1.1 Graphs and Transformations 1

Curve Sketching

1 MI/2009Prelim/I/5a

The curve C has equation $y=\dfrac{3x+2}{x-4}$.

(i) Write down the equations of all the asymptotes of C. [2]
(ii) Sketch the curve C, indicating all turning points, asymptotes and axial intercepts, if any. [3]
(iii) Find the set of values of k for which the line $y = kx$ intersects C at 2 points. Leave your answers to 3 significant figures. [4]

2 IJC/2009Prelim/II/1

The curve C has equation

$$y=\frac{x^2+9x+16}{x+2}.$$

(i) Find the equations of the asymptotes of C. [2]
(ii) Find, leaving your answers in exact form, the range of values of k for which the equation $k=\dfrac{x^2+9x+16}{x+2}$ has no real roots. [4]
(iii) Sketch C. Show, on your diagram, the equations of the asymptotes, the coordinates of the stationary points, and the coordinates of the points of intersection of C with the x- and y- axes. [3]

3 DHS/2009Prelim/I/3(b)part

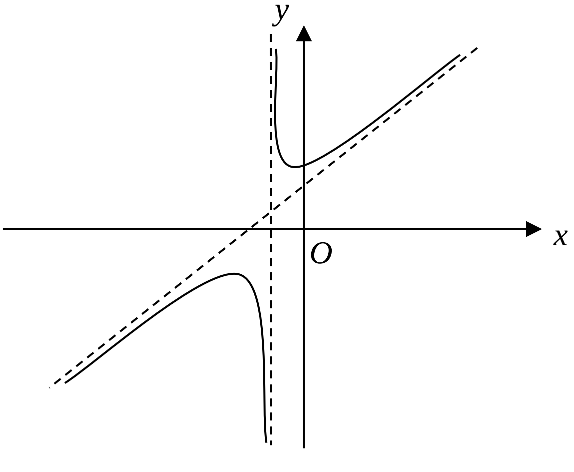
The curve C has the equation

$$y=\frac{x}{2}+\frac{A}{x-1}, \text{ where } A \text{ is a non-zero constant.}$$

(i) State the equations of the asymptotes of C. [2]
(ii) Find the range of values of A such that C does not have any stationary points. [2]

4 MJC/2011Promo/2

A sketch of the curve $y=\dfrac{2x^2+ax+b}{x+c}$, where a, b and c are constants, is shown, not to scale, in the diagram. Given that the asymptotes of the graph intersect at the point $(-1,1)$ and the curve passes through the point $(0,5)$,



(i) write down the value of c and find the values of a and b. [4]
(ii) hence determine the number of roots of the equation $2x+5=\dfrac{2x^2+ax+b}{x+c}$. [1]

5 NYJC Promo 9758/2018/Q4

Sketch the graph of $y=\dfrac{4\lambda-x^2}{x^2+\lambda}$ in each of the following two cases:

(i) $\lambda>0$, [2]

(ii) $\lambda<0$. [3]

Your sketches should include clear labelling of asymptotes, the exact coordinates of any points where the curve crosses the *x*- and *y*- axes intercepts and stationary point(s).

By using your graph in **(ii)** and considering a suitable graph whose cartesian equation is to be stated, find the positive value of *h* such that the equation

$$\left(\frac{4\lambda-x^2}{hx^2+h\lambda}\right)^2=1+\frac{x^2}{\lambda}$$

where $\lambda<0$, has only one real root.

State the value of the real root for this value of *h*. [3]

6 NJC/2015Promo/5

A curve *C* has equation

$$y=\frac{x^2-4x+8}{x-2}.$$

(i) Sketch *C*, labelling the equations of the asymptotes and the coordinates of the turning points. [3]

(ii) State the range of values of *x* for which *C* is concave upwards. [1]

(iii) Find the range of values of *x* for which $\dfrac{x^2-4x+8}{x-2}\geq 5$. [2]

(iv) State the range of values of *m* such that $\dfrac{x^2-4x+8}{x-2}=m(x-2)$ has exactly two real roots. [1]

7 RI/2010Promo/3

Sketch the graph of $y=\dfrac{2x^2+1}{x-1}$, showing clearly the coordinates of any turning point(s) and axial intercept(s), and the equation(s) of any asymptote(s).

Hence, state the range of values for *a* such that there are no real solutions to the equation $2x^2+1=a\left(x-1\right)\cos\left(bx+c\right)$, $x\neq 1$, for all values of *b* and *c*. [6]

8 JJC/2014Promo/6

The curve C_1 has equation $x^2-y^2+6y+16=0$ The curve C_2 has equation $\left(\dfrac{x-p}{5}\right)^2-\left(\dfrac{y-q}{5}\right)^2=1$, where p and q are constants.

