
$$\rho_w V_{water\ displaced}\, g = m_{ice}\, g$$
$$\rho_w V_{water\ displaced} = m_{ice}$$

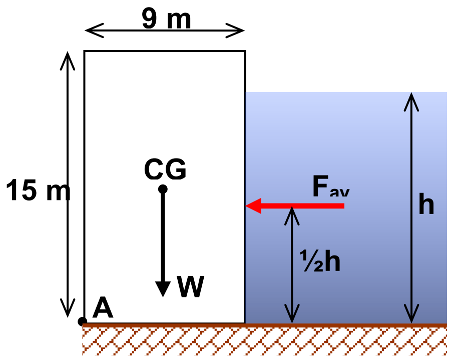
When the ice has fully melted, $m_{ice} = m_{water\ from\ melted\ ice}$

Hence,

$$\rho_w V_{water\ displaced} = m_{water\ from\ melted\ ice}$$
$$\rho_w V_{water\ displaced} = \rho_w V_{water\ from\ melted\ ice}$$
$$V_{water\ displaced} = V_{water\ from\ melted\ ice}$$

Thus the water level in the beaker remains the same.

15. Considering the situation just before the dam topples:



As the block is in equilibrium,
$\sum \vec{\tau} = 0 \Rightarrow \sum \vec{M}_{\text{Every Point}} = 0$

Taking moments about point A,
Anticlockwise moment = Clockwise moment

$$F_{av}\left(\frac{h}{2}\right) = W\left(\frac{9}{2}\right)$$

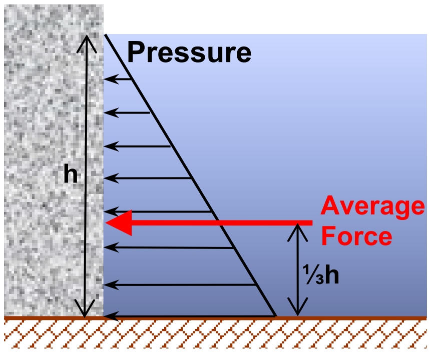
$$P_{ave} A\left(\frac{h}{2}\right) = \rho_C V_C g\left(\frac{9}{2}\right)$$

$$\left(\rho_W g \frac{h}{2}\right)(lh)\left(\frac{h}{2}\right) = \rho_C (15 \times 9 \times l) g\left(\frac{9}{2}\right)$$

$$1000(9.81)\frac{h^3}{4} l = 2200(15 \times 9 \times l)(9.81)\left(\frac{9}{2}\right) \Rightarrow h = 17.49m$$

h = 17.5 m

Since water pressure, P at a depth h is given by P = ρgh, the water pressure acting on the dam may be assumed to increase linearly with h, as the density of the water does not change significantly within a small variation of depth.



As such, the average force due to the water pressure will not act at mid-depth, but for this case of a triangular pressure distribution, it acts at ⅓ depth from the base.

Thus the overturning moment on the dam due to the water pressure assumed in part (b) is an overestimate and the height of the water calculated in part (b) is an underestimate.

For the same volume of concrete to be used, the design as shown on the right would be more efficient as

1) the restoring moment arm of the weight of the dam about point A would be larger.
2) the overturning moment arm of the water pressure about point A would be smaller, as compared to the rectangular design.