ANGLO-CHINESE JUNIOR COLLEGE
MATHEMATICS DEPARTMENT

MATHEMATICS
Higher 2

9740 / 01

Paper 1

23 August 2012

JC 2 PRELIMINARY EXAMINATION

Time allowed**: 3 hours**

Additional Materials: List of Formulae (MF15)

READ THESE INSTRUCTIONS FIRST

Write your Index number, Form Class, graphic and/or scientific calculator model/s on the cover page.
Write your Index number and full name on all the work you hand in.
Write in dark blue or black pen on your answer scripts.
You may use a soft pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphic calculator.
Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphic calculator are not allowed in the question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
At the end of the examination, fasten all your work securely together.

This document consists of **5** printed pages.

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ANGLO-CHINESE JUNIOR COLLEGE
MATHEMATICS DEPARTMENT
JC2 Preliminary Examination 2012

MATHEMATICS 9740
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Paper 1

/ 100

Index No: | | | | |

Form Class: ___________

Name: ___________________________

Calculator model: ___________________

Arrange your answers in the same numerical order.

Place this cover sheet on top of them and tie them together with the string provided.

Question No.	Marks
1	/7
2	/4
3	/8
4	/8
5	/5
6	/8
7	/7
8	/7
9	/5
10	/7
11	/10
12	/7
13	/5
14	/12

1 **(a)** Find $\int \frac{x}{x^2+4x+7}\,\mathrm{d}x$. **[3]**

(b) Find $\int_0^a x\sqrt{a-x}\,\mathrm{d}x$ by using integration by parts. **[4]**

2 The functions f and g are defined by

$$\mathrm{f}: x \mapsto \frac{x^2+2x}{2x^2-5x+2},\ x\in\mathbb{R}^+,\ x\neq\frac{1}{2},2,$$
$$\mathrm{g}: x \mapsto \frac{-3}{2x^2-5x+2},\ x\in\mathbb{R}^+,\ x\neq\frac{1}{2},2.$$

Without the use of the graphic calculator, find the subset of the domain for which the value of $\mathrm{f}(x)$ is greater than or equal to the value of $\mathrm{g}(x)$. **[4]**

3 The function f is defined by

$$\mathrm{f}: x \mapsto \ln x^2 - (\ln x)^2, \qquad x>0.$$

Sketch the graph of $y=\mathrm{f}(x)$, indicating clearly all points of intersection with the axes. Explain why the inverse function f^{-1} does not exist. **[3]**

The function h is defined by

$$\mathrm{h}: x \mapsto \ln x^2 - (\ln x)^2, \qquad 0<x\leq a.$$

State the greatest possible value of a for which the inverse function h^{-1} exists. Find the inverse function h^{-1} in similar form. **[4]**

The function g is defined by

$$\mathrm{g}: x \mapsto 3\mathrm{e}^x, \qquad x\in\mathbb{R}.$$

Find the range of the composite function gh. **[1]**

4 Tom and Jerry each start a new job. Tom is paid $30,000 in the first year, and in every subsequent year, his annual pay will increase by $1,500. Jerry is paid $25,000 in the first year, and in every subsequent year, his annual pay will increase by 5%.
If they both start working at the beginning of 2013,

(i) what is the first year in which Jerry will be paid more than Tom? **[4]**

(ii) at the end of which year will Jerry's total income first exceed Tom's total income? **[4]**

[Turn Over

5 A metal tank with an open top is in the form of a right circular cylinder. The total cost of metal used to make the tank is a fixed amount \$*A*. The metal used to make the base costs \$*B* per unit area and the metal used to make the curved surface costs \$*C* per unit area, where *B* and *C* are constants.
Given that the radius of the base of the tank is *r*, express the volume *V* of the tank in terms of *r*, *A*, *B* and *C*.
Use differentiation to find, in terms of *A*, the cost of the metal used to make the base which gives a tank of maximum volume. **[5]**

6 If $u_0 = \frac{1}{4}$ and $u_{n+1} = \frac{3u_n}{2u_n + 1}$ for any non-negative integer *n*, write down the values of u_1, u_2, u_3 and u_4 in the form $\frac{\mathrm{f}(n)}{1+\mathrm{f}(n)}$, where $\mathrm{f}(n)$ is expressed in terms of *n*. Hence formulate a conjecture for u_n in terms of *n*. **[3]**
Prove your conjecture by the method of mathematical induction for any non-negative integer *n*. **[5]**

