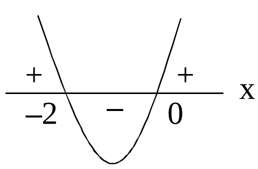
1 The mass of a radioactive substance reduces by half every 4 hours. It is given that m_0 is the mass of the substance at a particular time and that m is the mass of the substance t hours later. Calculate the value of the constant k in the relationship $m = m_0 e^{-kt}$. [3]

$m = m_0 e^{-kt}$ i.e when $t = 4$ sub $m = \frac{1}{2} m_0$ $\frac{1}{2} m_0 = m_0 e^{-4k}$ $\ln \frac{1}{2} = -4k \ln e$ $k = -\frac{1}{4} \ln \frac{1}{2}$ or $\frac{1}{4} \ln 2$ or 0.173 (3.s.f.)	Note: m_0 is a constant When t = 0, initial $m = m_0$

2 Solve the equation $3^{2x+1} - 6(3^x) = 27 - 2(3^{2x})$. [4]

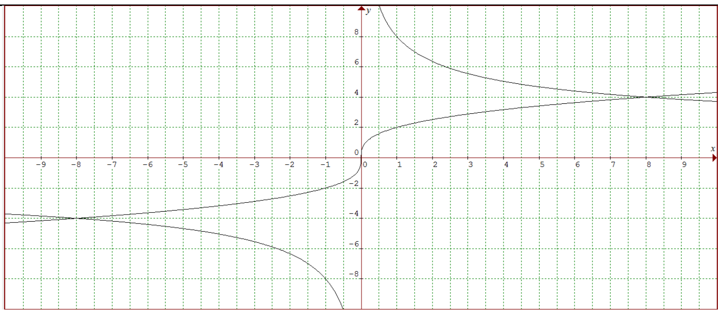
$3^{2x} \times 3 - 6(3^x) = 27 - 2(3^{2x})$ Let $u = 3^x$, $3u^2 - 6u = 27 - 2u^2$ $5u^2 - 6u - 27 = 0$ $(u-3)(5u+9) = 0$ $u = 3$, $u = -\frac{9}{5}$ $3^x = 3$, $3^x = -\frac{9}{5}$ (no solution) $x = 1$	

3 The function f is defined, for all values of x, by $f(x) = x^2 e^x$.
Find the range of values of x for which f is a decreasing function. [6]

$f'(x) = e^x \cdot 2x + x^2 e^x$ $= e^x(2x + x^2)$ $f'(x) < 0$ Since $e^x > 0$ for all real values of x, $2x + x^2 < 0$ $x(2+x) < 0$ $-2 < x < 0$	

4 **(i)** On the same diagram, sketch the curves $y = 2x^{\frac{1}{3}}$ and $y = 8x^{-\frac{1}{3}}$. [2]

(ii) Find the coordinates of the intersection points of the two curves. [3]

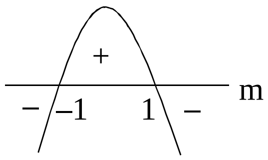
(i)		
(ii)	$2x^{\frac{1}{3}} = 8x^{-\frac{1}{3}}$ $\dfrac{x^{\frac{1}{3}}}{x^{-\frac{1}{3}}} = 4$ $x^{\frac{2}{3}} = 4$ Take cube $x^2 = 4^3 = 64$ $x = \pm 8$ When $x = 8$, $y = 4$ When $x = -8$, $y = -4$ Coordinates: (8, 4) and (–8, –4)	Note: $(x^{\frac{2}{3}})^{\frac{3}{2}} = 4^{\frac{3}{2}}$ $x = 4^{\frac{3}{2}} = 8$ Only 1 answer

5 The equation of a curve is $y = \dfrac{x-8}{5-2x}$. A particle moves along the curve in such a way that the y-coordinate of the particle is decreasing at a constant rate of 2 units per second. Find the possible y-coordinates of the particle at the instant when the x-coordinate of the particle is increasing at $1\frac{7}{11}$ units per second. [6]

