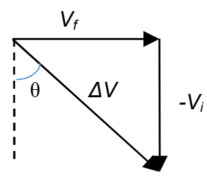
Solutions to 2023 Measurement Discussion Questions

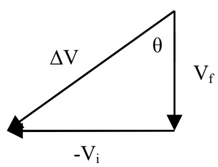
	Physical Quantities and Units
D1.	(A)
	Unit of v = m s^{-1}
	Unit of $\sqrt{g\lambda} = \left[\left(\text{m s}^{-2}\right)\left(\text{m}\right)\right]^{1/2} = \text{m s}^{-1}$
	Note the use of the word "could" in the question. We do not know whether the expression is indeed correct, but to stand a chance of being correct, it has to be of the correct unit! Note also the presentation: "Unit of … = …", and not "v = m s^{-1}".
D2.	(D). Unit of C = unit of αT = unit of βT^3 = J K^{-1} Unit of α = unit of C / unit of T = = J K^{-1} / K = J K^{-2} Unit of β = unit of C / unit of T^3 = J K^{-1} / K^3 = J K^{-4}.
	This is on homogeneity. Take note of the working presentation of such questions: "Unit of …"
D3.	(C). Homogeneous implies that the units of all the terms separated by "+", "-" or "=" are the same.
D4.	(D). Since we can only add or subtract quantities of the same units, *b* should have the same unit as *v*, hence the unit of *b* is m s^{-1}. Rearrange the equation and make *k* the subject, $k = \frac{2P}{d(v-b)^3}$ Unit of *k* = (kg m^2 s^{-3})(kg^{-1} m^3) / (m^3 s^{-3}) = m^2
D5.	(a) ampere, mole, kelvin, candela (any two)
	(b) The unit of energy, the joule, can be expressed in terms of base units.
	(c) (i) (1) $\text{density} = \frac{\text{mass}}{\text{volume}}$ Unit of density, $\rho = \text{kg m}^{-3}$ (2) $\text{pressure} = \frac{\text{force}}{\text{area}}$ Unit of pressure, p = $\frac{kg\ m\ s^{-2}}{m^2}$ = kg m^{-1}s^{-2} (shown)
	(ii) unit of c = $\left(\frac{\text{kg m}^{-1}\text{s}^{-2}}{\text{kg m}^{-3}}\right)^{1/2} = \text{m s}^{-1}$ (iii) It might be the speed of the gas molecules.

D6.	(D) • Mass of a typical watermelon is about 4 to 6 kg. • Power output of a domestic electric kettle is normally from 500 to 1000 W. • Human reaction time is about 0.2 to 0.5 s. • Weight of a typical one year old baby is about 60 to 100 N (remember to multiply by *g*!) • Height of the overhead bridge outside Hwa Chong Institution (College) from the road surface is about 6 to 8 m
	What is order of magnitude?
D7.	(C) A smartphone is about 150 g, which corresponds to a weight of 0.15 × 9.81 ≈ 1.5 N
	centi: 10^{-2} and deci: 10^{-1}.
	<u>Errors and Uncertainties</u>
D8.	(B). Because the student did not correct for zero error, his reading is off the mark by four divisions, or $\frac{0.08}{2.16} \times 100\% = 3.7\%$. With the help of the markings on the instrument, the uncertainty of his reading must be smaller than one division. Taking one division, $\frac{0.02}{2.16} \times 100\% = 0.93\%$ He is precise but not accurate.
D9.	(B). The mean is close to the true value (small systematic error). The spread of readings is quite large (not precise).
D10.	(D). ΔV = 0.01 × 4.072 + 0.010 = 0.05 (1sf). Value should be expressed to the tenths place, same as the uncertainty.
	An unusual way to present the uncertainty. Option C can be eliminated straightaway since its uncertainty was expressed to 2 sf.

D11.	(D) $L = \frac{y^2}{x}$ Percentage uncertainty, $\frac{\Delta L}{L} \times 100\% = \left[2\left(\frac{\Delta y}{y}\right) + \left(\frac{\Delta x}{x}\right) \right] \times 100\%$ $= 2 \times 3\% + 1\%$ $= 7\%$
	Make L the subject first.
D12.	Making k the subject: $k = 4\pi^2 \frac{m}{T^2}$. Express k in terms of the given variables: $k = 4\pi^2 \frac{m}{\left(\frac{t}{10}\right)^2} = 400\pi^2 \frac{m}{t^2}$ $k = 400\pi^2 \frac{0.150}{6.2^2} = 15.405$ N m^{-1} $\frac{\Delta k}{k} = \frac{\Delta m}{m} + 2\frac{\Delta t}{t}$ $\frac{\Delta k}{15.405} = 0.01 + 2\frac{0.2}{6.2}$ $\Delta k = 1.148$ N m^{-1} $k = 15 \pm 1$ N m^{-1}
	(1) Make the term of interest the subject and express it in terms of the given variables. *(2) Note that* $\frac{\Delta T}{T} = \frac{\Delta t}{t}$*, since T = t/10. See that the* $\frac{\Delta k}{k}$ *expression stays the same, even if one were to express k in terms of m and T.*
D13.	$\frac{\Delta \rho}{\rho} = 2\frac{\Delta d}{d} + \frac{\Delta V}{V} + \frac{\Delta L}{L} + \frac{\Delta I}{I}$ (a) $2\frac{\Delta d}{d} = 0.017; \ \frac{\Delta I}{I} = 0.033; \ \frac{\Delta L}{L} = 0.010; \ \frac{\Delta V}{V} = 0.020$ L has the smallest contribution to the uncertainty. (b) $\rho = \frac{\pi d^2 V}{4LI} = \frac{\pi (1.20)^2 (5.0)}{4(100)(1.50)} = 0.037699$ Ω cm $\frac{\Delta \rho}{0.037699} = 2\frac{0.01}{1.20} + \frac{0.1}{5.0} + \frac{1}{100} + \frac{0.05}{1.50}$

	$\Delta\rho = 3\times10^{-3}\ \Omega\text{ cm}$ $\rho = (38\pm3)\times10^{-3}\ \Omega\text{ cm}$
	One could obviously express it in Ω m too, in which case the answer would be *$(38 \pm 3) \times 10^{-5}$ Ω m.*
D14.	(A). <u>Best Method</u> $f=\left(\frac{1}{u}+\frac{1}{v}\right)^{-1}$ Smallest value of $f=\left(\frac{1}{47}+\frac{1}{195}\right)^{-1}=37.87\text{ mm}$ Largest value of $f=\left(\frac{1}{53}+\frac{1}{205}\right)^{-1}=42.11\text{ mm}$ $\Delta f=\frac{42.11-37.87}{2}=2.1\text{ mm}$ <u>Alternative method:</u> $\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$. Hence, $\Delta\left(\frac{1}{f}\right)=\Delta\left(\frac{1}{u}\right)+\Delta\left(\frac{1}{v}\right)$. Analyse the term in *u*: $\frac{\Delta\left(\frac{1}{u}\right)}{\left(\frac{1}{u}\right)}=\frac{\Delta u}{u}$ thus $\Delta\left(\frac{1}{u}\right)=\frac{\Delta u}{u^2}$ --------(1) Similar to (1): $\Delta\left(\frac{1}{v}\right)=\frac{\Delta v}{v^2}$ and $\Delta\left(\frac{1}{f}\right)=\frac{\Delta f}{f^2}$ Thus $\frac{\Delta f}{f^2}=\frac{\Delta u}{u^2}+\frac{\Delta v}{v^2}$ Re-arrange: $\Delta f=\left(\frac{\Delta u}{u^2}+\frac{\Delta v}{v^2}\right)f^2=\left(\frac{3}{50^2}+\frac{5}{200^2}\right)(40)^2=2.1\text{ mm}$
	If f is made the subject, f= uv/(u+v), this form would not be appropriate for the normal error calculation, since u and v appear in both *the numerator and the denominator. One cannot in a single physical situation maximise the numerator and minimise the denominator (or minimise the numerator and maximise the denominator) at the same time.*

<table>
<tr><td></td><td><u>Scalars and Vectors</u></td></tr>
<tr><td>D15.</td><td>(C). By definition, $\Delta v = v_f - v_i$


Change in velocity = $\sqrt{(8^2 + 6^2)} = 10$ m s^{-1}
tan θ = 6/8 = 37°</td></tr>
<tr><td></td><td>One could also say 10 m s^{-1} 53° south of east. In the exam, please provide a sketch as well to make yourself clear.</td></tr>
<tr><td></td><td></td></tr>
<tr><td>D16.</td><td>(i) By definition, $\Delta v = v_f - v_i$

</td></tr>
<tr><td></td><td>(ii) Change in speed = 25 – 30 = - 5 m s^{-1}</td></tr>
<tr><td></td><td>(iii) Change in velocity = $\sqrt{(30^2 + 25^2)} = 39$ m s^{-1}, 50° West of South or a bearing of 230°.

Note that θ = tan^{-1}(30/25) = 50°</td></tr>
<tr><td></td><td>Note the difference in the computations for the change in speed (a scalar) and the change in velocity (a vector). The negative sign for change in speed does not indicate direction; it merely means the speed decreases. In the exam, please provide a sketch as well to make yourself clear.</td></tr>
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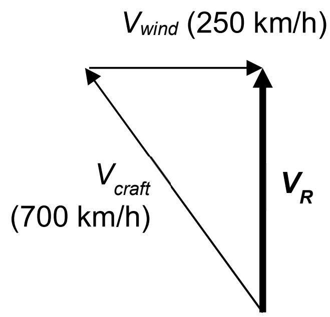
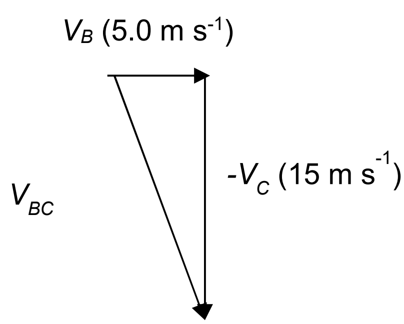
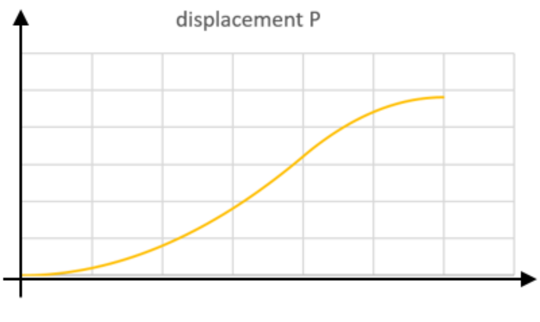
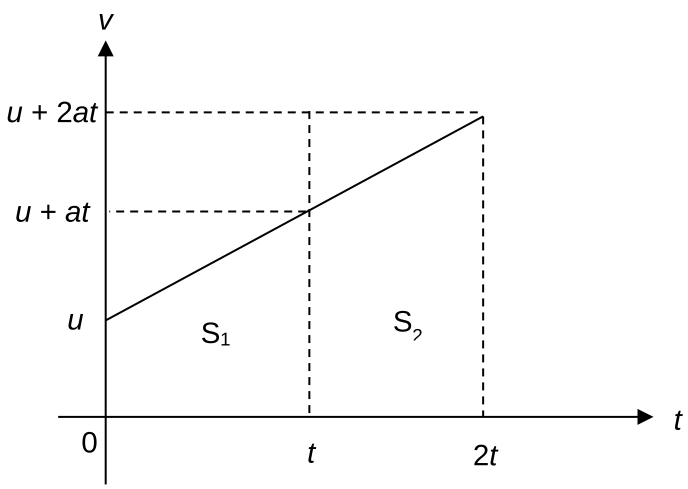
D17.	(B). The plane flies slightly into the wind so that its resultant velocity is directed northward. $V_R = V_{craft} + V_{wind}$  $v_R = \sqrt{700^2 - 250^2} = 654$ km/h
D18.	(C). $V_{BC} = V_B - V_C$ 
	From the car's perspective, the bicycle is moving eastward at 5.0 m s^{-1} and moving southward at 15 m s^{-1}.

Tutorial 2A: Kinematics Discussion Questions (Suggested Solution)

D1	(a) Yes. An object in uniform circular motion moves at constant speed but its direction of motion is changing all the time (its velocity is changing) and hence it has acceleration. Circular Motion will be covered in greater detail in chapter 6. (b) No. The magnitude of the velocity is the speed of an object. If the velocity is constant, both magnitude and direction of velocity have to be constant. Hence, speed cannot change. (c) Yes. Projectile motion of a projectile under free-fall, with no air resistance. The direction of motion (velocity) changes with time. The acceleration, g, is constant. (d) Yes. The object can be instantaneously at rest and the next moment its velocity increases or decrease. Eg an object thrown vertically upwards and at its highest point, it is instantaneously at rest but it is still accelerating downwards with g. (e) Yes. That happens when the object is slowing down. (f) No. Consider an object initially moving with some velocity then resting for some time then continuing to move with some velocity. The average velocity is not zero as net displacement is not zero but during this interval it was at rest at some point.
D2	At max height, velocity is zero and gradient at that point is equal to acceleration of free fall. Also, point A is the point it first hits the ground. NOTE: area of triangle ABC should be equal size to area of triangle CD since the ball rises and falls through the same distance after the first bounce. Ans: C
D3	The area under velocity-time graph gives the displacement. Both objects have the similar acceleration rates throughout. You may sketch the displacement versus time graph for both objects.   The displacement for P is obviously much greater. In fact, the area under velocity-time for Q suggests that the displacement of Q is zero at the end of time t_2. To compare the distance travelled by P and Q, we could make use of the area under velocity-time graphs Object P: $\frac{vt_2}{2}$

	Object Q: $\left(\frac{1}{2}\frac{v}{2}\frac{t_1}{2}\right)\times 2+\left(\frac{1}{2}\frac{v}{2}\frac{(t_2-t_1)}{2}\right)\times 2=\frac{vt_2}{4}$. The distance travelled is also different. Ans: A
D4	The area under velocity-time graph gives the displacement of the cars. At t = 0, both are at the same position where Car X takes over. At t = T, both cars must also be at the same position so that Car Y could take over. The displacement of Car X at t = T is P + Q + R and that of car Y is S + Q + R. The displacements must be the same. Hence, P is equal to S. Ans: A
D5	Area under the graph represents change in velocity. At point C, the area (from start to point C) is greatest. Beyond point C there is a negative change in velocity, which means the velocity is decreasing. Ans: C
D6	**Method I (equations of motion)** Let t = 0 be when the ball passes by light gate 1. At light gate 2, $s_1=ut+\frac{1}{2}at^2$ (1) At light gate 3, $s_1+s_2=u(2t)+\frac{1}{2}a(2t)^2$ (2) (2) - 2×(1): $s_2-s_1=\frac{1}{2}a(4t^2-2t^2)$ $a=\frac{s_2-s_1}{t^2}$ Ans: A **Method II (graphical)** Assume the speed of the ball is u when it passes the light gate 1 at time t = 0 and accelerates constantly with a. Sketching v-t graph,



The area under the *v-t* graph is displacement, hence

$$S_1 = \frac{1}{2}(u+u+at)(t-0) = \frac{1}{2}\left(2ut+at^2\right)$$

$$S_2 = \frac{1}{2}(u+at+u+2at)(2t-t) = \frac{1}{2}\left(2ut+3at^2\right)$$

$$S_2 - S_1 = \frac{1}{2}\left(3at^2-at^2\right) \rightarrow a = \frac{S_2-S_1}{t^2}$$

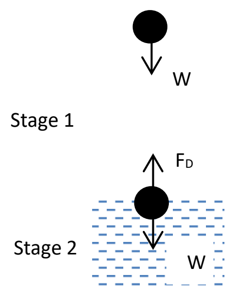
D7

Stage 1

=> sphere in air, with negligible air resistance and upthrust

=> free-fall, $a = g$

=> option A & E not possible



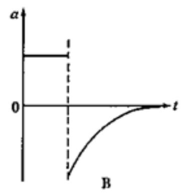
Stage 2

=> sphere enters fluid, there is drag force.

=> 2 possible cases➔ Case 1: $F_D > mg$, Case 2: $F_D < mg$

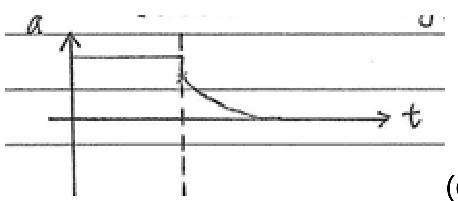
<u>Case 1</u>: $F_D > mg$

Sphere slows down as a is negative, reaches terminal velocity when a = 0 m s^{-2}



<u>Case 2</u>: $F_D < mg$

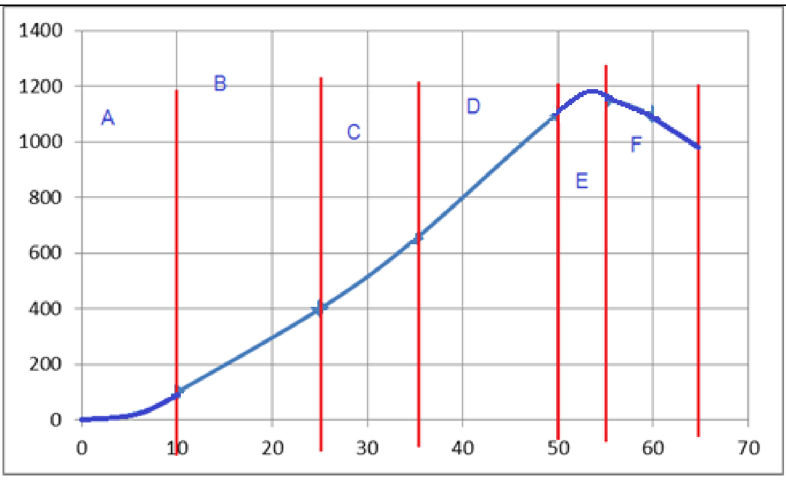
Sphere continues to accelerate downwards but at $a < g$. Velocity increases, F_D increases until $F_D = mg$. Net force on sphere is zero and a = 0. Sphere reaches terminal velocity.



