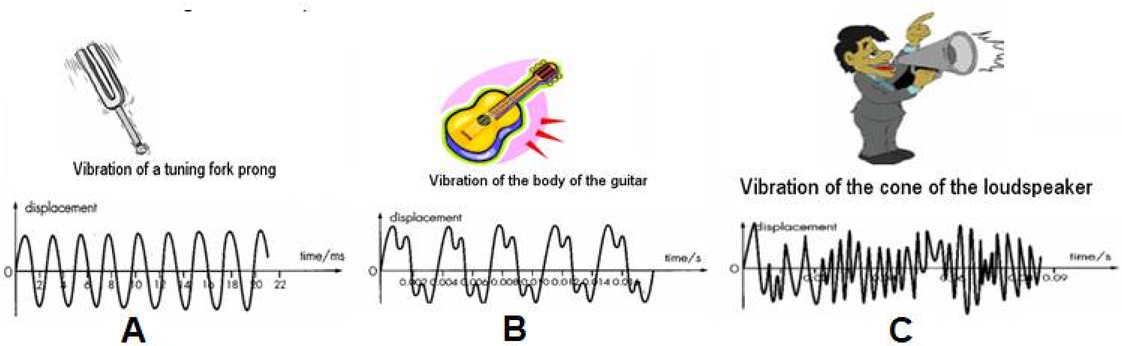
Oscillations 2022
Worked Examples

Example 8.1: Periodic Motion

Which of the following cases are periodic oscillations?



A and B are examples of periodic oscillations as we can identify a pattern that when repeated can reproduce theentire graph but C is not.

Example 8.2: Describing SHM in terms of time

(a)(i) x_0 = 0.12 m

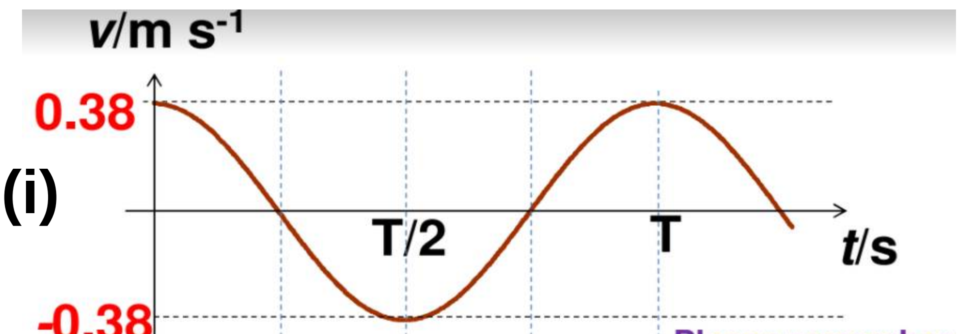
(ii) $\omega = \frac{2\pi}{T} = \frac{2\pi}{2}$ = 3.1rad s^{-1}

(iii) $v_0 = \omega x_0$ = 0.38 m s^{-1}

(iv) $a_0 = \omega^2 x_0$ = 1.2 m s^{-2}

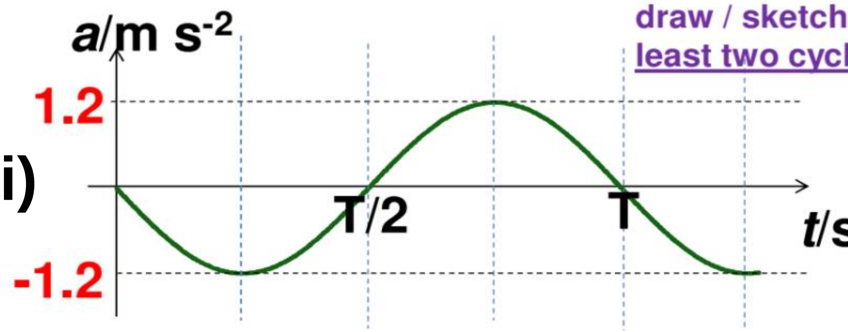
(b) x = 0.12sin(3.1t)

(c)(i)



Please remember that you have to draw / sketch at least two cycles.

(c)(ii)



Example 8.3: Displacement-time Relationship for SHM

(a) $x_0 = 3.5$ m

(b) $\omega = 4.0$ rad s^{-1}

(c) $x = 3.5 \sin 4.0(0.20) = 2.5$ m

(d) $v = 3.5(4.0) \cos 4.0(0.50) = -5.8$ m s^{-1}

Example 8.4: Relationship between acceleration and displacement for SHM

(a) x_0 = 30 mm

(b) From the graph, maximum acceleration

$a_0 = \omega^2 x_0$ = 0.48 m s^{-2}

hence $\omega = \sqrt{\frac{0.48}{0.03}}$ = 4 rad s^{-1}

(c) period $T = \frac{2\pi}{\omega} = \frac{\pi}{2}$ = 1.6 s

Example 8.5: Relationship between acceleration and displacement for SHM

	A	B	C	D	E
x	+	+	0	–	–
v	+	–	–	–	0
a	–	–	0	+	+

Example 8.6: Relationship between velocity and displacement for SHM

(a) The object could be moving in either direction when it passes through the equilibrium position.