$\dfrac{dy}{dx} = \dfrac{(5-2x)(1)-(x-8)(-2)}{(5-2x)^2}$ $= \dfrac{5-2x+2x-16}{(5-2x)^2}$ $= \dfrac{-11}{(5-2x)^2}$ $\dfrac{dy}{dt} = \dfrac{dy}{dx} \times \dfrac{dx}{dt}$ $-2 = \dfrac{-11}{(5-2x)^2} \times \dfrac{18}{11}$ $\dfrac{-11}{9} = \dfrac{-11}{(5-2x)^2}$	Note: mistake in expansion

$9=(5-2x)^2$ $5-2x=\pm 3$ $x=1 \quad , \quad x=4$ $y=-2\frac{1}{3} \quad , \quad y=1\frac{1}{3}$	

6 A curve has equation $y^2=-4x$ and a line has equation $y=m(x-1)$. Find the range of values of m for which the line $y=m(x-1)$ intersects the curve $y^2=-4x$ at two distinct points. [5]

$-4x=(mx-m)^2$ $-4x=m^2x^2-2m^2x+m^2$ $m^2x^2+x(4-2m^2)+m^2=0$ $b^2-4ac>0$ $(4-2m^2)^2-4(m^2)(m^2)>0$ $16-16m^2+4m^4-4m^4>0$ $1-m^2>0$ $(1-m)(1+m)>0$ $-1<m<1$	

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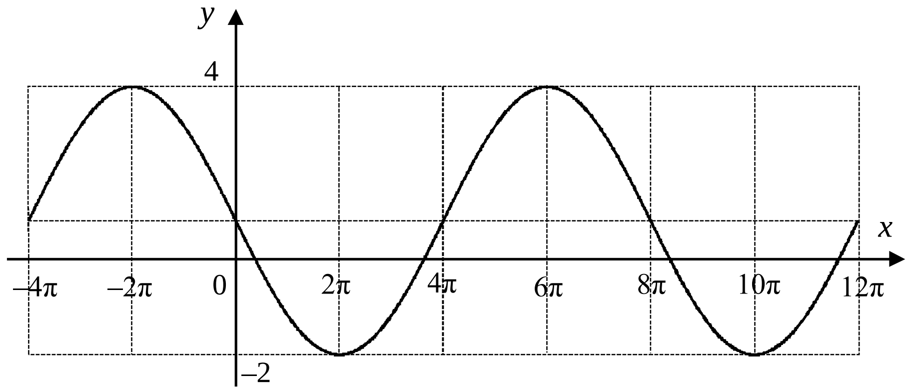


The diagram shows part of the graph of $y=a\sin\left(\frac{x}{b}\right)+c$.

(i) State the amplitude and period. [2]

(ii) Find the value of each of the constants a, b and c. [3]

(i)	Amplitude = 3 Period = 8π	Period $\neq \frac{1}{4}$
(ii)	$a=-3$ $8\pi=\frac{2\pi}{\left(\frac{1}{b}\right)}$ $\frac{1}{b}=\frac{1}{4}$	Note: a is negative based on the inverted sine curve