(option not available)

Ans: B

D8	Option D & E are not possible as *H* does not change after some time, implying that ball bearing stays stationary which is not possible. Since ball is released, its initial velocity is zero and hence gradient of *h*-*t* graph at t = 0 s should be zero. At t > 0 s, as ball bearing speeds up, F_D increases. $a=\frac{mg-F_D}{m}$, $a<g$, ball bearing continues to speed up at slower rate. Eventually, $F_D = mg$, net force acting on ball bearing = 0, a = 0 and ball bearing reaches terminal velocity (speed constant, gradient constant). Ans: A
D9	(a)(i) 20 m s^{-1} (ii) $a_A =$ gradient of *v*-*t* graph $=\frac{20-0}{10-0}$ $=2.0$ m s^{-2} (iii) $a_E =$ gradient of *v*-*t* graph $=\frac{(-5)-30}{55-50}$ $=-7.0$ m s^{-2} (iv) $s_B =$ area in section B $=20\times15$ $=300$ m (v) $s_C =$ area of trapezium in C $=\frac{1}{2}\times(20+30)\times10$ $=250$ m (b) At t = 50 s, the object decelerates uniformly at a rate of 7.0 m s^{-2} from 30 m s^{-1} until it comes to an instantaneous rest at $t \approx 54$ s. It then moves back (in opposite direction to its original motion) and accelerates at 7.0 m s^{-2} until it reaches a speed of 5 m s^{-1} at t = 55 s. It continues at a uniform speed of 5 m s^{-1} in the negative direction for the next 10 s until t = 65 s. (c)



Note: Sections B, D and F are straight. The peak occurs at about 54.5 s.

D10

(a) There is air resistance, which increases with the speed of the object.
At a large enough speed, the air resistance equals the weight of the object.
Net force becomes zero since weight and air resistance act in opposite directions, hence acceleration becomes zero and velocity becomes constant.

(b) (i) By Newton's 2nd law,

$$F_{net} = mg - kv$$
$$ma = mg - kv$$
$$(g - a) = kv / m$$

(ii) At v = 0 m s^{-1}, a = 9.81 m s^{-2} because the object experiences no air resistance when it is not moving.
At v = 40 m s^{-1}, a = 0 m s^{-2} because the object is at terminal velocity and net force is zero.

The acceleration at v = 30 m s^{-1} is calculated from the gradient of the tangent at v = 30 m s^{-1}.

velocity v/m s^{-1}	acceleration a/m s^{-2}	$(g - a)$/m s^{-2}
0	9.8	0.0
20	8.2	1.6
30	5.7	4.1
40	0.0	9.8

(iii) If the student's suggestion is correct, then $\frac{g-a}{v}$ is constant.

At v = 20 m s^{-1}, $\frac{g-a}{v} = \frac{1.6}{20} = 0.080$

At v = 30 m s^{-1}, $\frac{g-a}{v} = \frac{4.1}{30} = 0.137$

The student cannot be correct since the percentage difference between the values is large.

$$[\frac{0.137-0.080}{\left(\frac{0.137+0.080}{2}\right)}\times 100\% = 53\%]$$

D11

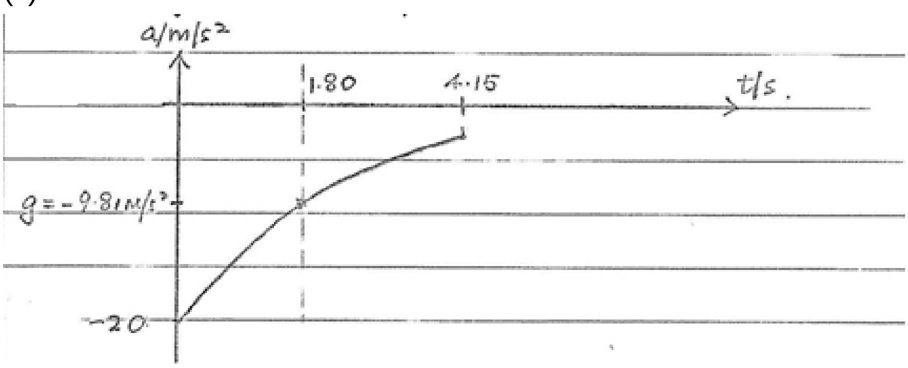
(c) (i) At maximum height, the ball is instantaneously at rest.
From the graph, v = 0 m s^{-1} at t = 1.80 s.

(ii) The acceleration at an instant can be determined from the gradient of the tangent line at that instant.

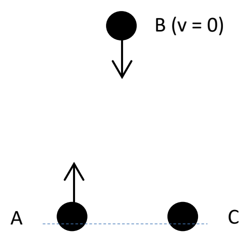
(iii)It could be because there is significant air resistance acing on the ball. Air resistance is present as the ball possesses velocity at the point which it is thrown. The air resistance is acting in the opposite direction as its velocity, which is in the same direction as its weight. Hence the downward acceleration is greater than the gravitational pull of g.

(iv)At t = 1.80 s, the ball is at rest and air resistance is zero, hence only its weights acts on it. It experiences an acceleration of g.

(v)



(d) On its flight up, the drag force acting on the ball is in the same direction as its weight. On its flight down, the drag force acting on the ball is in opposite direction to its weight. Hence, the average acceleration on its flight up a_{AB} is greater than the average acceleration on its flight down a_{BC}.



Using $v^2 - u^2 = 2as$,

From A to B, $0^2 - {v_A}^2 = 2a_{AB}s$

$${v_A}^2 = -2a_{AB}s \text{ ----- (1)}$$

From B to C, ${v_C}^2 - 0^2 = 2a_{BC}s$

$${v_c}^2 = 2a_{BC}s \text{ ----- (2)}$$

Since $a_{BC} < a_{AB}$, comparing (2) and (1), the final speed reached by the ball when it falls back onto the hand v_c is smaller than the initial speed of projection v_A.

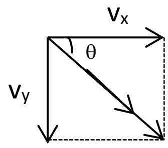
Hence, the average speed of the ball on its way up is greater than the average speed of the ball on its way down.

Since the distance travelled up = distance travelled down, the time taken to reach max height is less than the time taken to return to the starting point.

	(e) (i) KE = $\frac{1}{2}mv^2$ 54 = $\frac{1}{2}m \times 26^2$ m = 0.160 kg (ii) The ball starts with an initial kinetic energy of 54 J. As it moves up, its kinetic energy reduces till it reaches 0 J at the highest point. As it falls back, it increases its kinetic energy again. However, as air resistance is present, the ball loses kinetic energy to the surrounding throughout its flight and its final kinetic energy when it reaches back to its starting point is less than 54 J.

Tutorial 2B: Kinematics Discussion Questions (Suggested Solution)

<table>
<tr><td>D1</td><td>Assuming that horizontal speed of projection of bullet does not change.
Using $(\rightarrow) s_x = u_x t$, when s_x doubles, t doubles. Hence $t' = 2t$

For vertical motion, $(\downarrow) s_y = u_y t + \frac{1}{2} a_y t^2$
$s_y = \frac{1}{2} a_y t^2$
$s_y \propto t^2$
$\frac{s_y'}{s_y} = \frac{t'^2}{t^2}$
$s' = (\frac{2}{1})^2 \times 5.0 = 20 \text{ mm}$
<u>Alternative:</u> Solve using similar triangles by drawing v-t graph.
Ans: D</td></tr>
<tr><td>D2</td><td>$(\rightarrow) v_x = v$
$(\downarrow) v_y^2 = u_y^2 + 2a_y s_y$
$v_y^2 = 2gh$

$\tan\theta = \frac{v_y}{v_x} = \frac{\sqrt{2gh}}{v}$
Hence largest ratio of $\frac{\sqrt{h}}{v}$ will produce greatest θ.
Ans: B</td></tr>
<tr><td>D3</td><td>Initially, the ball is moving with the trolley. When it is released, it will have an initial horizontal speed of 10 m/s to the left. It also falls with a downward acceleration of g.
$a_x = 0$, $a_y = g$, $u_x = 10$ m/s & $u_y = 0$ m/s
Ans: C</td></tr>
<tr><td>D4</td><td>(a) Same u_y. By $v_y{}^2 = u_y{}^2 + 2a_y s_y$, since the three trajectories have the same v_y, a_y and s_y, they must have the same u_y.
(b) Same time of flight. By $v_y = u_y + a_y t$, since v_y, u_y and a_y are the same, t must be the same.
(c) a, b, c . Since they have same time of flight and $s_x = u_x t$, the larger the range the larger the u_x.
(d) $u = \sqrt{u_x^2 + u_y^2}$ => since u_y are the same, determined by u_x
a, b, c.</td></tr>
<tr><td>D5</td><td>(a) $(\rightarrow) s_x = u_x t$
$60 = (u\cos\theta) \times 3.0$
$3u\cos\theta = 60$ ---- (1)</td></tr>
</table>

$(\uparrow)\ s_y = u_y t + \frac{1}{2} a_y t^2$

$-15 = u\sin\theta \times t - \frac{g}{2}t^2$

$6u\sin\theta = 9g - 30$ --- (2)

(2)/(1): $\dfrac{6u\sin\theta}{3u\cos\theta} = \dfrac{9g-30}{60}$

$\theta = 26.0^o$

(b) Subst. $\theta = 26.0^o$ into (1),

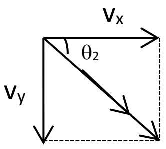
$3u\cos 26.0^o = 60$

u = 22.3 m/s

(c) Angle of landing ramp should match angle of velocity on landing, θ_2

$(\rightarrow)\ v_x = u_x = 22.3\cos 26.0^o = 20.0$ m/s

$(\uparrow)\ v_y = u_y + a_y t$



$v_y = 22.3\sin 26.0 + (-9.81)3.0$

$v_y = -19.7$ m/s

$\tan\theta_2 = \dfrac{v_y}{v_x} = \dfrac{19.7}{20.0}$

$\theta_2 = 45^o$

(d) landing speed, $v = \sqrt{v_x^2 + v_y^2} = \sqrt{20.0^2 + (-19.7)^2} = 28.1$ m s^{-1}

D6

Time of flight from considering the horizontal motion;
Horizontal displacement of cannon = 50.0 m
t (15.0 cos30^o) = 50.0
t = 3.85 s

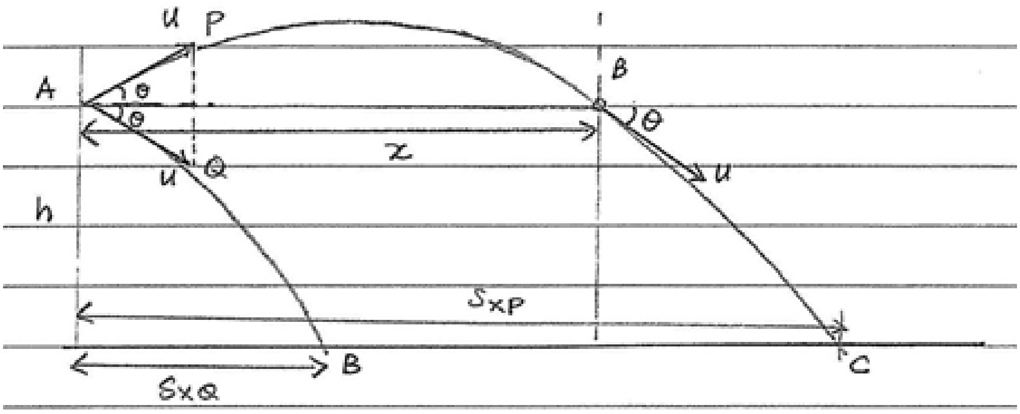
Taking upward to be positive and displacement with respect to sea-level,
Vertical displacement of cannon = Vertical displacement of balloon
$s_{oc} + u_c t + \frac{1}{2} a t^2 = s_{ob} + u_b t$
100.0 + (15.0 sin30^o) t + ½ (-9.81) t^2 = 30.0 + u t
u = 6.8 m s^{-1}

D7

(a) As $a_x = 0$, the horizontal distance between the balls are constant. The 2 balls are projected with same velocity, therefore they follow identical paths. As the balls fall, the vertical distance between them increases. Hence, the balls are closest at the point when the 2nd ball was projected.

(b) $v = \sqrt{v_x^2 + v_y^2} = \sqrt{u_x^2 + v_y^2}$

u_x are the same for both ball. v_y for the first ball > v_y for the second ball. Hence, the first will always be travelling faster.

	(c) Both have the same time of flight; hence they are one second apart between impact. (d) No, it cannot. For time of flight, consider vertical motion. $(\downarrow)\ s_y = u_y t + \frac{1}{2} a_y t^2$ $s_y = \frac{1}{2} a_y t^2$ When the balls are thrown off horizontally from the same height, the time of flight for both are the same.
D8	(i) 1. $(\rightarrow) s_x = u_x\ t$ $s_x = 10\cos(60°)\ t$ ---- (1) 2. $(\downarrow) s_y = u_y\ t + \frac{1}{2} a_y t^2$ $s_y = -10\sin(60°)\ t + \frac{1}{2}(9.81)t^2$ ----(2) (ii) Since $s_y = s_x$, (1) = (2), $t = 2.78\ s$ (iii) $d = \frac{s_y}{\cos(45°)} = 19.7\ m$
D9	(i)  (ii) Horizontal distance between points of impact, $x = u_{X,P} t$ (Path AB is identical to path BC. Distance between points of impact = $s_{XP} - s_{XQ} = x$) Let *t* be the time taken for P to reach B. $(\uparrow) v_y = u_y + a_y t$ $t = \frac{(-u\sin\theta) - u\sin\theta}{-g}$ $= \frac{2u\sin\theta}{g}$ Therefore, $x = u_{X,P} t = (u\cos\theta)(\frac{2u\sin\theta}{g}) = \frac{u^2\sin 2\theta}{g}$.

2023 Dynamics Tutorial - Suggested Solutions for Discussion Questions

Part 1: Newton's Laws, Inertia, Force, Momentum, Impulse

D1

D

Constant velocity implies $a = 0$, $F_{net} = 0$. Hence, the first part of the graph should be zero.
Constant deceleration implies a = constant, F_{net} = constant, so the last part must be constant.

Comment:

Students tend to get confused or distracted by the term "deceleration" and leap to the conclusion that the graph must be sloping downwards which is actually how the velocity is changing and not the acceleration or net force.

D2

D

Constant force implies uniform acceleration.
Applying equations of motion: $v^2 = u^2 + 2as$. The railway carriage starts from rest, so $u = 0$
Thus $v = \sqrt{(2as)}$
$\Rightarrow p = m\sqrt{(2as)}$, i.e. $p \propto \sqrt{s}$
Note: Slight error in the answer option.
$\lim_{s \to 0} \frac{dp}{ds} = 0$ (i.e. limit of the gradient) should approach infinity as s approaches zero.

Problem-solving skills:

A repeat of skills taught in kinematics regarding the use of an equation relating the 2 axes is deployed here.

Common mistake:

Students may choose A, thinking they are tested on force = rate of change of momentum.

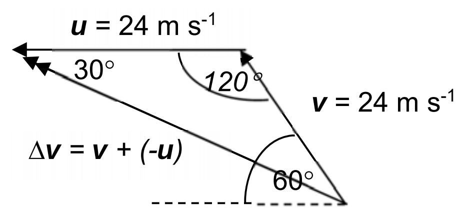
Extension:

Try to plot momentum-time graph.

D3

A

$\boldsymbol{u} + \Delta\boldsymbol{v} = \boldsymbol{v}$
$\Rightarrow \Delta\boldsymbol{v} = \boldsymbol{v} + (-\boldsymbol{u})$



Using the sine rule, $\frac{\Delta v}{\sin 120^\circ} = \frac{24}{\sin 30^\circ}$

$\Delta v = 48 \sin 120^\circ$

Alternatively, use the cosine rule.