(i) Sketch C_1, stating the coordinates of any points of intersection with the *x*- and *y*-axes and the equations of any asymptotes. [3]

(ii) The line $y=mx+c$ intersects C_1 twice. State the range of values of *m*. [2]

(iii) Given that C_1 and C_2 have the same asymptotes, determine the values of p and q.[2]

(iv) On the same diagram in part **(i)**, sketch C_2. [1]

(v) Let *n* be the number of points of intersection of the curve $x^2+\left(y-3\right)^2=r^2$ where $r>0$, with the curve C_2. Determine the possible values of *n*. [1]

9 EJC Promo 9758/2018/Q9

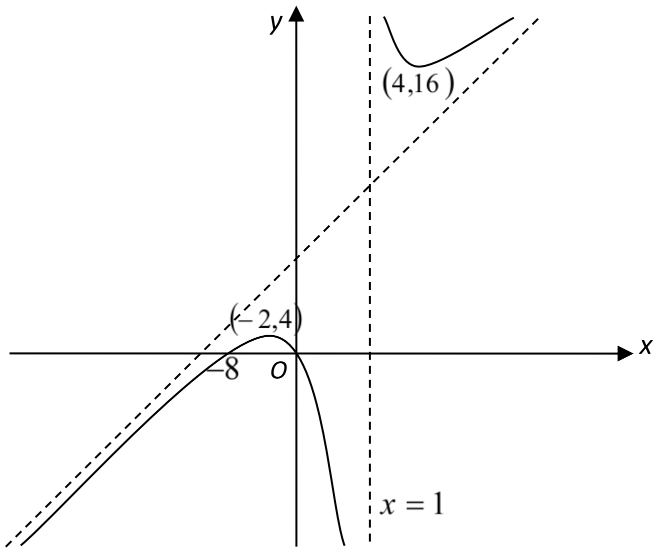
The curve C has equation $y=\dfrac{x^2+ax-4}{x+b}$, where a and b are constants, and $x\neq -b$. It is given that the asymptotes of C are $y=x-2$ and $x=1$.

(i) Find the values of a and b. Hence sketch C, stating the coordinates of the point(s) where the curve crosses the axes, and the equations of the asymptotes. [5]

(ii) By drawing a suitable curve on the same diagram in part **(i)**, find the number of real distinct roots of the equation $x^2-2x-20+21\left(\dfrac{x^2+ax-4}{x+b}\right)^2=0$. [3]

10 ACJC/2011MYE JC1/6

(a) The diagram below shows a sketch of the graph of $y=\dfrac{x^2+8x}{x+k}$, where k is a constant. The graph has stationary points at (−2, 4) and (4, 16). It also passes through the point $(-8,0)$ and has a vertical asymptote of $x=1$.



(i) State the value of k and the equation of the oblique asymptote. [2]

(ii) Sketch the graph of $y=\dfrac{x+k}{x^2+8x}$, labelling all asymptotes, axial intercepts and turning points, if any. [4]

(b) The curve C is given by the equation $(x-1)^2+(y-10)^2=r^2$, where r is a positive integer.

Write down the smallest value of r such that C intersects the graph of $y=\dfrac{x^2+8x}{x+k}$ at four points. [1]

11 NYJC/2010Promo/10

The curve C has equation $y=\dfrac{x^2+a}{x-a}$, $x\neq a$ and a is a non-zero constant.

(i) Given that the oblique asymptote of C is $y = x + 2$, find the value of a. [1]

(ii) Find the set of values of a if C has two turning points. [3]

(iii) Using the value of a found in part **(i)** and using an algebraic method, show that C cannot exist for $y_1 < y < y_2$ where values of y_1 and y_2 are to be determined in exact form. [3]

(iv) Hence sketch the graph of C, showing clearly the equations of the asymptotes and coordinates of the stationary points. [3]

12 NJC/2011MYE JC1/10

The equation of a curve C is given by $y=\dfrac{(px+q)^2}{x+r}$, where p, q and r are non-negative real constants. It is also given that C has a vertical asymptote $x=0$ and an oblique asymptote of $y=9x+\lambda$, where λ is a positive real constant.

(i) State the value of r and show that $q=\dfrac{\lambda}{6}$. [4]

(ii) Determine the coordinates and the corresponding nature of the stationary point(s). Express your answer in terms of λ. [4]

For $\lambda=18$,

(iii) sketch C. Label clearly the coordinates of the stationary point(s) and axial intercept(s) (if any). [3]

The curve D is defined parametrically by

$$x=a\sin\theta+1\ ,\ y=a\cos\theta+18\text{, where } a,\ \theta\in\mathbb{R}.$$

Find the Cartesian equation of D in terms of a. [2]

Deduce the least integer value of a for which C and D intersect each other more than once. [1]

13 NJC Promo 9758/2021/Q6

A curve C_1 has equation

$$x^2+2y^2=100$$

and a curve C_2 has parametric equation

$$x=2\mathrm{e}^{-t}-4\mathrm{e}^{2t},\quad y=3\mathrm{e}^{-t}+\mathrm{e}^{2t}.$$