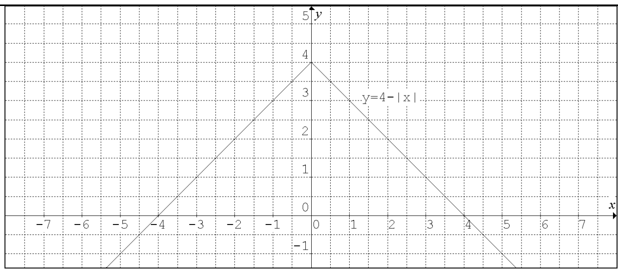
	$b = 4$ $c = 1$	

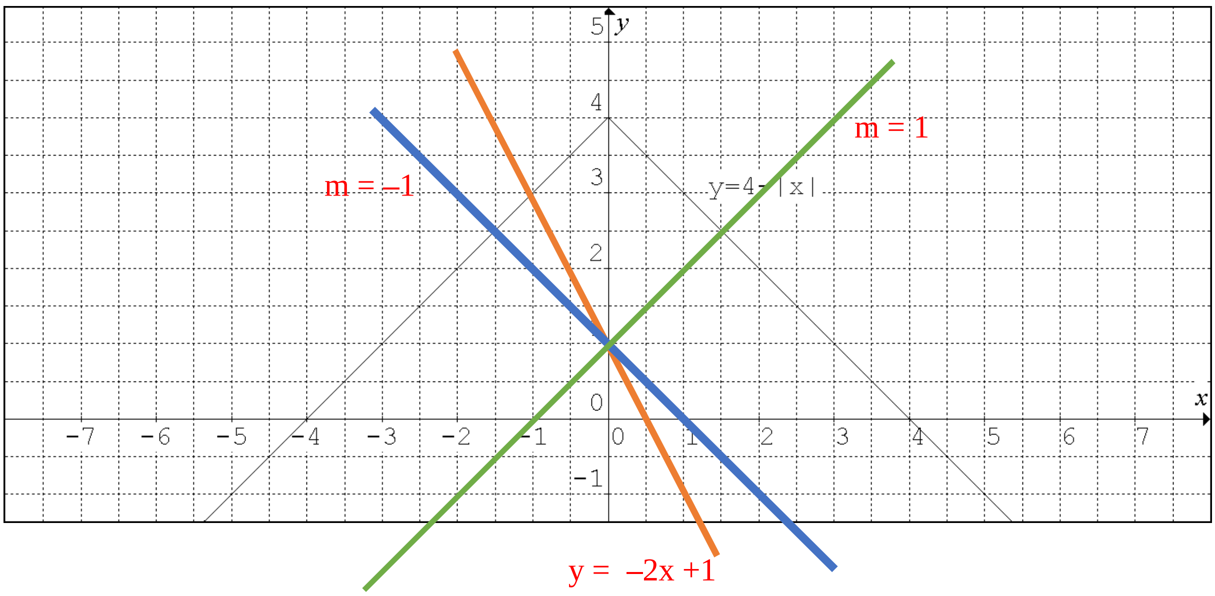
8 **(i)** Sketch the graph of $y = 4 - |x|$. [2]

A line of gradient m passes through the point (0, 1).

(ii) In the case where $m = -2$, find the coordinates of any point of intersection of the line and the graph of $y = 4 - |x|$. [3]

(iii) Determine the set of values of m for which the line intersects the graph of $y = 4 - |x|$ at one point. [2]

(i)		Steps to sketch 1) When $x = 0$, $y = 4$ 2) When $y = 0$, $\|x\| = 4$, $x = 4$ or $x = -4$ Need to show correct x and y-intercepts (0, 4), (4, 0), (–4, 0)
(ii)	$y = -2x + 1 \;\ldots(1)$ $y = 4 - x \;\ldots(2)$ or $y = 4 + x \;\ldots(3)$ (1) = (2), $-2x + 1 = 4 - x$ $x = -3$ and $y = 7$ (n.a.) (1) = (3), $-2x + 1 = 4 + x$ $3x = -3$ $x = -1$ and $y = 3$ Intersection point is (–1, 3)	Note: (–3, 7) is (n.a.) because the y-coordinate is greater than the maximum y-value of the modulus graph.
(iii)	$m \le -1$ or $m \ge 1$	Read from graph



9 **(i)** Prove that $\tan A + \dfrac{\cos A}{1+\sin A} = \sec A$. [3]

(ii) Hence, find the exact solutions of $\tan 2x + \dfrac{\cos 2x}{1+\sin 2x} = cosec\ x$ for $0 \le x \le 2\pi$. [5]