By Newton's 2nd Law, or impulse-momentum relationship

$$\langle F \rangle = \frac{\Delta p}{\Delta t} = \frac{m \Delta v}{\Delta t} = \frac{(0.11)(48 \sin 120^\circ)}{0.025} = 180 \text{ N}$$

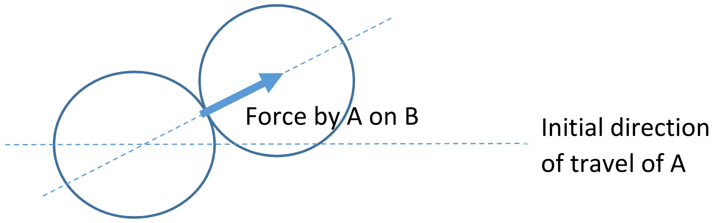
Comment:

Students who are not as strong in their maths or vectors can choose to solve the components separately. Taking rightwards and upwards as positive:

- *horizontally: $\Delta v_x = -24 \cos 60^o - 24 = -36$ m s^{-1};*
- *vertically: $\Delta v_y = 24 \sin 60^o = 20.8$ m s^{-1};*
- *total change in momentum is $\Delta p = m\ \Delta v = m \sqrt{36^2 + 20.8^2} = 0.11 \times 41.6 = 4.57$ N s;*
- *average force is <F> = $\Delta p / \Delta t = 4.57 / 0.025 = 183$ N = 180 N (2 s.f.)*

D4

At the instant of collision, if the collision is not head-on (line joining center of A and B is not along the initial direction of travel of A) the contact force between them will have a component perpendicular to the initial direction of travel of A. This means that there will be a change in momentum of sphere B in the perpendicular direction and the initial velocity of sphere B will no longer be along the direction of travel of A.



D5

At terminal velocity, $(F_{net} = 0)$

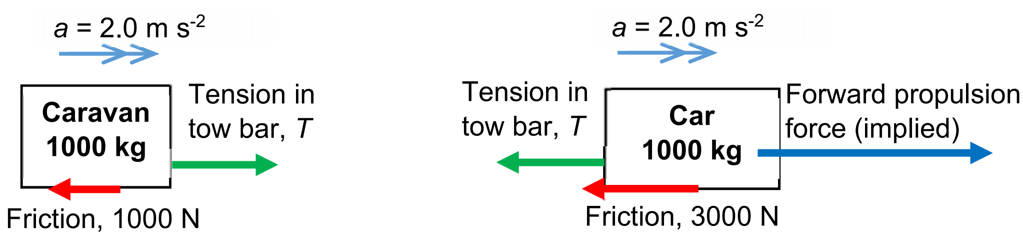
$$mg = R$$
$$(3.0)(9.81) = 0.60v$$
$$v = 49.1 \text{ m s}^{-1}$$

At $v = 12 \text{ m s}^{-1}$, $(F_{net} = ma)$

$$mg - R = ma$$
$$(3.0)(9.81) - 0.60(12) = 3.0a$$
$$a = 7.41 \text{ m s}^{-2}$$

D6

a)



- As the light tow bar is under tension, it exerts forces of equal magnitude on the caravan as well as the car. The tension in the tow bar pulls the caravan forward and the car back.
- Applying Newton's second law on the caravan, taking rightwards as positive,
  $F_{net} = ma$
  $T - 1000 = (1000)(2.0)$
  $T = 3000$ N
- The force by the tow bar on the caravan is 3000 N to the right.
- Hence the force by the tow bar on the car is **3000 N to the left**.

b) When the vehicles are moving at constant speed, net force on each of them must be zero.

Hence, referring to caravan again, the force by the tow bar on the caravan is equal in magnitude to the friction on the caravan. Hence, the force by the tow bar on the caravan is 1000 N to the right.

By Newton's third law, the force by the tow bar on the car is **1000 N to the left**.

D7

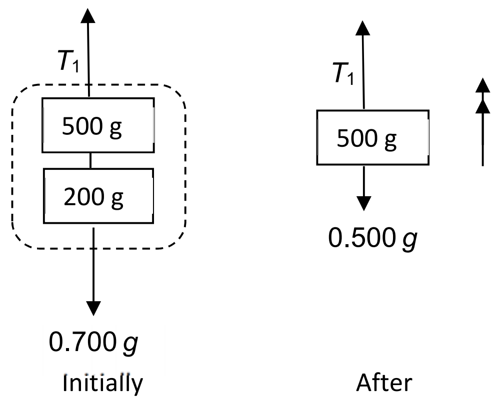
B

Initially, the system is in equilibrium.

$T_1 = 0.700\,g$

Immediately after the thread is cut,
Applying Newton's 2[nd] Law, $\sum F = ma$

$T_1 - 0.500\,g = 0.500\,a$
$a = 2/5\,g$
$= 0.4\,g$



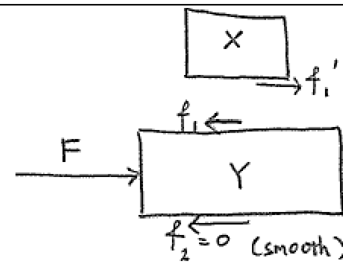
<u>Note:</u>
Questions like these come up quite regularly in exams. An object (in this case the block of 500 g) is initially at rest, when suddenly one of the forces "disappears". The new net force on the object <u>immediately after</u> the force disappears is given by $F_{net,new} = -F_{disappeared}$. The force that disappears is the tension in the lower string, which is equal to the weight of the block of 200 g.

D8

Applying Newton's second law

For X & Y: $F = (m + 3m)\,a = 4m\,a$
$a = F/4m$

For X alone: $f_1' = m\,a$
$= m\,(F/4m)$
$= \mathbf{F/4}$



<u>Note:</u>
The "m" that occurs in Newton's second law is in general the mass of the object that is being accelerated. However, in the first part of this working, the mass as indicated in Newton's second law is actually equal to 4m.

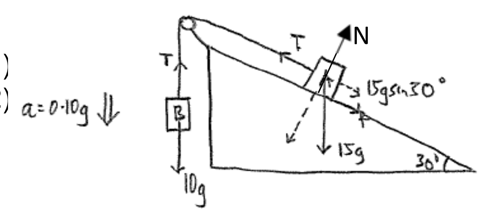
D9

A

Consider the blocks separately.

B: $10\,g - T = 10\,(0.10\,g)$ -------------- (1)
A: $T - 15\,g \sin 30° - f = 15\,(0.10\,g)$ -------------- (2)

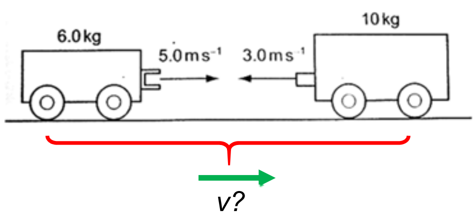
$10\,g - 15\,g \sin 30° - f = (10 + 15)\,0.10\,g$
$f = 0$ N



<u>Note:</u>
This is a difficult question for most students. The pulley confuses them. Most students would choose downwards as positive and then get stuck at forming the equation for the block on the slope.

Part 2: Conservation of Linear Momentum/Collisions

D10 **C**



$\sum p_i = 6.0\,(5.0) + 10(-3) = 0$ N s

$\sum p_f = 0$ N s

For inelastic collision,

$(6.0 + 10)\, v = 0$

Final velocity $= 0$ m s^{-1}

Consider the 6.0 kg trolley,

$$\langle F \rangle = \frac{\Delta p}{\Delta t} = \frac{|0 - 6.0(5.0)|}{0.20} = 150\ \text{N}$$

D11 Let initial velocity of bullet be u, velocity of bullet after emerging from 1.2 kg block be v.

Before hitting 1.2 kg block, $\sum p_i = 0.0035\, u$ ------------------- (1)

After bullet emerging from 1.2 kg block, $\sum p_f = 1.2(0.63) + 0.0035\, v$ ------------------ (2)

After bullet stuck in 1.8 kg block, $\sum p_f' = 1.2(0.63) + (1.8 + 0.0035)(1.4)$

$= 3.2809$ ------------- (3)

(a) By the principle of conservation of linear momentum,

(2) = (3): $0.0035\, v = 2.5249$

$v = 721$ m s^{-1}

(b) By the principle of conservation of linear momentum,

(1) =(3): $0.0035\, u = 3.2809$

$u = 937$ m s^{-1}

<u>Note</u>:

You are strongly encouraged to draw “before” and “after” diagrams to apply COLM.

D12

a) (i) refer to lecture notes

(ii) refer to lecture notes

b) Elastic: total kinetic energy is conserved

Head-on: The motions of the molecules after the collision will be along the same straight line of motions before the collision.

c) For elastic collision,

relative speed of separation = Relative speed of approach = $1.88 \times 10^3 + 405 = 2285$ m s^{-1}.

After the collision,

(ii) Applying the principle of conservation of linear momentum,
taking rightwards as positive,

$2.00u\,(1.88 \times 10^3) + 32.0u\,(-405) = 2.00u\, v_H + 32.0\, u\, v_O$

$v_H + 16\, v_O = -4600$ --------------------------(1)

(iii) Relative speed of approach = relative speed of separation,
taking rightwards as positive

$u_H - u_O = v_O - v_H$

$v_O - v_H = 2285$ -----------------------(2)

(1) + (2): $17\, v_O = -2315$

$v_O = -136$ m s^{-1}

Sub into (2): $v_H = -2420$ m s^{-1}

Hence, the velocity of the oxygen molecule is 136 m s^{-1} to the left and the velocity of the hydrogen molecule is 2420 m s^{-1} to the left.

Note:

It is simpler to use relative speeds to solve the simultaneous equations, rather than the conservation of total kinetic energy.

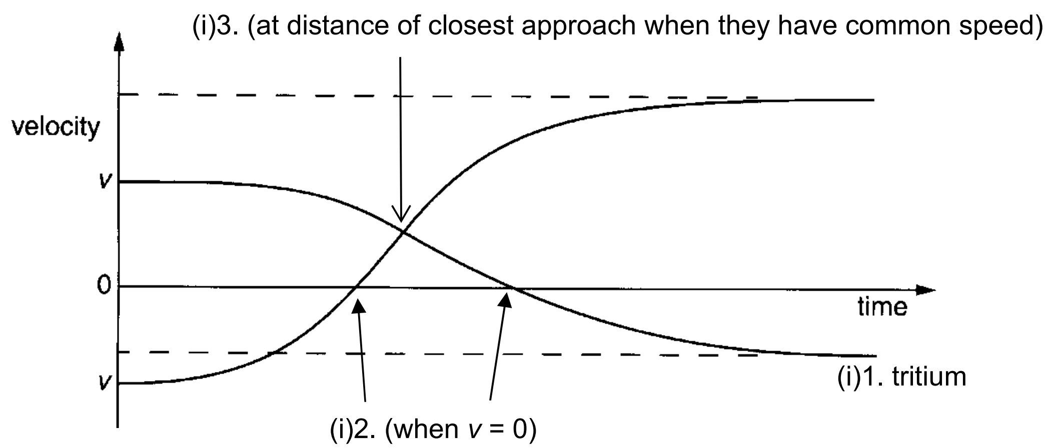
D13

a)
In this case, total momentum before collision = 3*mv* – 2*mv* = *mv* to the right.
By the principle of conservation of linear momentum (PCLM), the total momentum in a closed system is conserved.
Thus at any instant, the total momentum of both the nuclei together is *mv*.
If both the nuclei were to stop at the same instant, total momentum would be zero and the principle would be violated.

b) By Principle of conservation of linear momentum,

$$(3m + 2m)\,v' = mv$$
$$v' = 0.20\,v$$

c)



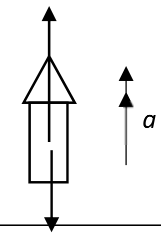
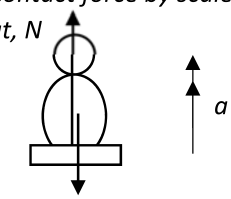
(ii)

By the principle of conservation of linear momentum, taking rightwards as positive,
$m_T u_T + m_D u_D = m_T v_T + m_D v_D$
$(3m)\,v + (2m)(-v) = (3m)v_T + (2m)v_D$
$v = 3v_T + 2v_D$ ----- (1)

For an elastic collision, relative speed of separation equals relative speed of approach,
$u_T - u_D = v_D - v_T$
$v - (-v) = v_D - v_T$
$2v = v_D - v_T$ ----- (2)

Solving (1) and (2), we find $v_T = -0.60v$ and $v_D = 1.40v$.

Hence, the final speed of the tritium atom is **0.60*v*** (directed to the left), while the final speed of the deuterium atom is **1.40*v*** (directed to the right).

D14	**(i)**	For the rocket, taking upwards as positive, $(28.6 \times 10^6) - (2.0 \times 10^6)(9.81) = (2 \times 10^6)a$ a = **4.49 m s^{-2} upwards** 
	(ii)	(ii) By Newton's 2nd Law, $F = m\,a$ $N - 65\,g = 65\,a$ $N = 65\,(a + g)$  Note that the above working only shows normal contact force on man by scale. Thus, it is necessary to apply Newton's third law. By Newton's 3rd Law, the force exerted on scale by man (apparent weight) is 65 (a +g) downwards. The force exerted downwards is (65) (4.49 + 9.81) = 929.5 N downwards. The scale will give a reading in kilograms; it will read (929.5) / (9.81) = 94.8 kg.
	(iii)	For the rocket, the thrust can be shown to be given by $$F = \frac{dm}{dt}u_{rel}$$ Hence, $(2.0 \times 10^4)\,v = 2.86 \times 10^7$ $v = 1.43 \times 10^3\ \text{ms}^{-1}$

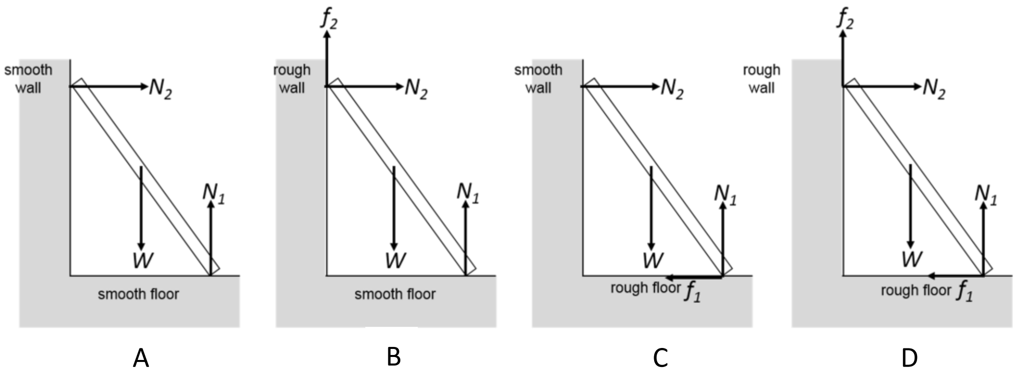
D15	a)	The gases and the car are an isolated system, hence according to the principle of conservation of linear momentum, the total momentum of the system must remain constant all the time. As the rocket expels the exhaust gas backwards, there is an increase in momentum of these gases along their direction of travel. Therefore, there should be an increase in momentum of the car (i.e. car, rocket engine and remaining fuel) in the opposite direction (i.e. forward direction of the car) as the total momentum of the system is conserved. Since there is a change in momentum of the car, from Newton's second law, there must be a forward force acting on the car.
	b)(i)1.	Gradient of the tangent to the velocity-time graph at t = 2.0 s represents the acceleration at that time. Acceleration = (15.2 – 2.8) / (4.60 – 0.10) = 2.76 m s^{-2}
	b)(i)2.	$\Sigma F = m\,a$ $4.6 - F_D = (0.440)(2.76) \Rightarrow F_D = 3.4$ N
	b)(ii)	The gradient of the velocity-time graph decreases with increase of speed. Hence net force decreases with increase of speed. Net force = Driving force – Resistive force With the driving force kept at a constant, the resistive force must increase with increase of speed.
	c)	Estimate the value of the gradient of the velocity-time graph at t = 0 s. From the v-t graph, acceleration at t = 0, a_o = (11.2 – 0.0) / (1.8 – 0.0) = 6.2 m s^{-2}. At t = 0 s, there is no resistive force. $\Sigma F = m\,a$ $4.6 = m_o\,a_o \Rightarrow m_o = 4.6 / 6.2 = 0.74$ kg Upward thrust = 4.6 N Initial weight of car = $m_o g$ = (0.74)(9.81) = 7.28 N Thus, initial weight of the car > initial upward thrust and the rocket cannot be launched.
	Note: *It is hard to estimate the gradient at t = 0 accurately.* *The argument as to whether the rocket could be launched or not should be based on a comparison between weight of rocket and upward thrust.*	

Learning Outcomes	**Discussion Question**
(a) state and apply each of Newton's laws of motion.	D3, D14
(b) show an understanding that mass is the property of a body which resists change in motion (inertia).	
(c) describe and use the concept of weight as the force experienced by a mass in a gravitational field.	D6, D9, PTT10
(d) define and use linear momentum as the product of mass and velocity.	D2, D10, PTT4
(e) define and use impulse as the product of force and time of impact.	PTT2, PTT9
(f) relate resultant force to the rate of change of momentum.	D1, D3, D10
(g) recall and solve problems using the relationship F = ma, appreciating that resultant force and acceleration are always in the same direction.	D1, D4, D5, D6, D7, D8, D9, PTT1, PTT3, PTT7, PTT8, PTT10
(h) state the principle of conservation of momentum.	
(i) apply the principle of conservation of momentum to solve simple problems including inelastic and (perfectly) elastic interactions between two bodies in one dimension. (Knowledge of the concept of coefficient of restitution is not required.)	D4,D15, D10, D11, D12, D13, PTT6
(j) show an understanding that, for a (perfectly) elastic collision between two bodies, the relative speed of approach is equal to the relative speed of separation.	D12, D13, PTT5
(k) show an understanding that, whilst the momentum of a closed system is always conserved in interactions between bodies, some change in kinetic energy usually takes place.	