7 A curve has parametric equations $x = 2t, \quad y = \mathrm{e}^{-t^2}$.
Sketch the curve, stating the exact coordinates of any points of intersection with the axes and the equations of any asymptotes. **[2]**
Find the equation of the normal to the curve at the point $\left(2p, \mathrm{e}^{-p^2}\right)$. **[2]**
The normal at the point $C\left(2, \frac{1}{\mathrm{e}}\right)$ meets the *x*-axis at *A* and the *y*-axis at *B*. Find the value of the ratio $|OA| : |OB|$, where *O* is the origin, giving your answer in terms of e. **[3]**

8 The equation of a curve is $y = \frac{x^2 - 3a^2}{x - a}$, where *a* is a positive constant. Find the equations of the asymptotes of the curve and show that there are no stationary points on the curve. **[3]**
Sketch the graph, stating, in terms of *a*, the coordinates of any points of intersection with the axes. **[2]**
By considering the graph of $y = \mathrm{f}'(x)$, where $\mathrm{f}(x) = \frac{x^2 - 3a^2}{x - a}$, or otherwise, state, in terms of *a*, the range of values of *x* for which the function f' is increasing. **[2]**

9 The complex number *z* satisfies the relations $|z| \le 2$ and $\frac{\pi}{6} \le \arg z \le \frac{\pi}{2}$. Illustrate both of these relations on a single Argand diagram as the region *R*. **[3]**
Given that *l* is the locus of points such that $\arg\left(z - \sqrt{3}\right) = k$, where $-\pi < k \le \pi$, find the set of values of *k* such that *l* and *R* have no common point. **[2]**

10 **(i)** Show that $\dfrac{1}{\sqrt{a}+\sqrt{b}} = \dfrac{\sqrt{a}-\sqrt{b}}{a-b}$. **[1]**

(ii) Find $\displaystyle\sum_{r=1}^{2500} \frac{1}{\sqrt{4r-3}+\sqrt{4r+1}}$ exactly. **[3]**

(iii) Hence show, without using a calculator, that $\displaystyle\sum_{r=0}^{2500} \frac{1}{\sqrt{4r+1}+\sqrt{4r+5}} > 24$. **[3]**

11 $OABC$ is a rhombus. The points A and C are such that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$. The point M is the centre of the rhombus and angle AOC is 60°.

(i) By using scalar products, show that the length of projection of $\overrightarrow{OM}$ on $\overrightarrow{OC}$ is $\dfrac{3}{4}|\mathbf{c}|$. **[4]**

(ii) By using a vector product, show that the area of triangle OMC can be written as $k|\mathbf{c}|^2$ where k is a constant to be found. **[3]**

(iii) The point D lies on OC produced such that $OC : CD = 2:3$. Show that the shortest distance from D to the line OA can be written as $t|\mathbf{c}|$ where t is a constant to be found. **[3]**

12 **(i)** Show, by using the substitution $w = \dfrac{y}{t^2}$, that the general solution of the differential equation $t\dfrac{\mathrm{d}w}{\mathrm{d}t} = w^2t^3 + 2wt - 2w$ is $w = \dfrac{1}{t^2}\left(\dfrac{2A\mathrm{e}^{2t}}{1-A\mathrm{e}^{2t}}\right)$. **[5]**

(ii) Sketch, on a single diagram, two members of the family of solution curves corresponding to $A = 0$ and to $A > 0$ respectively. **[2]**

13 Find the set of values of θ lying in the interval $-\dfrac{\pi}{2} < \theta < \dfrac{\pi}{2}$ such that the sum to infinity of the geometric series

$$1 + \tan\theta + \tan^2\theta + \tan^3\theta + \cdots$$

exists and is less than $\dfrac{3+\sqrt{3}}{2}$. Give your answer(s) in exact form. **[5]**

14 **(a)** Find, in a simplified form, the cartesian equation of the locus of the point P representing the complex number z where $|z+3| = 2\,\mathrm{Re}(z)$. Sketch the locus on an Argand diagram. **[5]**

(b) Write down the modulus and argument of the complex number w, where $w = -2 + \left(2\sqrt{3}\right)\mathrm{i}$. Given that w^n is real, find the possible values of n, where n is a positive integer. **[4]**

Express $w^{50} - (w^*)^{50}$ in the form $k\mathrm{i}$, where k is real, giving the exact value of k in non-trigonometrical form. **[3]**

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