(i)	LHS $= \dfrac{\sin A}{\cos A} + \dfrac{\cos A}{1+\sin A}$ $= \dfrac{\sin A + \sin^2 A + \cos^2 A}{\cos A(1+\sin A)}$ $= \dfrac{\sin A + 1}{\cos A(1+\sin A)}$ $= \dfrac{1}{\cos A}$ $= \sec A$ = RHS	
(ii)	$A = 2x$ $\sec 2x = \text{cosec } x$ $\dfrac{1}{\cos 2x} = \dfrac{1}{\sin x}$ $\cos 2x = \sin x$ $1 - 2\sin^2 x = \sin x$ $2\sin^2 x + \sin x - 1 = 0$ $(\sin x + 1)(2\sin x - 1) = 0$ $\sin x = -1$, $\sin x = \dfrac{1}{2}$ $x = \dfrac{3\pi}{2}$ ref $\angle = \dfrac{\pi}{6}$ $x = \dfrac{\pi}{6}, \dfrac{5\pi}{6}$	Sub part (i) into LHS, RHS remains Must be exact values

10 A particle, moving in a straight line, passes through a fixed point O with a velocity of 3 m/s . The acceleration, a m/s^2, of the particle, t seconds after passing through O is given by $a = -0.6e^{0.2t}$.

(i) Find the value of t when the particle is at instantaneous rest. [4]

(ii) Find the distance travelled by the particle in the first 5 seconds. [4]

(i)	$a = -0.6e^{0.2t}$ $v = \int -0.6e^{0.2t} dt$ $v = -3e^{0.2t} + c$ Sub v =3, t=0 $c = 6$ $v = -3e^{0.2t} + 6$ Sub v = 0, $-3e^{0.2t} + 6 = 0$ $3e^{0.2t} = 6$ $\ln e^{0.2t} = \ln 2$ $t = \frac{\ln 2}{0.2} = 3.465 =$ 3.47s	
(ii)	$s = \int -3e^{0.2t} + 6 \ dt$ $s = -15e^{0.2t} + 6t + c$ Sub s = 0, t = 0 $c = 15$ $s = -15e^{0.2t} + 6t + 15$ At $t = \frac{\ln 2}{0.2}$, $s = 5.7944$ At $t = 5$, $s = 4.226$ Distance travelled $= 5.7944 + (5.7944 - 4.226) = 7.3628 =$ 7.36 m Alternative method Distance travelled $= \left\| \int_0^{5\ln 2} \left(-3e^{0.2t} + 6\right) dt \right\| + \left\| \int_{5\ln 2}^{5} \left(-3e^{0.2t} + 6\right) dt \right\|$ $= \left\| \left[-15e^{0.2t} + 6t\right]_0^{5\ln 2} \right\| + \left\| \left[-15e^{0.2t} + 6t\right]_{5\ln 2}^{5} \right\|$ $= \left\|-15e^{\ln 2} + 30\ln 2 + 15 - 0\right\| + \left\|-15e + 30 + 15e^{\ln 2} - 30\ln 2\right\|$ $= \left\|-15 + 30\ln 2\right\| + \left\|60 - 15e - 30\ln 2\right\|$ =5.7944 +1.5686 =7.36 m	

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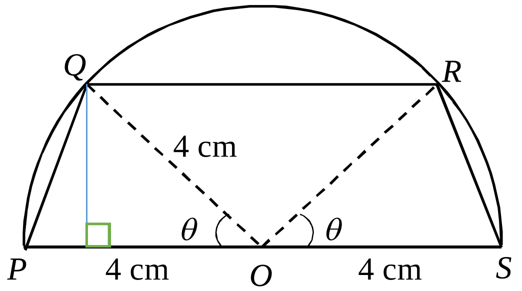

The diagram shows a trapezium *PQRS* inscribed in a semicircle with centre *O*. The radius of the semicircle is 4 cm. Angle *POQ* = angle *SOR* = θ radians.