Tutorial 4 Forces

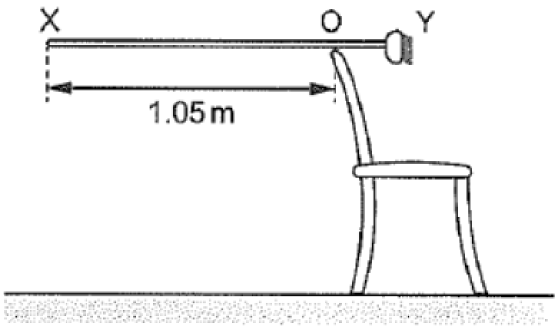
Discussion Questions (Suggested Solutions)

D1. On static equilibrium of a ladder



W : weight of ladder
N_1 : normal contact force by floor on ladder
N_2 : normal contact force by wall on ladder
f_1 : frictional force, by floor on ladder
f_2 : frictional force, by wall on ladder
In scenarios A and B, the ladder will definitely slip, as there is a net force: the horizontal forces are not balanced.

D2. Application of the Principle of Moments



Initially, the centre of gravity (c.g.) is at O. After the mass is tied to the handle, is 27 cm to the right of O, producing a clockwise moment about O.
Similarly, the mass is initially 95 cm to the left of O. After the broom is moved, it is 95 – 27 = 68 cm to the left of O, producing an anticlockwise moment about O.
Taking moments about O, by the principle of moments,
sum of anticlockwise moments = sum of clockwise moments

$(0.200 \times 9.81)(0.68) = (9.81m)(0.27)$

The mass of the broom m = 0.504 kg = 500 g (to 2 s.f.)

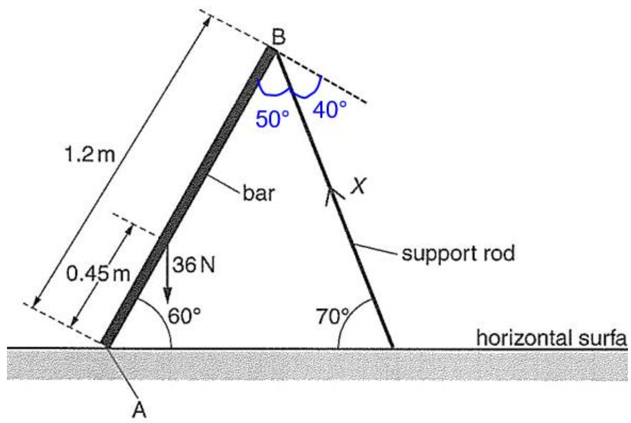
D3. On static equilibrium

(a) The component of X perpendicular to the bar is $X\cos 40°$.

Taking moments about A:
CW moment by 36 N = ACW moment by X

$(36)(0.45\cos 60°) = (X\cos 40°)(1.2)$

$X = 8.81\text{ N}$



bi) Approach 1

For translational equilibrium, the net force acting on the bar must be zero.
The vector sum of the weight and X is a leftward and downward force. There must be a rightward and upward force at A.

Approach 2

For rotational equilibrium, the net moment acting on the bar about any point must be zero.

Taking moments about the C.G., *X* exerts an ACW moment. There must be a force at A that produces a CW moment.

bii) Method 1

Vertically, net force must be zero: $F_y + X\sin 70° = 36$ N

$F_y + (8.81)\sin 70° = 36$

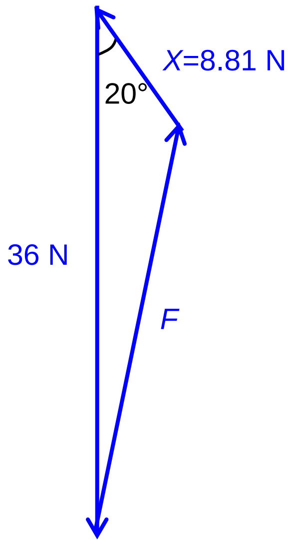
$F_y = 27.7$ N

Horizontally, net force must be zero: $F_x = X\cos 70°$

$F_x = \cos 70° = 3.01$ N

$|F| = \sqrt{F_x^{\ 2} + F_y^{\ 2}} = \sqrt{3.01^2 + 27.7^2} = 27.9$ N

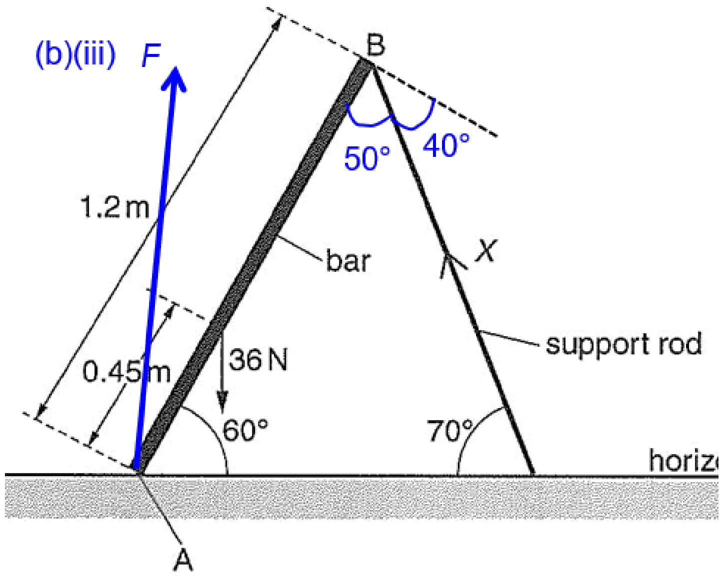
Method 2



Using cosine rule:

$F^2 = 36^2 + 8.81^2 - 2(36)(8.81)\cos 20°$

$F = 27.9$ N

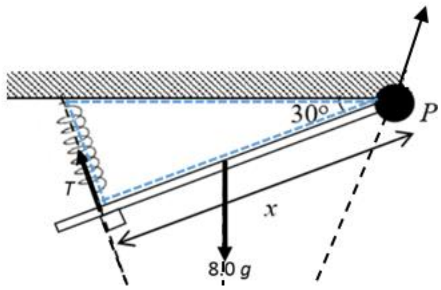


biii) See drawing.

COMMENT: Part (b)(ii) should be used to guide the drawing of *F*. The force's line of action must pass above the bar (because it is supposed to produce a CW moment about the CG to counter the ACW moment of *X*)

You may also recall that in a 3-force system, the lines of action of all 3 forces should intersect.

D4. Application of Hooke's Law. Static equilibrium of a rigid extended body.



Taking moments about P,

sum of clockwise moments = sum of anticlockwise moments

$T\,x = (8.0 \times 9.81)\,(0.25)\cos 30°$

$(500\,e)\,x = (8.0 \times 9.81)\,(0.25)\cos 30°$ ----- (1)

where e is the extension of the spring.

From the blue dashed triangle, using trigonometry,

$\tan 30° = (0.20 + e) / x$

$x \tan 30° = 0.20 + e$

$e = x \tan 30° - 0.20$ ----- (2)

Combining (1) and (2),

$16.99 = 500\,(x \tan 30° - 0.20)\,x$

$16.99 = 500\,x^2 \tan 30° - 100\,x$

$289x^2 - 100x - 16.99 = 0$

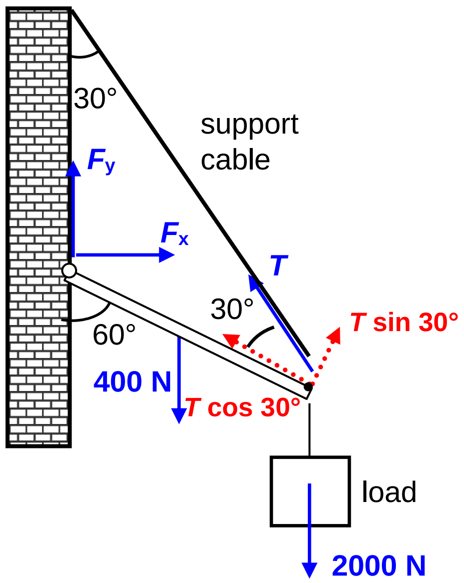
$x = 0.47$ m (accept) or $x = -0.12$ m (reject)

D5. On static equilibrium of a rigid extended body. [RJC/2009/Prelim/P3/Q1]

(a) The moment of a force about a point is the product of the (magnitude of the) force and the perpendicular distance from the line of action of the force to the point.

(b) (i) Taking moments about the hinge,

sum of clockwise moments = sum of anticlockwise moments



$$(400)\left(\frac{L}{2}\sin 60^\circ\right)+(2000)\left(L\sin 60^\circ\right)=T\sin 30^\circ\,(L)$$

$$T = 3810\text{N} \quad (3\text{sf})$$

(ii) Analyse vertical components of forces:

$$\sum F_y = 0$$

$$F_y + T\cos 30^\circ = 2400$$

$$F_y = -900\text{N}$$

$$F_y = 900\text{N (downwards)}$$

Analyse horizontal components:

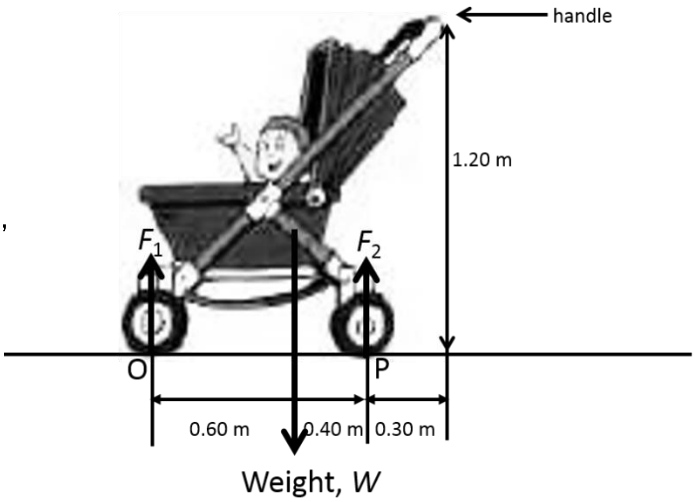
$$\sum F_x = 0$$

$$F_x = T\sin 30^0 \approx 1910\text{N (right)}$$

D6. On static equilibrium of a rigid extended body. [HCI/BT/05/P2/Q3(b)]

(i) Taking moments about the hind wheels,
sum of anticlockwise moments = sum of clockwise moments,
(22.0) (9.81) (0.40) = F_1 (1.00)
F_1 = 86.3 N

Taking moments about the front wheels,
sum of anticlockwise moments = sum of clockwise moments,
F_2 (1.00) = (22.0) (9.81) (0.60)
F_2 = 129 N



(ii) If the stroller topples, it will topple about point P. At the instant of toppling, the normal contact force from the ground acting on the front wheels (F_1) is zero.

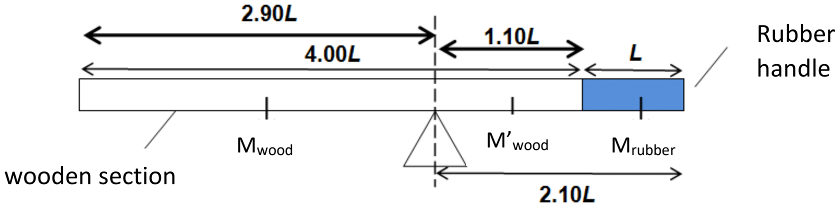
Taking moments about the hind wheels,
sum of anticlockwise moments = sum of clockwise moments,
(22.0) (9.81) (0.40) = W (0.30)
The weight of the groceries is W = 288 N.
Because the perpendicular distance from the handle to point P is 3/4 of the perpendicular distance from the centre of mass to the handle, the load can be 4/3 times as large as the combined weight of the stroller and the baby.

(iii) The baby may shift the centre of gravity towards P, which reduces the anticlockwise moments due to the baby about P.
Parents may lean on the handle and inadvertently press it down, creating an additional clockwise moment about P.
On an upward slope, the anticlockwise moment due to the weight of the stroller would decrease, but the clockwise moment due to the weight of the groceries would increase.

D7. Application of the principle of moments.

Answer: B

Method 1



Let A be the cross-sectional area of the rod.
Taking moments about the pivot,
$M_{wood}g(1.45L) = M'_{wood}g(0.55L) + M_{rubber}g(1.60L)$
$\rho_{wood}(2.90L)A(1.45L) = \rho_{wood}(1.10L)A(0.55L) + \rho_{rubber}(L)A(1.60L)$
$3.600\rho_{wood} = 1.60\rho_{rubber} \Rightarrow$ Ratio = 2.25

Method 2

Note that the centre of gravity of the rod is at the pivot. Hence, the pivot exerts an *upwards* force of magnitude W_{rod} at the pivot.
Taking moments about the left end of the rod,
$W_{rod}\ (2.90L) = W_{wood}\ (2.00L) + W_{rubber}\ (4.50L)$
$(4\rho_{wood} + \rho_{rubber})\ (2.90L) = (4\rho_{wood})\ (2.00L) + \rho_{rubber}\ (4.50L)$
$\rho_{rubber} / \rho_{wood} = 3.6 / 1.6 = 2.25$

D8. On upthrust.

Assume that in air, the upthrust by the air on the solid is negligible. Hence, W_1 is the weight of the solid.
When immersed in a liquid, let the upthrust by the liquid on the solid be U.
Thus, $W_2 = W_1 - U$
Since the solid is totally immersed in the liquid, by Archimedes' Principle, the upthrust is equal to the weight of the liquid displaced by the solid, $U = \rho V_{solid} g$.
Thus volume of the solid, $V_{solid} = (W_1 - W_2) / (\rho g)$

D9. On upthrust.

Answer: C

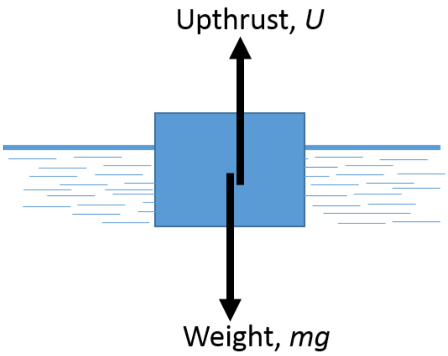
Upthrust acting on bubble = weight of air + viscous drag force
By Archimedes' principle, the upthrust acting on the bubble is equal to the weight of the water displaced,

weight of water displaced = weight of air + viscous drag force
$(1000)\ (2.370 \times 10^{-8})\ (9.81) = (1.290)\ (2.370 \times 10^{-8})\ (9.81)$ + viscous drag force

Hence, the viscous drag force is $F_{VD} = 2.322 \times 10^{-4}$ N

D10. On upthrust.

(a)



By the principle of flotation,

upthrust acting on wood = weight of wood

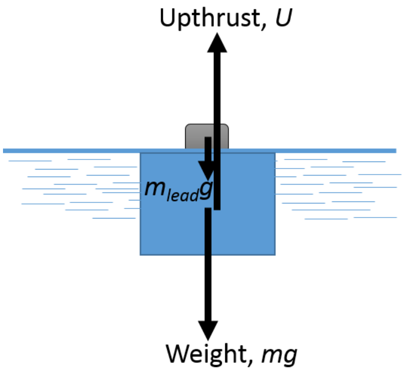
$$U = mg$$

$$\rho_{\text{water}} g A h_{\text{in water}} = \rho_{\text{wood}} g A h_{\text{wood}}$$

$$h_{\text{in water}} = \frac{\rho_{\text{wood}}}{\rho_{\text{water}}} h_{\text{wood}} = \frac{0.65 \times 10^3}{1.00 \times 10^3} 20 = 13 \text{ cm}$$

$$h_{\text{above water}} = 20 - 13 = 7.0 \text{ cm}$$

(b)



By the principle of flotation,

upthrust = weight of wood + weight of lead

$$U = m_{\text{wood}} g + m_{\text{lead}} g$$

$$\rho_{\text{water}}\, g\, A\, h = \rho_{\text{wood}}\, g\, A\, h + m_{\text{lead}} g$$

$$m_{\text{lead}} = 2.8 \text{ kg}$$

D11. On upthrust.

Answer: D

Identify the forces *X*, *Y*, *Z*.

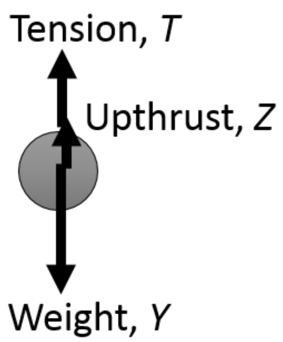
X: weight of the beaker and water

Y: weight of the solid

Z: weight of water displaced by object immersed in water = upthrust on object (by water)

Sketch the FBD of the object immersed in water.