(b) At zero velocity, the object is at the furthest point from the equilibrium position (maximum displacement) when its about to reverse direction, there are two such locations on either side of the equilibrium position.

(c) Since maximum velocity is given by $v_0 = \omega x_0$,

taking readings from the graph, $\omega = \frac{0.9}{0.003}$ = 300rad s^{-1}

Hence $v = \pm\omega\sqrt{x_0^2 - x^2} = \pm 300\sqrt{0.003^2 - x^2}$

Example 8.7: Relationship between velocity and displacement for SHM

(a) period, $T = \frac{1}{f} = \frac{1}{20} = 0.050$ s

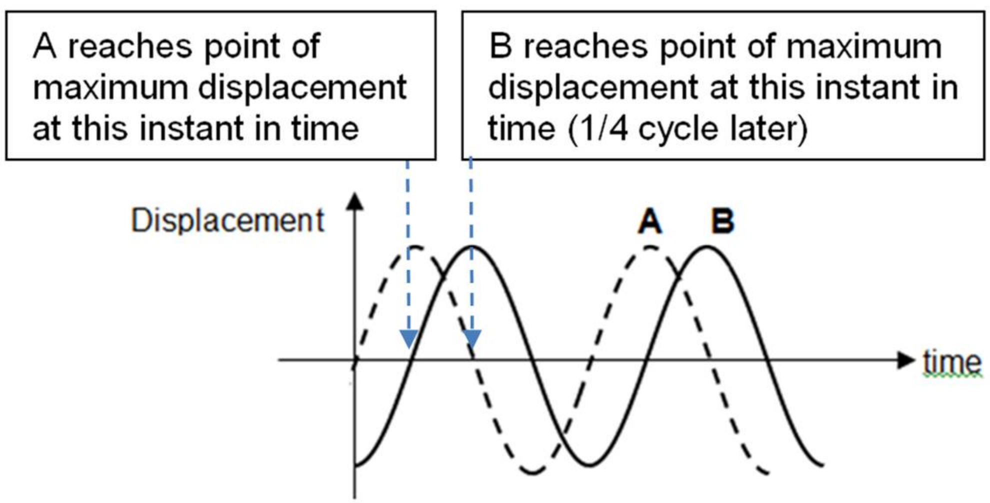
(b) At the equilibrium point, the object moves at maximum velocities,

$v = \pm\omega x_0 = \pm(2\pi f)x_0 = \pm(2\pi)(20)(0.020) = \pm 2.51$ m s^{-1}

At position of maximum negative displacement, the object is instantly at rest.

Example 8.8: Phase Difference

After A reached the maximum displacement (peak of graph), it takes a quarter of a cycle for B to also reach its maximum displacement. Hence A leads B by $\pi/2$. It can also be stated that A lags B by $3\pi/2$.



Example 8.9: Relationship between Energies and Displacement

(a) $E_T = \frac{1}{2}m\omega^2 x_0^2 = \frac{1}{2}m\left(\frac{k}{m}\right)x_0^2 = \frac{1}{2}(20)(0.03)^2 = 0.009$ J

(b)(i) $E_K = \frac{1}{2}m\omega^2(x_0^2 - x^2) = \frac{1}{2}(20)(0.03^2 - 0.02^2) = 0.005$ J

(ii) $E_p = E_T - E_K = \frac{1}{2}m\omega^2 x^2 = \frac{1}{2}(20)(0.02)^2 = 0.004$ J

Example 8.10: Relationship between Energies and Time

Answer: A.

As the bob is released from rest, the initial kinetic energy of bob is zero. Also, note that in one cycle of SHM, the energy graph goes through two cycles.

Example 8.11: Car Suspension System

Answer: D.

Critical damping is desirable in car suspension system because it returns the car body to its equilibrium position in the **shortest amount of time** without overshooting. D is the graph of critical damping.

A and C are underdamped, the car cannot afford to still oscillate after hitting a bump.

B is heavily damped, it takes much longer for the car body to return to the equilibrium position which poses a high risk.

Example 8.12: Damped Oscillation

Answer: D.

For light damping the resonance frequency is approximately the same as the natural frequency of the spring mass system.

Due to loss of energy as a result of damping, the resonance peak is lower and broader.