(i) Show that the area, A cm^2, of the trapezium *PQRS* is given by $A = 16\sin\theta + 8\sin 2\theta$. [2]

(ii) Given that θ can vary, find the value of θ for which the area of the trapezium is a maximum. [6]

(i)	Horizontal = $4\cos\theta$ Vertical height = $4\sin\theta$ Area = $\frac{1}{2}(4\sin\theta)(8+2\times 4\cos\theta)$ $=16\sin\theta+16\sin\theta\cos\theta$ $=16\sin\theta+8\sin 2\theta$	Area = $2\times\frac{1}{2}(4)^2\sin\theta+\frac{1}{2}(4)^2\sin(\pi-2\theta)$ Using supplementary ∠s, Area = $2\times\frac{1}{2}(4)^2\sin\theta+\frac{1}{2}(4)^2\sin(2\theta)$
(ii)	$\frac{dA}{d\theta}=16\cos\theta+16\cos 2\theta$ $16\cos\theta+16\cos 2\theta=0$ $\cos\theta+\cos 2\theta=0$ $\cos\theta+2\cos^2\theta-1=0$ $2\cos^2\theta+\cos\theta-1=0$ $(\cos\theta+1)(2\cos\theta-1)=0$ $\cos\theta=-1$, $\cos\theta=0.5$ $\theta=\pi$ (NA) ref $\angle=\frac{\pi}{3}$ $\theta=\frac{\pi}{3}$ or 1.05 (3s.f.) Check for max $\frac{d^2A}{d\theta^2}=-16\sin\theta-32\sin 2\theta$ At $\theta=\frac{\pi}{6}$, $\frac{d^2A}{d\theta^2}=-41.6<0$, max value	Note: different angles, do not use R formula

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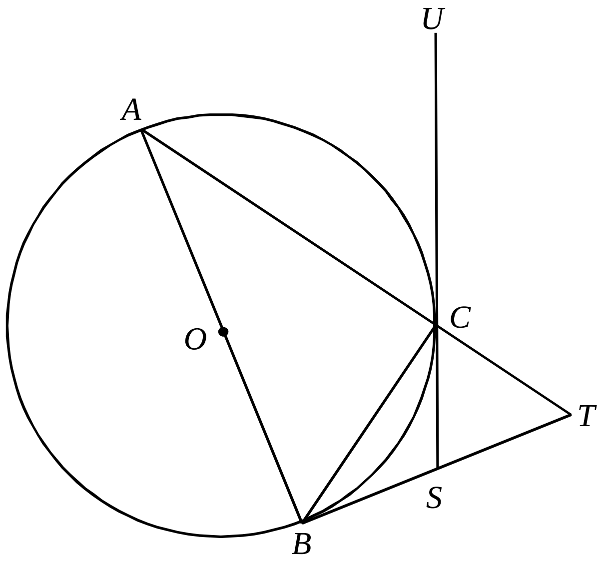

In the diagram, AB is a diameter of the circle with centre O. CS and BT are the tangents to the circle at C and B respectively. UCS, ACT and BST are straight lines. Prove that

(i) triangle ABC and triangle ATB are similar, [2]

(ii) angle SCT = angle CTS, [3]

(iii) S is the midpoint of BT. [2]

(i)	$\angle BAC = \angle TAB$ (common angle) $\angle ACB = 90°$ (right angle in semi-circle) $\angle ABT = 90°$ (tangent perpendicular to radius) $\therefore \angle ACB = \angle ABT = 90°$ $\therefore$ triangle ABC and triangle ATB are similar.	
(ii)	$\angle ABC = \angle ATB = \angle CTS$ (ΔABC similar ΔATB, proven in i) $\angle ABC = \angle ACU$ (tangent chord theorem) $\angle ACU = \angle SCT$ (vertically opposite $\angle$s) $\therefore \angle CTS = \angle SCT$	
(iii)	BS = CS (tangents from external point) Since $\angle CTS = \angle SCT$ (proven in ii) CS = ST (base $\angle$s of isosceles triangle) $\therefore$ BS = ST $\therefore$ S is the midpoint of BT.	