T: tension by the spring balance on object (determine the spring balance reading)

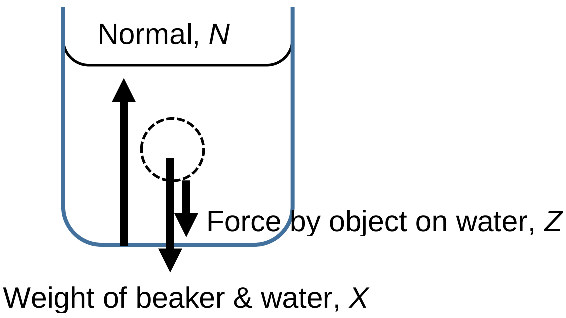


$$T = Y - Z$$

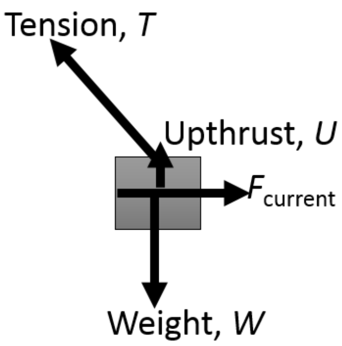
Sketch the FBD of the beaker & water.

N: normal force by weighing machine on Beaker & water (determine the weighing machine reading)

N = *X* + *Z*



D12. On upthrust.



$$\sum F_x = 0 \rightarrow F_{\text{current}} - T \sin 20 = 0$$

$$\rightarrow F_{\text{current}} = T \sin 20 \;\; (1)$$

$$\sum F_y = 0 \rightarrow T \cos 20 + U - W = 0$$

$$\rightarrow T = (W - U) \tan 20 \;\; (2)$$

Substituting equation (2) into equation (1),

$$F_{\text{current}} = (W - U) \tan 20$$

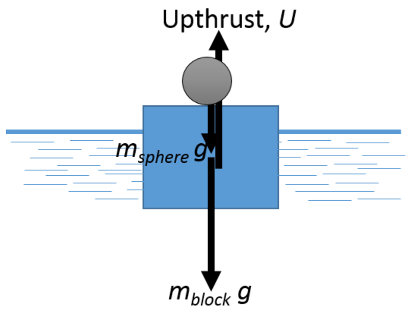
$$= \left(m_{\text{copper}} g - \rho_{\text{water}} g V_{\text{copper}}\right) \tan 20$$

$$= \left(0.50 \times 9.81 - 1 \times 10^3 \times 9.81 \times \frac{0.50}{9 \times 10^3}\right)$$

$$\rightarrow F_{\text{current}} = 1.59 \text{ N}$$

D13. On upthrust.

i.



ii. $\sum F_y = 0 \rightarrow m_{\text{sphere}}g + m_{\text{block}}g = U$

$$U = \rho_{\text{sphere}} V_{\text{sphere}} g + 0.050 \times g$$

$$\rightarrow U = 7.85 \times \frac{4}{3}\pi(0.70)^3 \times 9.81 \times 10^{-3} + 0.050(9.81)$$

Upthrust acting on the system (by the oil), $U = 0.601\ \text{N}$

iii. By Archimedes' Principle,

upthrust on system by oil = weight of oil displaced by block

$$U = \rho_{\text{oil}} g V_{\text{displaced}}$$

Hence, the volume of oil displaced by the wooden block $V_{\text{displaced}} = \frac{0.601\times10^3}{0.83\times9.81} = 73.8\ \text{cm}^3$

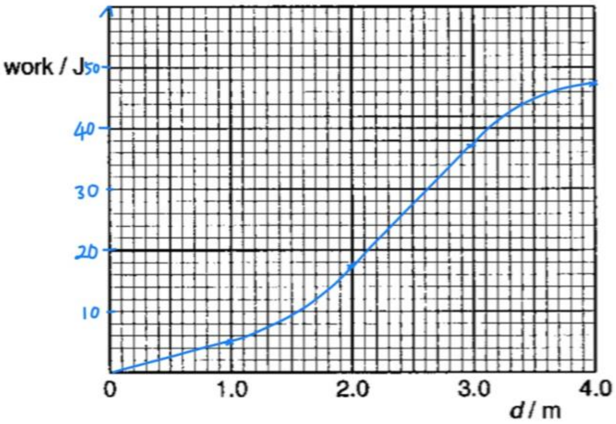
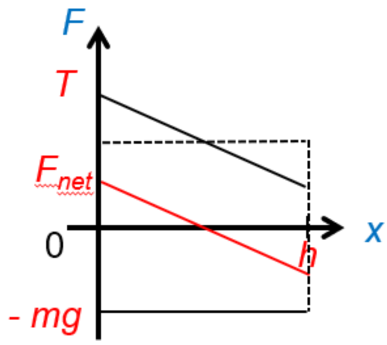
iv. Before the spherical ball dropped into the oil, the upthrust on the block was $U = m_{sphere}g + m_{block}g$. After it dropped into the oil, the upthrust on the block becomes $U = m_{block}g$.

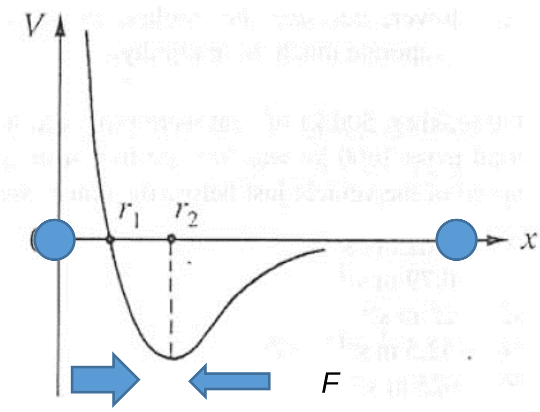
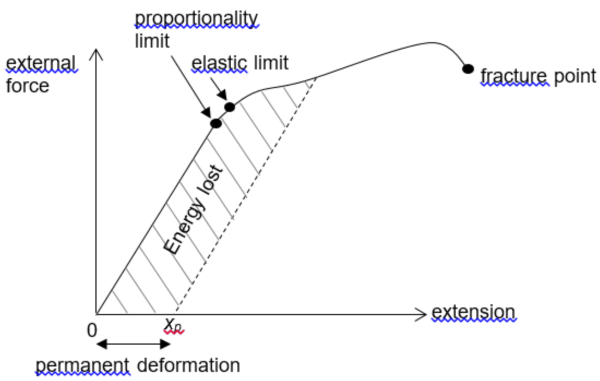
The spherical ball dropped into oil is still displacing some oil, so it still experiences an upthrust. However, this upthrust is no longer supporting the ball's weight, so the upthrust acting on the ball is much smaller than the ball's weight. Hence, the total upthrust provided by the oil is smaller than before, which means that less oil is displaced than before: the oil level is lower than before.

Numerical solution:

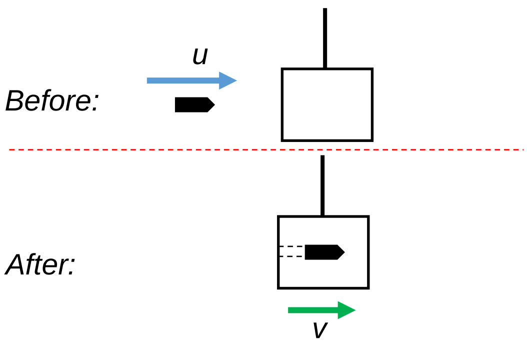
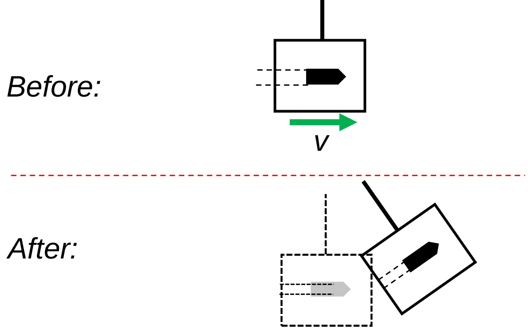
	Upthrust (while the sphere is on the block)	Volume of oil displaced by block (while the sphere is on the block)	Upthrust (after the sphere fell into the oil)	Volume of oil displaced by sphere and block (after the sphere fell into the oil)
Sphere	0.11 N (weight of sphere)		$\rho_{\text{oil}} \times g \times 1.44$ = 0.012 N	$\frac{4}{3}\pi(0.7)^3$ = 1.44 cm^3
Block	0.4905 N (weight of block)		0.4905 N	$0.4905 = \rho_{\text{oil}} \times g \times V_{\text{disp}}$ $V_{\text{disp}} = 60.2$ cm^3
Total	**0.601 N (calculated earlier)**	**73.8 cm^3 (calculated earlier)**	**0.502 N**	**61.6 cm^3**

Topic 5 Work Energy Power
Suggested Solutions to Discussion Questions

Qn	Ans	Explanation
D1	**[2008/P3/Q1(part)]**  F = gradient of W-d graph, W = area under F-d graph, From d = 0 to 1.0 m, W increases linearly (as F is positive & constant), from 0 to 1.0 x 5.0 = 5.0 J From d =1.0 to 2.0 m, W increases at increasing rate (as F is positive & increasing), from 5.0 to [5.0 + (½)(15)(1.0)] = 17.5 J From d = 2.0 to 3.0 m, W increases linearly (as F is positive & constant), from 17.5 J to [17.5 + 1.0(20)] = 37.5 J From d = 3.0 to 4.0 m, W increase at decreasing rate (as F is positive & decreasing), from 37.5 J to [37.5 + (½)(1.0)(2.0)] = 47.5 J	
D2	**C**	**[CIE June 2003]** There is no change in kinetic energy, since the trolley starts and finishes at rest, so the weight is only gaining gravitational potential energy. work done by tension force (trolley) $= \Delta E_P = mg\Delta h = Wq$ To better appreciate and understand the situation, one possible scenario is as shown in the F- x graphs. Tension force, T is a varying force, weight mg acts in opposite direction. Comparing the area under each graph, positive work done by T = negative work done by mg Total work done = work done by net force F_{net} = 0 J thus, satisfying the condition that the weight is lifted from rest and finishes at rest with no change in KE. 

D3	**C**	*F = -dU/dx or the negative of the gradient of U-x graph.* The force on the body is the negative gradient of the potential energy-position graph. Since the gradient of the graph is positive and constant, the force is negative and constant, implying that the force acts in the opposite direction to the displacement. In other words, the force acts towards the origin, in the direction towards lower potential energy.
D4	**E**	$F = -dU/dx$ The force between the molecules is the negative of the gradient of the potential energy (of the 2 molecules)-distance graph.  When $x < r_2$, the gradient of tangent line at a point on the graph is negative, the intermolecular force is thus positive (in the direction of +x), it is repulsive. When $x > r_2$, the gradient of tangent line at a point on the graph is positive, the intermolecular force is negative (in the opposite direction to +x), it is attractive. Further reference: https://www.schoolphysics.co.uk/age16-19/Properties%20of%20matter/Elasticity/text/Intermolecular_forces/index.html
D5	**D**	**[H1 N2009/P1/Q6 & 2015 P1 Q11]** Recall Work done by a force = area under the force-displacement graph. Hence option D is correct.  As for option A, work done by *F* is equal to elastic potential energy stored in the wire only when the wire is within its elastic limit. Since the wire is stretched beyond its elastic limit, (as seen on the left) not all work done by *F* becomes elastic potential energy, the energy is lost and cannot be recovered when releasing the load.
D6	**C**	**[H1 2010/P1/Q11]** Area under graph from $x = 0$ to extension up to point Q = $\int F dx$ which is the work done by the applied force. When the force is released, only Z is returned as energy released by the wire. Since Z is the energy released by wire, the EPE stored in wire at Q is Z. Y is usually energy lost as heat in the material.

D7	**A**	**[J1987/P1/Q5]** When an object rises to the top, its KE will be totally converted into GPE: Loss in KE = Gain in GPE $\frac{1}{2}mv^2 - 0 = mgh \Rightarrow h = \frac{v^2}{2g}$ From this we can see that $h \propto v^2$. $\Rightarrow \frac{h_1}{h_2} = \left(\frac{v_1}{v_2}\right)^2$ $\frac{h}{h_2} = \left(\frac{v}{\frac{1}{2}v}\right)^2$ $h_2 = \frac{1}{4}h$	
	when there is no work done by external force, by conservation of energy, *total initial energy = total final energy* i.e. $KE_{initial} + PE_{initial} = KE_{final} + PE_{final}$ (1st possible starting point) i.e. $KE_{initial} - KE_{final} = PE_{final} - PE_{initial}$ or $KE_{final} - KE_{initial} = PE_{initial} - PE_{final}$ i.e Loss in KE = Gain in PE or Gain in KE = Loss in PE (2nd possible starting point) i.e. $-\Delta KE = \Delta PE$ or $\Delta KE = -\Delta PE$ i.e. $\Delta KE + \Delta PE = 0$ (3rd possible starting point) Realise that all the above Energy equations are equivalent, but we choose the one that is more convenient for our problem solving.		
D8	**[College Physics by Serway & Faughn]**		
	Loss in GPE of mass-spring system = 2.00 x 9.81 x 0.200 x sin 37° =2.3615 J		M1
	Gain in EPE of mass-spring system = ½ $k\,x^2$ = ½ x 100 x 0.200^2 = 2.00 J		B1
	By Conservation of Energy, Loss in GPE = Gain in EPE + WD against friction WD against friction = Loss in GPE – Gain in EPE F_{fr} x 0.200 = 2.3615 – 2.00 = 0.3615 $\Rightarrow F_{fr}$ = 1.81 N.		A1
	OR Work done by friction = change in GPE + change in EPE $W = \Delta E_p + \Delta E_s$ $F_{fr} s \cos 180° = mg\Delta h + \frac{1}{2}kx^2$ $F_{fr}\left(-\frac{20}{100}\right) = (2.00)(9.81)\left(-\frac{20}{100}\right)(\sin 37°) + \frac{1}{2}(100)\left(\frac{20}{100}\right)^2$ $F_{fr} = 1.81\text{ N}$		

D9(a)	**A**	**[J1988/P1/Q6]** Heat generated by friction = work done against friction = $F x$ (since the small block would have moved up the slope by distance x when the large block had moved down by x)
D9(b)	By the Principle of Conservation of Energy, Loss in GPE of M = Gain in KE of M & m + Gain in GPE of m + Work done against friction Gain in KE of M & m = Loss in GPE of M – Gain in GPE of m – Work done against friction $= Mgx - mgx \sin\theta - Fx$	
D10	**C**	**[2016 Specimen P1Q6]** $E_k + E_p = E_{Total}$ where E_{Total} is constant At the starting point, $E_{Total} = E_k = ½ mu^2$, at the top of flight, v = u cos 45° $E_k = ½ m(u^2 \cos^2 45°) = ½ \times ½ mu^2 = ½ E_{Total}$, $E_p = E_k$
D11	**C**	**[HCI Promo 1999/1/6]** Consider the block and bullet before and immediately after collision:  Note: KE of bullet is not conserved as the collision with the block is an inelastic collision. By principle of conservation of momentum $(\rightarrow)\ 0.010u + 0 = (2.010)(v)$ $u = 201v \cdots\cdots(1)$ Consider the block (with bullet) rising up to maximum height:  By principle of conservation of energy, Loss in KE = Gain in GPE $\frac{1}{2}m(v^2 - 0) = mg(0.20)$ $v^2 = 0.40g$ $v = \sqrt{0.40g} \cdots\cdots(2)$ Sub (2) to (1): $u = 201\sqrt{0.40g} = 398 \text{ m s}^{-1}$

D12	**B**	**[N2012/1/10, modified]** The GPE obviously decreased as the final position is lower than the starting point. Hence the correct option is either choice A or B. We know that there must be stored energy in the spring, so it is a matter of how much. Since the spring came to rest after some time due to resistive forces, there must be some loss of energy to the surrounding (and also heat in the spring). The gain in EPE must hence be less than the loss in GPE. Hence, choice B. If we want to verify the answer, we can do so too. At equilibrium position, spring force = weight = *mg* Hence, $E_s = \frac{1}{2}kx^2 = \frac{1}{2}Fx = \frac{1}{2}(mg)x = (0.5)(4.2)(9.81)(0.29) = 6.0$ J	
D13	**[H1 2009 P2 Q7]**		
(ai)	Before the rope becomes taut, man is in free fall. Assume negligible air resistance, Take vectors downwards as positive. $s = ut + ½ at^2$ $41 = 0 + ½ (9.81) t^2$ t = 2.89 s or 2.9 s (2 s.f.)		M1 A1
(aii)	There is no significant difference between the theoretical time and the 2.9 s quoted. Therefore, air resistance is insignificant		B1
(bi)	Extension = 73 – 41 = 32 m		A1
(bii)	From the area under the F-x graph from x = 0 to x = 32, E_s = ½ (32)(3400) = 54400 J		M1 A1
(ci)	(see table below)		
(ii)	**1.** He has maximum KE when he reaches his equilibrium position, where net force = 0, acceleration is zero, velocity is maximum. At equilibrium point, net force = 0 Tension = Weight = (75)(9.81) = 740 N From graph, Extension, e = 7 m when tension is 740 N Total distance fallen = 41 + 7 = 48 m		 M1 A1

(ci)