(i) On the same diagram, sketch C_1 and C_2, labelling the coordinates of the points where both curves cross the x- and y-axes. [5]

(ii) Show that C_2 has a Cartesian equation of the form

$$(ax+by)^2(cx+dy)=k$$

for some integer constants a, b, c, d and k to be determined. [3]

14 NJC/2014Promo/12

(a) The curve C_1 has parametric equations

$$x = t^2 + t,\ y = 4t - t^2,\ -1 \le t \le 1.$$

(i) Sketch C_1, labelling the coordinates of the end-points and the axial intercepts (if any) of this curve. [2]

(ii) Calculate the gradient of the curve C_1 at the point where $x = \frac{5}{16}$. [3]

(iii) The curve C_2 is defined parametrically by the equations

$$x = t^2 + t,\ y = 4t - t^2,\ t \in \mathbb{R}.$$

Find a Cartesian equation of C_2. [2]

(b) The curve C_3 has equation $y = \frac{x-1}{x+1}$. The curve C_4 has equation $\frac{x^2}{20} - \frac{y^2}{5} = 1$.

Sketch C_3 and C_4 on the same diagram, stating the exact coordinates of any points of intersection with the axes and the equations of any asymptotes. [4]

Hence find the number of solutions to the equation $x^2 - \frac{4(x-1)^2}{(x+1)^2} = 20$. [2]

15 NJC/2010Promo/6

Sketch the graph of $y = \frac{2}{x(x-\mu)}$, where μ is a positive constant, indicating clearly any axial intercept(s), asymptote(s) and coordinates of turning point(s). [3]

Given $a = \frac{\mu}{2}$, deduce the range of values of b in terms of μ, such that the graphs $b(x-a)^2 + a^2y^2 = a^2b$ and $y = \frac{2}{x(x-\mu)}$ intersect twice, where a and b are positive real numbers. [3]

16 NYJC/2012Promo/11

The curve C has equation $y = \frac{x^2 + kx + 1}{x - 2}$, $x \ne 2$, where k is a constant.

(i) Find the range of values of k if C has 2 stationary points. [3]

(ii) Given that $y = x$ is an asymptote of C, find the value of k. [2]

(iii) Sketch the graph of $y = \frac{x^2 + kx + 1}{x - 2}$ for $k = -2$, stating clearly the equations of any asymptotes, coordinates of any turning points and axial-intercepts. [2]

(iv) On the same diagram, sketch the graph of $9y^2 = 9a^2 - a^2(x-3)^2$, where a is a constant, showing clearly the coordinates of the axial intercepts. [3]

(v) Deduce the possible values of a if the equation $9(x-1)^4 = a^2(x-2)^2\left(9-(x-3)^2\right)$ has exactly 3 real roots. [2]

Answers

Curve Sketching

1)(i) $y = 3$; $x = 4$, (iii) $\{k \in \mathbb{R} : k < -1.66 \text{ or } k > -0.339, k \neq 0\}$

2)(i) $y = x + 7$; $x = -2$, (ii) $5 - \sqrt{8} < k < 5 + \sqrt{8}$

3)(i) $y = \frac{x}{2}$; $x = 1$, (ii) $A < 0$.

4)(i) $c = 1 \quad b = 5 \quad a = 5$, (ii) <u>One</u> root

5) $h = 4$, $x = 0$

6)(ii) $x > 2$, (iii) $x \geq 6$ or $2 < x \leq 3$, (iv) $m > 1$

7) $-0.899 < a < 0.899$

8)(ii) $m < -1$ or $m > 1$. (iii) $p = 0,\ q = 3$, (v) $n = 0, 2, 4$

9)(i) $a = -3$, $b = -1$, (ii) 4 distinct real roots

10)(a)(i) $k = -1$, Oblique Asymptote is $y = x + 9$, (b) Smallest value of $r = 7$

11)(i) $a = 2$, (ii) $\{a \in \mathbb{R} : a < -1 \text{ or } a > 0\}$, (iii) $4 - 2\sqrt{6} < y < 4 + 2\sqrt{6}$

12)(i) $r = 0$, (ii) $\left(\frac{\lambda}{18}, 2\lambda\right)$ is minimum point. $\left(-\frac{\lambda}{18}, 0\right)$ is maximum point,

(iii) Least integer value of $a = 19$

13)(ii) $(x + 4y)^2 (2y - 3x) = 2744$

14)(a) (ii) $\frac{7}{3}$, (iii) $(x + y)^2 = 5(4x - y)$ or $y = \pm 5\sqrt{x + \frac{1}{4}} - \frac{5}{2} - x$; $x = 10 \pm 5\sqrt{4 - y} - y$

(b) Number of solutions = 2

15) $b > \frac{64}{\mu^4}$

16)(i) $k > -\frac{5}{2}$, (ii) $k = -2$, (v) $a = 4$ or $a = -4$