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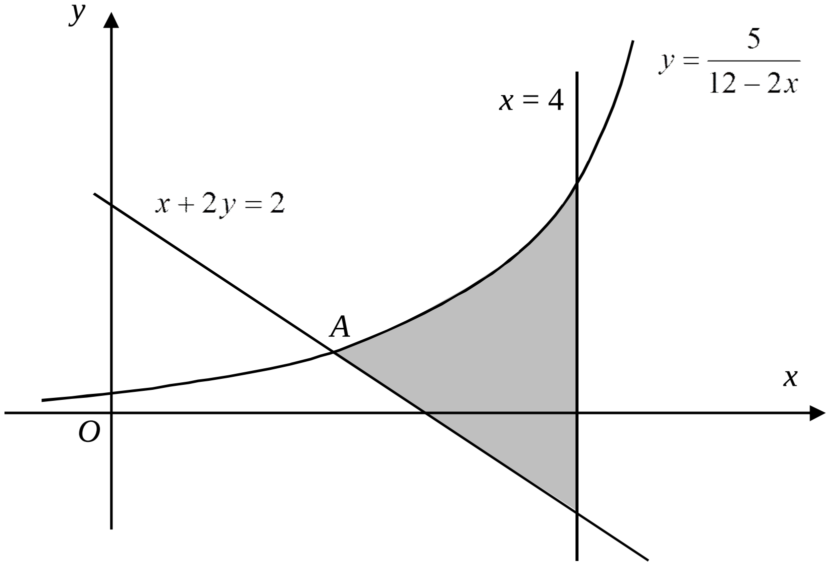

The diagram shows part of the curve $y = \frac{5}{12-2x}$ and the line $x + 2y = 2$.

The line intersects the curve at point A.

(i) Find the coordinates of A. [3]

(ii) Find the area of the shaded region bounded by the curve $y = \frac{5}{12-2x}$, the lines $x = 4$ and $x + 2y = 2$. [5]

(i)	$\frac{5}{12-2x} = \frac{2-x}{2}$ $10 = 24 - 12x - 4x + 2x^2$ $2x^2 - 16x + 14 = 0$ $x^2 - 8x + 7 = 0$ $(x-7)(x-1) = 0$ $x = 1$, $x = 7$ $y = 0.5$ $y = -2.5$ (NA) A is (1, 0.5)	
(ii)	Method 1 Area= $\int_1^4 \frac{5}{12-2x} - \frac{2-x}{2} \quad dx$ $= \int_1^4 \frac{5}{12-2x} - 1 + \frac{1}{2}x \quad dx$ $= \left[\frac{5\ln(12-2x)}{-2} - x + \frac{x^2}{4}\right]_1^4$ $= \left(-\frac{5}{2}\ln 4 - 4 + 4\right) - \left(-\frac{5}{2}\ln 10 - 1 + \frac{1}{4}\right)$ $= 3.0407 =$ 3.04 sq units	Method: y1 – y2

	<u>Method 2</u> Sub $y = 0$, $\therefore x = 2$ Area of $\Delta\,1 = \frac{1}{2}\times 1\times\frac{1}{2} = \frac{1}{4}$ sq units Sub $x = 4$ into straight line, $2y = 2 - 4$ $y = -1$ Area of $\Delta\,2 = \frac{1}{2}\times 2\times 1 = 1$ sq unit Area under curve $= \int_1^4 \frac{5}{12-2x}\ dx$ $= \left[\frac{5\ln(12-2x)}{-2}\right]_1^4$ $= -3.4657 - (-5.7564)$ $= 2.2907$ Total area $= 2.2907 + 1 - 0.25$ $= 3.04$ sq units	