	At the top	After falling 41 m	After falling 73 m (i.e. when stopped)
Gravitational potential energy / J	54 000	54 000 – 30 000 = 24 000	**0**
Elastic potential energy/ J	0	0	54 000
Kinetic energy / J	0	75 x 9.81 x 41 = 30 000	0

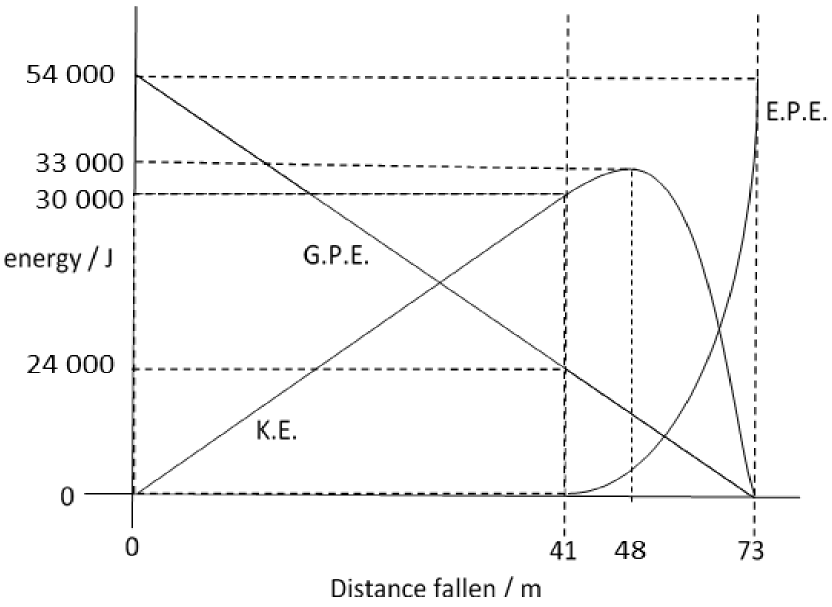
2. By conservation of energy,

Loss in E_p = Gain in E_k + Gain in E_s — M1

$(75)(9.81)(48)$ = Max. KE + $\frac{1}{2}k(7^2)$ — B1

Maximum KE = 33000 J — A1

(iii) — B4



To associate the graphs to theory:

$E_p = mgh$ ∴ GPE varies linearly with fallen distance

$E_s = 0$ (before rope is taut)

$E_s = \frac{1}{2}kx^2 = \frac{1}{2}k(\text{total fallen distance} - 41)^2$ ∴ Varies quadratically

E_k = Total energy – GPE – EPE

= constant – linear – quadratic ∴ Varies quadratically

D14 **D** **[H2 2010/P1/Q11]**

$P = Fv = mav$

a constant $\Rightarrow$ $v^2 = 2as$ since $u = 0$.

Thus $P = ma\sqrt{2as}$ $\Rightarrow$ $P \propto \sqrt{s}$

D15 **D** **[N2009/P1/Q12]**

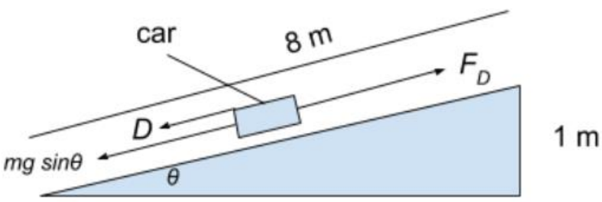
Since the car is moving at a constant speed along the horizontal road.

acceleration and therefore net force = 0

Drag force at this speed, D = driving force = 200 N

On the incline, since the car is travelling at the same speed, drag force D remains the same.

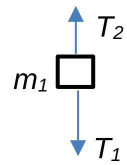
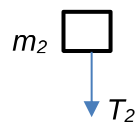
Let the new driving force be F_d

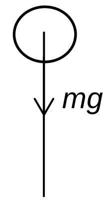


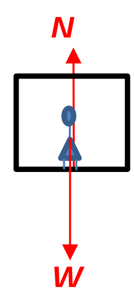
		Along the slope : $\sum F = 0$ $F_d - D - mg\sin\theta = 0$ $F_d = D + mg\sin\theta = 200 + (800)(9.81)(1/8) = 1181$ N Power supplied when traveling with this speed = $F_d v$ = 1181(20) = 23 600 W Alternatively, As the car is traveling at a constant speed on the level road resistive force, D = driving force = 200 N When the car rises 1 m for each 8 m of travel along the road on the slope, Rate of change of GPE along the slope = $mgh/t = mgv\sin\theta = \frac{1}{8}mgv$ ∴Total power = rate of work done against resistive force + rate of increase GPE $= Dv + \frac{1}{8}mgv$ $= 200 \times 20 + \frac{1}{8}(800)(9.81)(20) = 23\,600\text{W} = 24\text{ kW}$
D16	**D**	**[N2009/P1/Q13]** Since $P = Fv$, and drag force is proportional to the square of the velocity, power must be proportional to cube of the velocity, meaning $P = kv^3$, where k is some constant. The situations of 2 engines and 1 engine yield the following two equations. $72000 = k(12)^3$ $36000 = kv^3$ Solving them gives us $\frac{v}{12} = \sqrt[3]{\left(\frac{1}{2}\right)}$, or v = 9.52 m s^{-1}.
D17	**B**	**[CIE/N2011/1/16, modified]** $\langle P \rangle = \frac{\Delta E_P}{\Delta t} = \frac{mg\Delta h}{\Delta t} = mgv$
D18	**D**	**[HCI/2007/BT2/P1/Q7]** Consider a column of air of length L and cross-sectional area A approaching the windmill perpendicularly. KE per unit time contained in this column = $(\frac{1}{2}mv^2)/t = [\frac{1}{2}(\rho AL)v^2]/t = \frac{1}{2}\rho Av^3$ Since efficiency is 60%, power harnessed = $(0.60)(\frac{1}{2}\rho Av^3) = (0.60)(\frac{1}{2})(1)(60.0^2\pi)(8.00^3) = 1.74$ MW

Circular Motion Discussion Questions Suggested Solutions

D1		Given mass, $m = 2.0 \times 10^{-3}$ kg (S.I unit) and time for 3 revolutions, $3T = 3.14$ s $\Rightarrow T = 3.14/3$ s (a) $\theta = \omega t = \frac{2\pi}{3.14/3} \times (2) = 12.0 \text{ rad} = 12.0 - 2\pi = 5.7 \text{ rad}$ (b) $r = 0.05$ m $v = r\omega = \frac{2\pi}{3.14/3} \times 0.05 = 0.30\ ms^{-1}$ (c) $a = r\omega^2$ or $a = \frac{v^2}{r} = \frac{(0.3001)^2}{5.0 \times 10^{-2}}$ = 1.800 = <u>1.8 m s^{-2}</u> (d) Note: **Frictional force, *f*, provides the centripetal force, F_c.** $f = F_c = ma_c$ $= (2.0 \times 10^{-3})(1.800)$ $= 3.600 \times 10^{-3}$ = <u>3.6×10^{-3} N</u> Writing this statement is necessary to justify why friction is equated to the centripetal force in the next step of working.
D2		Tension, $T = kx = 40 \times 0.20 = 8.0$ N **Tension provides the centripetal force**, $T = \frac{mv^2}{r}$ $v = \sqrt{\frac{Tr}{m}} = \sqrt{\frac{8 \times 0.70}{0.050}} = 10.58 = 11 \text{ m s}^{-1}$ Answer: **A**
D3		The external forces acting on the pendulum are tension (strictly speaking, force of string on pendulum) and weight. The **horizontal component of the tension provides the centripetal force** for the pendulum to turn right. (Note that the **centripetal force should not be drawn as a third force**.) Answer: **C**
D4		**Frictional force <u>of road on car</u> *F* provides for centripetal force** for car to round the corner. $F = m\frac{v^2}{r}$ Since m and r remain the same, $F \propto v^2$ $\frac{F_{wet}}{F_{dry}} = (\frac{v_{wet}}{v_{dry}})^2$ $\frac{1}{2} = (\frac{v_{wet}}{20})^2$ $v_{wet} = \frac{20}{\sqrt{2}}$ Answer: **D**

D5	(a) (b) (c)	Diagram 1. **Frictional force of wall on person** keeps him/her from sliding down. $a_c = r\omega^2 = 3.00 \times 5.00^2 = 75.0\ \mathrm{m\,s^{-2}}$ $F = ma_c = 60 \times 75 = 4500\ \mathrm{N}$ **Normal contact force of wall on person provides for the centripetal acceleration**.
D6	(a) (b)	For m_1, $T_1 - T_2 = m_1 a_1$ (Newton's second law) $a_1 = \frac{T_1 - T_2}{m_1} = \frac{4.5 - 2.9}{2.5} = 0.64\ \mathrm{m s^{-2}}$  For m_2, $T_2 = m_2 a_2$ (Newton's second law) $a_2 = \frac{T_2}{m_2} = \frac{2.9}{3.5} = 0.83\ \mathrm{m s^{-2}}$  $a = \frac{v^2}{r}$ $v_1 = \sqrt{a_1 r_1} = \sqrt{0.64 \times 1.0} = 0.80\ \mathrm{m s^{-1}}$ $v_2 = \sqrt{a_2 r_2} = \sqrt{0.83 \times 1.3} = 1.04 = 1.0\ \mathrm{m s^{-1}}$
D7	(a) (b) (c)(i) (ii) (iii)	The **tension in the string** created by the weight, *W* of the washers **provides the centripetal force** for the bung to perform circular motion. If we let *M* = mass of washers and *m* = mass of rubber bung, then $Tension = W = Mg = mr\left(\frac{2\pi}{T}\right)^2$ $0.035 \times 9.81 = m(0.57)\left(\frac{2\pi}{\frac{18.2}{20}}\right)^2 = 12.6g$ If glass rod is twirled at just the right pace the paper clip can be maintained in a position just below the bottom of the glass tube. This ensures that the radius is kept constant at a known radius. $v = \sqrt{\frac{Mgr}{m}} = \sqrt{\frac{95 \times 10^{-3} \times 9.81 \times 0.57}{12.6 \times 10^{-3}}} = 6.49\ \mathrm{m\ s^{-1}}$ There must exist an upward vertical component of the tension to balance the weight of the rubber bung. If the string was purely horizontal no such component could exist. In the vertical direction, forces acting on rubber bung $T \sin\theta = \mathrm{mg}$ Hence, $\sin\theta = \frac{mg}{Mg} = \frac{12.6 \times 10^{-3}}{95 \times 10^{-3}}$ $\theta = 7.62^{\circ}$

D8	(a)	String is just taut when particle reaches C $\rightarrow$ Tension $T = 0$ **Weight of particle provides for centripetal force** $mg = \frac{m{v_C}^2}{L}$ $v_C = \sqrt{gL}$ (shown) 
	(b)	**By conservation of energy,** **Initial total energy at the bottom = Final total energy at the top** $\frac{1}{2}mv^2 = \frac{1}{2}m{v_C}^2 + mg(2L)$ $\frac{1}{2}v^2 = \frac{1}{2}gL + g(2L)$ $v^2 = gL + 4gL$ $v = \sqrt{5gL}$
D9	(a)	Let v be the tangential velocity at A and m be the mass of the roller coaster. Using conservation of energy, Initial KE + initial GPE = final KE + final GPE $\frac{1}{2}mv_o^2 + mgh = \frac{mv^2}{2} + \frac{2}{3}mgh$ $\frac{1}{2}mv_o^2 + \frac{1}{3}mgh = \frac{mv^2}{2} \quad ----(1)$ The resultant force on the roller coaster at A provides the centripetal force, Using N2L, $W - N = \frac{mv^2}{R}$ When $N = 0$, v is maximum $mg = \frac{mv^2}{R}$ $\frac{1}{2}Rmg = \frac{mv^2}{2} \quad ----(2)$ Sub (2) into (1), when v is maximum, v_o will also be maximum: $\frac{1}{2}mv_o^2 + \frac{1}{3}mgh = \frac{1}{2}Rmg$ $v_o^2 = Rg - \frac{2}{3}gh$ $v_o = \sqrt{g(R - \frac{2}{3}h)}$

	(b)	**By conservation of energy,** **Initial total energy = Final total energy at B** $\frac{1}{2}mv_o^2 + mgh = mgh'$ (just makes it to B $\Rightarrow$ final KE = 0) $gh' = \frac{1}{2}v_o^2 + gh$ $h' = \frac{1}{2}\frac{v_o^2}{g} + h$ $h' = \frac{1}{2}\frac{Rg - \frac{2}{3}gh}{g} + h$ $h' = \frac{2}{3}h + \frac{1}{2}R$
D9	(c)	The acceleration is not uniform.
D10		At point C, particle's velocity is entirely horizontal. i.e. $v = -3.0\ \text{m s}^{-1}$ By conservation of energy, Total energy at A = total energy at C $KE_A = GPE_C + KE_C$ $\frac{1}{2}m\left(v_A^2 - v_C^2\right) = mg(2R)$ $R = \frac{5.0^2 - (-3.0)^2}{4g} = 0.408\ \text{m}$ $a_c = \frac{v^2}{R} = \frac{3^2}{0.408} = 22.1\ \text{m s}^{-2}$ Note that $a_c = r\omega^2$ is not applicable in this context. This is because the angular speed of the particle is not constant throughout its vertical circular motion. The centripetal acceleration varies as the speed of the particle changes. Answer: **D**
D11	(a)	$v = \frac{2\pi r}{T} = \frac{2\pi \times 7.0}{20.0} = 2.20\ \text{m s}^{-1}$
	(b)	$F_c = \frac{mv^2}{r} = \frac{50 \times 2.2^2}{7.0} = 34.6\ \text{N}$
	(c)	Considering the forces acting on Sasha at the top of the ride and using N2L: $W - N = \frac{mv^2}{r}$  For Sasha to feel weightless at the top of the ride, $N = 0$ $\frac{mv^2}{r} = mg$ $v = \sqrt{rg} = \sqrt{7.0 \times 9.81} = 8.29\ \text{m s}^{-1}$
	(d)	At the bottom of the Ferris wheel, $\frac{mv^2}{r} = N - mg$ $N = mg + \frac{mv^2}{r} = 2mg = 2 \times 50 \times 9.81 = 981\ \text{N}$ O

D12

(a)

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{37 \times 60} = 2.8303 \times 10^{-3} = 2.83 \times 10^{-3} \text{ rad s}^{-1}$$

(b)

$$v = r\,\omega = \left(\frac{150}{2}\right) \times (2.8303 \times 10^{-3}) = 0.21227 = 0.212 \text{ m s}^{-1}$$

(c)

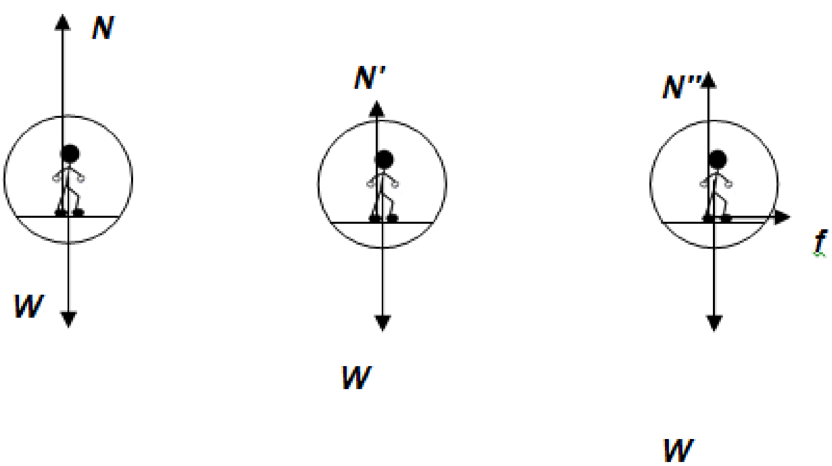
$$t = \frac{\Delta\theta}{\omega} = \frac{2 \times \left(\frac{2\pi}{32}\right)}{2.8303 \times 10^{-3}} = 138.75 = 139 \text{ s}$$

or

$$t = \frac{37 \times 60}{32} \times 2 = 139 \text{ s}$$

(d)

At the bottom | At the top | At the side (horizontal)



N significantly longer than *W* ($N \approx 1.5W$)

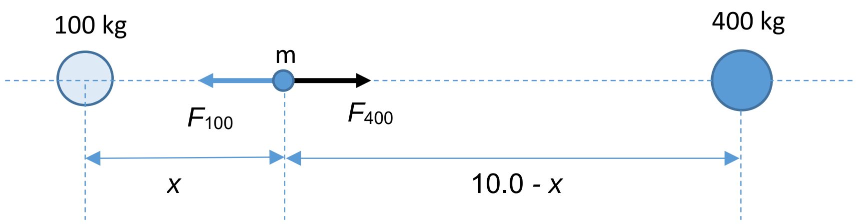
N' significantly shorter than *W* ($N \approx 0.5\,W$)

N'' = *W*
$f \approx 0.5\,W$, so about half its length

N: Normal contact force with the capsule floor
W: weight of passenger
f: friction between passenger's feet and capsule floor

Gravitation Tutorial 7A Discussion Questions Suggested Solutions

D1



$F_{100} = F_{400}$

$$G\frac{(100)m}{x^2} = G\frac{(400)m}{(10.0-x)^2}$$

OR simply $g_{100} = g_{400}$

$$G\frac{(100)}{x^2} = G\frac{(400)}{(10.0-x)^2}$$

x = 3.33 m

NOTE: The concept of finding the point where the resultant force acting on a mass due to the two masses equals zero is equivalent to finding the point where the resultant gravitational field strength due to the two masses equals zero.

D2 **B**

Vector sum of gravitational field strengths at B due to each of the 3 bodies = 0 at the neutral point

Gravitational field strength at a point due to a mass is proportional to the mass and inversely proportional to the square of the distance between the mass and the point.

Consider Earth and Moon's interaction first, the point where the resultant gravitational field strength is zero is nearer to the smaller mass (refer to D1 where g = 0 is nearer the 100 kg mass). Then consider Moon and Sun's interaction, which the neutral point will be shifted nearer to the Sun, but it will be still nearer to the smaller mass.

Good to know:

Mass of the Earth = 5.97 x 10^{24} kg,

mass of the Moon = 7.35 x 10^{22} kg,

mass of the Sun = 1.99 x 10^{30} kg

D3 $g_E = G\frac{M_E}{R_E^2} = 9.81\, m s^{-2}$

The concept of using ratio is a common method to solve many physics problems.

(a) $g \propto \frac{1}{R_E^2} \Rightarrow \frac{g_{new}}{g_E} = \frac{R_E^2}{R_{new}^2}$

$$g_{new} = \frac{R_E^2}{(2R_E)^2} g_E = \frac{1}{4} g_E = 2.45\, m s^{-2}$$

(b) $g = \frac{GM}{R^2} = \frac{G\left(\frac{4}{3}\pi R^3 \rho\right)}{R^2} = \frac{4}{3}\pi G \rho R$

$g \propto \rho R$

Same radius but twice the density → g' = 19.6 m s^{-2}

(c) Half radius but twice the density → g' = 9.81 m s^{-2}

D4 **B**

$g = \frac{GM}{r^2} = \frac{G\left(\frac{4}{3}\pi r^3 \rho\right)}{r^2} = \frac{4}{3}\pi G \rho r \;\Rightarrow\; g \propto \rho r$

The concept of using ratio again similar to D3.

$\frac{g_E}{g_M} = \frac{\rho_E r_E}{\rho_M r_M} \;\Rightarrow \frac{6}{1} = \frac{5}{3}\frac{r_E}{r_M} \;\Rightarrow \frac{r_E}{r_M} = \frac{18}{5} = 3.6$

D5 Apparent weight of zero means no force on its support → normal force by ground on it = 0

The entire gravitational force provides the centripetal force.

$W - N = ma_c$ (since N = 0)

$W = ma_c$

$mg = mr\left(\frac{2\pi}{T}\right)^2$

Taking g at the surface to be 9.81 m s^{-2}, and the given radius of Earth, T = 5100 s (2 s.f.)

D6 (2009 P3 Q5)

(a)(i) The gravitational field strength at a point is the **gravitational force per unit mass** acting on a small test mass placed at that point.

(ii) Newton's Law of Gravitation states that every point mass (or particle) attracts every other point mass (or particle) with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

Thus $F = \frac{GMm}{R^2}$

Since $F = mg$,

$mg = \frac{GMm}{R^2}$

$\therefore g = \frac{GM}{R^2}$

(b)(i) Average density = $\frac{\text{total mass}}{\text{total volume}} = \frac{5.2 \times 10^{30}}{\frac{4}{3}\pi\left(1.7 \times 10^{4}\right)^3} = 2.53 \times 10^{17}\ kg\,m^{-3}$

(ii) Gravitational force acts toward the centre of the neutron star; all the particles are pulled inwards. The outer layers compress the inner layers, resulting in increased density towards the centre.

(c)(i) $g = \frac{GM}{R^2} = \frac{\left(6.67\times10^{-11}\right)\left(5.2\times10^{30}\right)}{\left(1.7\times10^{4}\right)^2} = 1.20\times10^{12}\ m s^{-2}$

(ii) $a = r\omega^2 = (1.7 \times 10^4)\left(\frac{2\pi}{0.21}\right)^2 = 1.52 \times 10^7$ m s^{-2}

(iii) Suppose there is normal contact force *N* acting on the particle by the surface of the star. Applying Newton's 2nd Law, centripetal force = gravitational force – normal contact force, i.e. *ma = mg – N,*
Based on the computed values of *a* and *g*, ***N*** = *m (g – a)* **> 0**.
Thus the particle does not leave the surface, but <u>remain in contact with the surface</u>. The gravitational force is more than enough to provide the required centripetal force.

D7 CIE J96/III/2(part)

(c)(i) $F_g = \frac{GMm}{R^2} = \frac{\left(6.67\times10^{-11}\right)\left(5.98\times10^{24}\right)(1.00)}{\left(6370\times10^{3}\right)^2} = 9.83\,N$

(ii) $F_c = mr\omega^2 = (1.00)(6370\times10^3)\frac{4\pi^2}{(24\times60\times60)^2} = 0.0337\,N$

(iii) $F_g - T = F_c = 0.0337$
$T = 9.80$ N

(d)(i) 9.83 m s^{-2}

(ii) 9.80 m s^{-2}

(e) The acceleration due to gravity is actually larger than the measured acceleration by the amount of centripetal acceleration due to the Earth's rotation. What the student measured should be called the acceleration of free fall.

D8 2018 P1 Q11

C
Without firing its rocket, gravity or gravitational force is the only force acting on the spacecraft. Under the influence of gravity alone, it is possible for paths A, B (spacecraft in circular motion) and D (spacecraft projected away from Earth).

D9 2021 P2 Q4(b)

Gravitational force provides for centripetal force.

$$\frac{GMm}{r^2} = mr\omega^2$$

$$GM = r^3\omega^2$$

$$\left(6.67\times10^{-11}\right)\left(6.0\times10^{24}\right) = r^3\left(\frac{2\pi}{110\times60}\right)^2$$

$$r = 7.615\times10^6 \text{ m}$$

$$g = \frac{GM}{r^2} = \frac{\left(6.67\times10^{-11}\right)\left(6.0\times10^{24}\right)}{\left(7.615\times10^6\right)^2} = 6.9 \text{ N kg}^{-1}$$

D10 2017 P2 Q2 (Part)

2 (a)(i)

$$T = \frac{2\pi r}{v} = \frac{2\pi\left(1.75\times10^7\right)}{200} = 549778 \text{ s} = \frac{549778}{24\times3600} = 6.3632 = 6.36 \text{ days}$$

Note: the distance and velocity must be converted to S.I. units.

Since this is a "show" question, students should express the answer in terms of seconds first then show the steps to convert to days. Should also show that they have hit the calculator by writing the more exact value of 6.3632.

2 (a)(ii) 1. Charon will be above the same place on the surface of Pluto at all times.

2 (a)(ii) 2. The same face of Charon will be seen from Pluto at all times.

2 (b)

$$g_P = \frac{GM_{Pluto}}{r_{Pluto}^2} = \frac{\left(6.67\times10^{-11}\right)\left(1.31\times10^{22}\right)}{\left(1.20\times10^6\right)^2} = 0.607 \text{ N kg}^{-1}$$

D11 2016 P3 Q9 (part)

(i) The gravitational forces of attraction between the two stars provide the centripetal forces necessary for the stars to go in circular motion. This pair for forces constitute a Newton's 3rd law action-reaction pair, which are always equal in magnitude (and opposite in direction).

(ii) Assuming it is Earth year, $\omega = \frac{2\pi}{T} = \frac{2\pi}{4.0\times365\times24\times60\times60} = 4.98\times10^{-8}$ rad s^{-1}

(iii) Since the centripetal forces are equal in magnitude,

$$M_A r_A \omega^2 = M_B r_B \omega^2$$

Since they also have common ω, $r_B = d - r_A$, we can re-arrange the above equation:

$$r_A = \frac{M_B}{M_A}(d - r_A)$$

$$\frac{M_A}{M_B} = \frac{(d - r_A)}{r_A} = \frac{d}{r_A} - 1$$

$$\frac{M_A}{M_B} + 1 = \frac{d}{r_A}$$

$r_A = \frac{d}{\left(\frac{M_A}{M_B}+1\right)} = \frac{3.0\times10^{11}}{(3.0+1)} = 7.5 \times 10^{10}$ m

(iv) By Newton's 2nd Law, $F_{net} = ma$;

$$\frac{GM_A M_B}{d^2} = M_A r_A \omega^2$$

$$M_B = \frac{d^2 r_A \omega^2}{G} = \frac{(3.0\times10^{11})^2(7.5\times10^{10})(4.98\times10^{-8})^2}{6.67\times10^{-11}}$$

$M_B = 2.51 \times 10^{29}$ kg

Since $\frac{M_A}{M_B} = 3.0$,

$M_A = 3.0\ (2.51 \times 10^{29}) = 7.53 \times 10^{29}$ kg

Comments: students should distinguish clearly between the symbols for quantities related to each star.

(v) In each period, there will be two instants when the two stars form a straight line with Earth. At these instances, the intensity will dip because one star is behind the other. Therefore, the fluctuation in intensity is half the period of the orbit of the stars.

Gravitation Tutorial 7B Discussion Question Suggested Solutions

D1

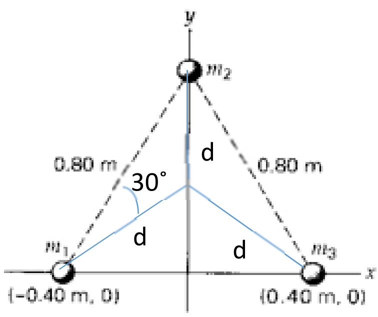
(a) Total gravitational potential energy of the configuration, U = Work done by the external force in assembling the system

$$U = -\frac{Gm_1m_2}{0.80} + \left(-\frac{Gm_1m_3}{0.80}\right) + \left(-\frac{Gm_2m_3}{0.80}\right)$$

$$= 3\times\left(-\frac{G(1.0)(1.0)}{0.80}\right)$$

$$= -2.50\times10^{-10}\ \text{J}$$

(b)



d cos30˚ = 0.80/2 → d = 0.4619

$$\phi_{centre} = 3\times\left(-\frac{G(1.0)}{0.4619}\right) = -4.332\times10^{-10}\ \text{J kg}^{-1}$$

$$WD = \Delta U$$

$$= m\left(\phi_{centre} - \phi_{\infty}\right)$$

$$= (1.0)\times\left(-4.332\times10^{-10} - 0\right)$$

$$= -4.33\times10^{-10}\ \text{J}$$

Gravitational force is **attractive** in nature and the gravitational **potential is set to be zero at infinity**. To move the 4th mass from infinity to the centre of the system, **the force exerted on the mass by the external agent will be in opposite direction to the displacement of the mass**. Thus negative work is done by the external force (agent).

D2 2020 P1 Q12

A

Increase in gravitational potential energy = $-\frac{Gm_1m_2}{(2r)} - \left(-\frac{Gm_1m_2}{r}\right) = \frac{Gm_1m_2}{2r}$

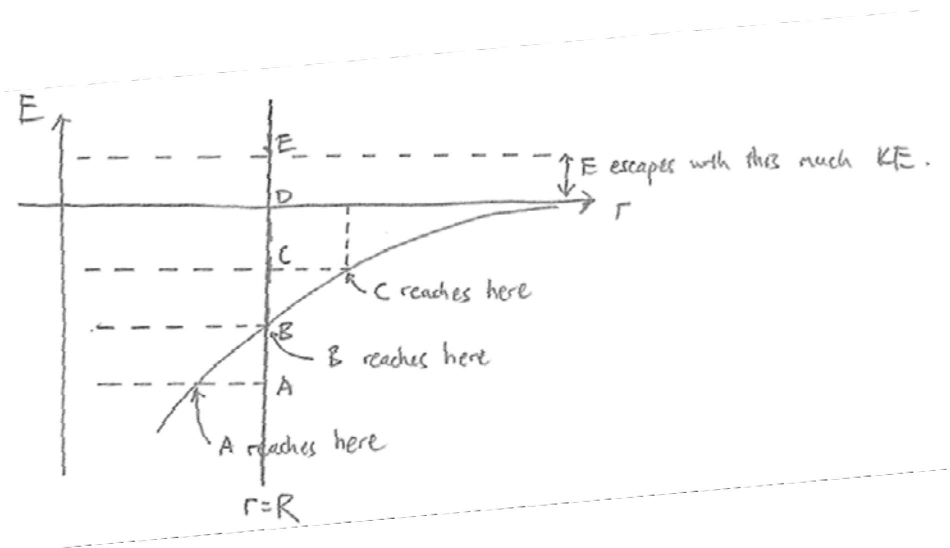
D3

(a) It represents the gravitational force on the body on Earth's surface. $F_g = -\frac{dE_p}{dr}$. The negative sign indicates that the gravitational force is in the direction of decreasing potential energy.

(b) B. This amount of total energy allows the mass to just reach r = R. At r = R, TE = GPE and KE = 0.

(c) C. The mass will go beyond r = R, comes to rest at r where the PE = TE and then turns back towards Earth – and that's why it is "falling towards the Earth".

(d) D and E. For D, the mass will just reach infinity. For E, the mass will reach infinity where GPE = 0 and still have some KE.



D4 2021 P3 Q2

(a) Gravitational force is attractive.
In displacing a mass from infinity towards M (at constant speed), an external force opposite in direction to the displacement needs to be applied. Work done by this external force is negative [OR positive work needs to be done by an external force to move a mass away from M, implying that the potential gets larger as one moves away from M.]
Infinity is assigned a potential-value of zero. Hence the potential at any other point is negative.

(b) (i) $\phi = -\frac{GM}{r} = -\frac{(6.67\times10^{-11})(6.2\times10^{23})}{\left(\frac{6.8\times10^{6}}{2}\right)} = -1.2\times10^{7}\text{ J kg}^{-1}$

(ii) To travel to infinity, the total energy must be greater than or equal to zero
At surface, GPE + KE = $(-1.22 \times 10^7)(2.8) + ½ (2.8)(3800)^2 = -1.4 \times 10^7$ J
Hence, it does not escape, it returns to the planet.

OR
Initial KE of rock = $½ (2.8)(3800)^2 = 2.0 \times 10^7$ J
To reach infinity, GPE to be gained = $m(\phi_f - \phi_i) = 2.8\left(0-\left(-1.22\times10^{7}\right)\right) = 3.4\times10^{7}$ J
Not enough KE to escape.

OR (calculate the escape speed)

Loss in KE = Gain in GPE

$KE_i - KE_f = GPE_f - GPE_i$

$$\frac{1}{2}(2.8)v^2 - 0 = 0 - (2.8)(-1.22\times10^7)$$

$v = 4.9\times10^3$ m s^{-1}

3.8×10^3 m s^{-1} is not enough to escape.

D5

Total energy of the space station in orbit = KE + GPE

= ½ GPE of the space station at the orbital radius

$= -\frac{GMm}{2r}$ [see example 10 in lecture notes]

Energy required

$$= -\frac{GMm}{2r_f} - \left(-\frac{GMm}{2r_i}\right) = \frac{GMm}{2}\left(\frac{1}{r_i} - \frac{1}{r_f}\right)$$

$$= \frac{6.67\times10^{-11}(6.0\times10^{24})(450000)}{2}\left(\frac{1}{6.378\times10^6 + 415\times10^3} - \frac{1}{6.378\times10^6 + 20200\times10^3}\right)$$

$= 9.9 \times 10^{12}$ J

D6

(a)(i) The gravitational force provides the centripetal force on the satellite.

$$\frac{GMm}{R^2} = \frac{mv^2}{R}$$

$$v^2 = \frac{GM}{R}$$

(ii) $E_k = \frac{1}{2}mv^2$

$$E_k = \frac{1}{2}m\frac{GM}{R} = \frac{1}{2}\frac{GMm}{R}$$

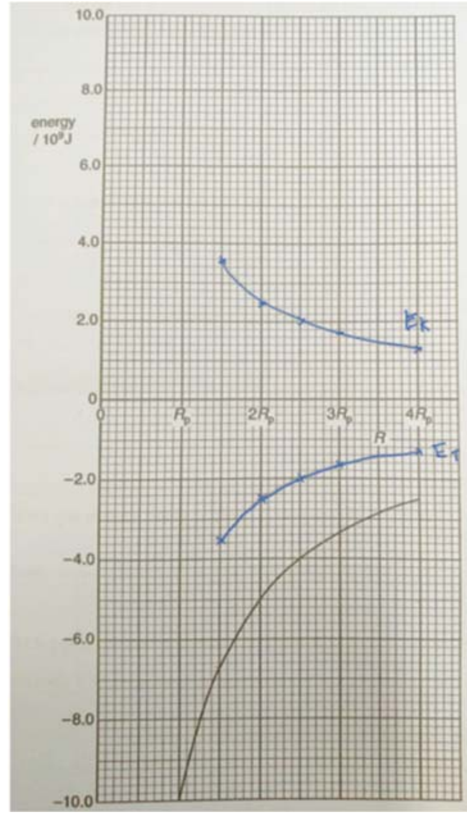
(b)(i) $E_p = -\frac{GMm}{R}$

[like D4(a)] Gravitational force is **attractive** in nature and the **potential is set to be zero at infinity**. To move a mass from infinity to a point in the field (of the source mass), **the force exerted on the mass by the external agent will be in opposite direction to the displacement of the mass**. Thus negative work is done by the external force (agent).

(ii) $\frac{E_p}{E_k} = -\frac{GMm}{R} \Big/ \frac{GMm}{2R} = -2$

(iii) $E_T = E_p + E_k = -\frac{GMm}{R} + \frac{GMm}{2R} = -\frac{GMm}{2R}$

(c)(i), (ii) See graphs on the right.



R	E_p /10^9 J	E_k/10^9 J	E_T/10^9 J
1.5 R_P	-6.70	3.35	-3.35
2 R_P	-5.00	2.50	-2.50
2.5 R_P	-4.00	2.00	-2.00
3 R_P	-3.38	1.69	-1.69
4 R_P	-2.55	1.28	-1.28

(d)(i) More negative, since total energy must decrease.

(ii)1. Radius decreases. Satellite does not have enough energy to stay at that altitude. Given the formula for total energy, radius must decrease.

2. Speed increases. Gravitational potential energy is converted into kinetic energy and work done against air resistance. According to the formula for kinetic energy, kinetic energy must increase.

(iii) At R = 4R$_p$,

$$E_k = 1.25 \times 10^9$$

$$\frac{1}{2}(1600)v_i^2 = 1.25 \times 10^9$$

$$v_i = 1250 \text{ m s}^{-1}$$

At R = 2R$_p$,

$$E_k = 2.50 \times 10^9$$

$$\frac{1}{2}(1600)v_f^2 = 2.50 \times 10^9$$

$$v_f = 1768 \text{ m s}^{-1}$$

Change in speed = 1768 – 1250 = 518 m s^{-1}

D7

(a) $V = -\dfrac{GM_E m}{r}$

(b) The potential gradient gives the magnitude of the gravitational field strength, the direction of which is towards decreasing potential, $g = -\dfrac{d\phi}{dr}$. The force on the tektite is given by the mass of the tektite multiplied by the potential gradient, and the direction of the force is towards decreasing potential.

(c) At P, the gradient is zero, implying Σ g = 0

$$g_M = g_E$$
$$\frac{GM_M}{X^2} = \frac{GM_E}{Y^2}$$
$$\left(\frac{X}{Y}\right)^2 = \frac{M_M}{M_E}$$
$$\frac{X}{Y} = \sqrt{\frac{7.4 \times 10^{22}}{6.0 \times 10^{24}}} = 0.11$$

(d) The tektite needs to be given enough energy to reach P, beyond which the resultant gravitational field towards the Earth will accelerate the tektite towards the Earth.

By the principle of conservation of energy,

From moon's surface to point P,

Loss in KE = Gain in GPE

$$\frac{1}{2} m V_0^2 = m\left(-1.3 - \left(-3.9\right)\right) \times 10^6$$
$$V_0 = 2280 \text{ m s}^{-1}$$

(e) Tektite will reach Earth with a speed greater than 2280 m s^{-1}. Earth's surface is at a lower potential than moon's surface. This larger loss in GPE is converted into a larger gain in KE when the tektite hits Earth.

D8

D. Gravitational force = -dE_p/dr. Near the earth's surface, the (gravitational) force is constant as the gravitational field strength is constant.

D9

B. Uniform g → magnitude of $g = \dfrac{\Delta \phi}{\Delta x}$

$$g = \frac{6.0}{10} = 0.60 \text{ N kg}^{-1}$$

$$mg\Delta h = 2.0\left(0.60\right)\left(2.5\right) = 3.0 \text{ J}$$

Oscillation Tutorial Hints & Suggested Solutions to Discussion Questions

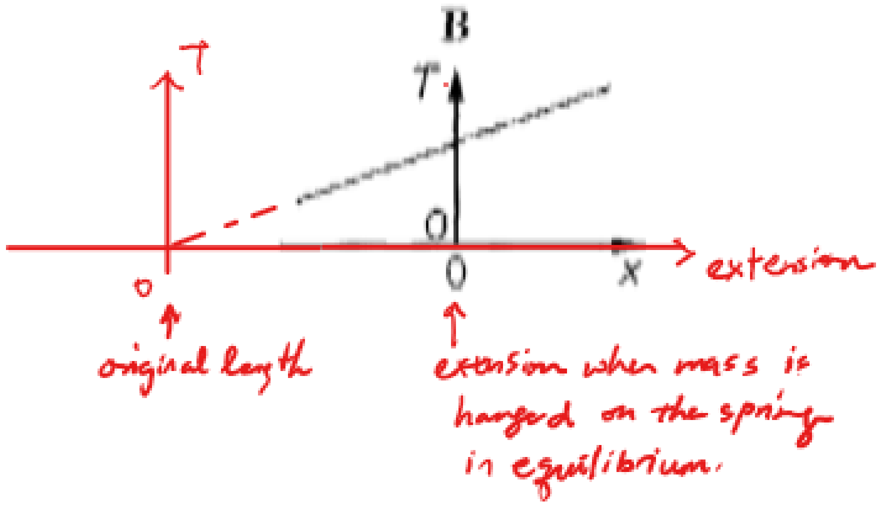
D1 Amplitude of oscillation = 50 mm
Period of oscillation = 2 s
$x = 50 \sin (2\pi /2)\, t$ (x in mm)
[Note: Taking to the right of 650 mm mark as positive direction]

when x = +25 mm
$25 = 50 \sin (2\pi /2)\, t$
Time duration for which shutter remained open, t = 1/6 s = 0.17 s

Key Concepts: *Characteristics of S.H.M? Able to write down the displacement-time function for a S.H.M. from initial condition (i.e. when t = 0 s).*
Note*: For SHM, displacement is NOT proportional to time.*

D2 ***Hints:*** *Check at equilibrium x = 0, is there tension? How is tension related to the extension of spring?*

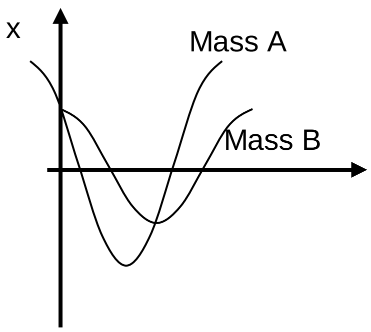
B.



Key Concepts: *Link to topic of Dynamics & Forces.*
Note: *Tension in spring, T is NOT proportional to the displacement, x.*
Qn D2, D10 & D11 is about oscillation of a vertical spring mass system.

D3 ***Hints:*** *Sketch the displacement, x vs phase angle, ϕ graph for the scenario.*
Note that at time t = 0 s, Mass A is released from its maximum displacement position (2 cm below equilibrium position). Mass B is released from it maximum displacement position (1 cm below equilibrium position) only when Mass A is at displacement of 1 cm below equilibrium position.

B.



$$1 = 2\cos\varphi \rightarrow \varphi = \frac{\pi}{3}$$

Key Concepts: Understand phase angle and phase difference for oscillations.

D4 ***Hints:*** ***i)*** *Can you write down the function that relates the acceleration, a to time, t?*
ii) Can you write down the function that relates the displacement, x to time, t?
iii) Can you write down the function that relates the velocity, v to time, t?

A.

From **a – t** graph, $a = a_o \cos \omega t$
We can then infer that displacement, $x = - x_o \cos \omega t$
Hence, velocity, $v = v_o \sin \omega t$

Sketch the $x – t$ and $v – t$ graphs on the $a – t$ graph (same time axis), and determine that at P when a is positive, x is negative and v is positive.

Key Concepts: *See the relationship between a-t, x-t & v-t graphs of SHM. Know the kinematics of SHM.*

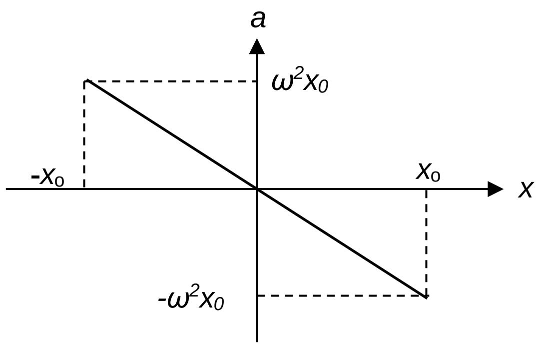
D5 ***Hints:*** *i) How do you know that the oscillation is SHM?*
ii) Where is the equilibrium position of the SHM?
iii) What is the amplitude of the SHM?
iv) What is the displacement of SHM when the depth of water is 1.5 m?
v) Can you write down the appropriate function that relates displacement, x to the time, t?

2.0 m is the equilibrium position.
Amplitude, $x_0 = 1.0$ m
$T = 12$ hrs
Depth of water, $x = - 1.0 \cos (2\pi/12)\, t + 2.0$

$1.5 = - 1.0 \cos (2\pi/12)\, t + 2.0$
Time duration that the boat will have to wait before entering, $t = 2.0$ hrs

Key Concepts: *Characteristics of S.H.M? Able to write down the displacement-time function for a S.H.M. from initial condition (i.e. when t = 0 s).*

D6 (b) (i)



Key Concepts: *How to identify SHM? Defining equation for a SHM.*

(ii) 1. $\omega = 2\pi f = 2\pi (13) = 82$ *rad s*$^{-1}$
2. when $a_{max} = g$
$| -\omega^2 x_o | = g$
Amplitude of oscillation, $x_o = 9.81 / 82^2$
$= 1.47 \times 10^{-3}$ m

(c) **Hint:** *i) Sketch a diagram of the plate and one particle of sand on plate. What forces act on the particle of sand? Which of these forces is dependent on the relative motion between sand and plate?*

ii) When the plate is at maximum displacement above the equilibrium position, what is the direction of acceleration of the plate? What is the direction of velocity of the plate just before it reach maximum displacement?

Suggested Solution:

If the amplitude of the oscillations of the plate exceeds the value calculated in (b)(ii)2, the maximum acceleration of the plate will be higher than the acceleration of free fall, *g*. Thus the sand will lose contact with the flat horizontal plate on its way up (pass the equilibrium point) as the plate slows down at a rate larger than the sand.

D7 (a) (i) $\theta = \omega t$

(ii) $ST = r sin\omega t$ *(How do you know that it is a sine function?)*

(b) ST is the displacement *x* of the shadow with respect to S. Comparing it with the standard expression for the displacement of an object in SHM ($x = x_0 sin\omega t$), we can see that the shadow of the peg is moving in simple harmonic motion about the point S with amplitude $x_0 = r$.

(c)

(i) amplitude of shadow's SHM is same as radius of the turntable 20 cm.

(ii) Since the turntable has angular speed $\omega = 3.5 = \frac{2\pi}{T}$, the period of the turntable is 1.8s which is also the period of the shadow's SHM.

(iii) Maximum speed of shadow at S is $v_{max} = \omega x_0 = (3.5)(0.2) = 0.7 m/s$

(iv) Maximum acceleration (when shadow at rest) is given by

$a_{max} = \omega^2 x_0 = (3.5)^2(0.2) = 2.45 m/s^2$

D8 **Hint:** *i) What is the expression of the total mechanical energy of a mass in SHM?*

Further qn: *Can you prove that the mass will oscillate in SHM?*

Total mechanical energy for S.H.M = max. K.E.

$= \frac{1}{2} M v_{max}^2$

$= \frac{1}{2} M (\omega a)^2$

$= \frac{1}{2} M (a\, 2\pi/T)^2$

$= 2\pi^2 Ma^2 / T^2$

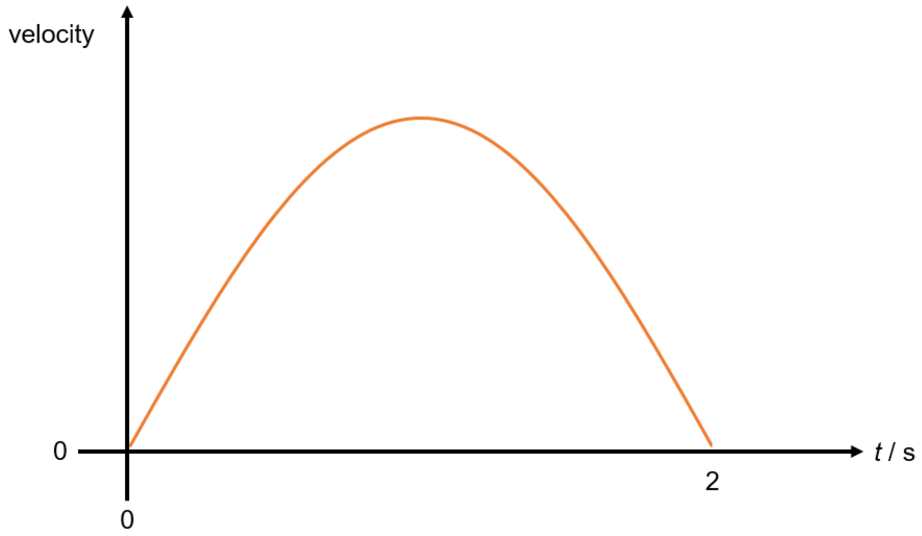
Another method:

Let the effective spring constant for this system be *k*.

From $\omega^2 = \frac{k}{m}$, we get $k = M\omega^2$

Total mechanical energy for SHM = $E_T = \frac{1}{2}kx_0^2 = \frac{1}{2}M\omega^2 a^2 = \frac{1}{2}M\left(\frac{4\pi^2}{T^2}\right)a^2 = \frac{2M\pi^2 a^2}{T^2}$

D9 It is given in the question that the potential energy versus time graph is for half a period. At *t* = 0, the potential energy is maximum. This suggests that the particle is at the maximum displacement where the velocity is zero. Half a period later, the particle is at the other maximum displacement where velocity is again zero. During this time, the velocity is unidirectional. Hence, the velocity-time graph should give a half of sine graph.



D10

$PE_{max} = KE_{max} = \frac{1}{2} m (\omega x_o)^2 = \frac{1}{2} m (x_o\, 2\pi/T)^2$

From graph, when $x_o = 0.2$ m, $U = 1.0$ J

$1.0 = \frac{1}{2} (4) (0.2 \times 2\pi/T)^2$

$1.0 = 2.0 \times 0.04 \times 4\pi^2 / T^2$

$T^2 = 2.0 \times 0.04 \times 4\pi^2$

$T = 1.8$ s

Key Concepts: *Understand and able to determine the potential energy vs displacement relation for a SHM.*

D11 **Hint:** *i) How do you know that the vertical spring-mass system will oscillate in SHM?*

(a) At the equilibrium point,

$$F_{res} = 0 \rightarrow kx - Mg = 0$$

$$kx = Mg \rightarrow k = \frac{Mg}{x}$$, where x is the extension of spring.

Any of these points or other correct points from the graph;
When a mass of 150 g is hung, the extension on the spring is 10.0 cm.
When a mass of 300 g is hung, the extension is 20.0 cm.
When a mass of 450 g is hung, the extension is 30.0 cm

$$k = \frac{0.150g}{0.100} = 14.7 \text{ N m}^{-1}.$$

(b) **Hint:** *i) What is the unstretched length, l_o, of the spring?*
ii) What is the length of the spring when 450 g is attached? What is the extension of the spring when 450 g is attached?
iii) What is the amplitude of SHM?

$$\omega = 2\pi f \rightarrow f = \frac{1}{2\pi}\sqrt{\frac{14.715}{0.450}} = 0.910 \text{ Hz (3 s.f.)}$$

(c) **Hint:** *i) What is the displacement of the oscillation when the length of spring is 80.0 cm?*

ii) What is the relationship (formula) that relates velocity, v with displacement, x?

$$\Delta GPE = Mg(extension) = 0.450 \times 9.81 \times 0.300 = 1.32435 \text{ J}$$

$$\Delta EPE = \frac{1}{2}k(e_2^{\,2} - e_1^{\,2}) = \frac{1}{2}(14.715)(0.400^2 - 0.100^2) = 1.103625 \text{ J}$$

$$\Delta KE = 1.32435 - 1.103625 = 0.220725 \text{ J}$$

$$0.220725 = \frac{1}{2}mv^2 \rightarrow v = 0.990 \text{ m s}^{-1}$$

OR

Note that question is asking for *v* for an oscillation with amplitude 20.0 cm at displacement of 10.0 cm.

$$v = \omega\sqrt{x_0^{\,2} - x^2} = 2\pi(0.910)\sqrt{0.200^2 - 0.100^2} = 0.990 \text{ m s}^{-1}$$

(d) Use $\omega = 2\pi f \rightarrow f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$, deduce that *f* is inversely proportional to $\sqrt{m}$. When *m* increases, *f* is reduced

OR The effective mass of the spring-mass system increases. The resonant frequency of heavier masses is at lower values of frequencies (or at greater periods). Hence, the frequency of the oscillation would be reduced.

D12 *Qn D2, D11 & D12 is about oscillation of a vertical spring mass system.*

(a) Frequency is the number of oscillations per unit time whereas angular frequency is the rate of change of phase angle. Since each complete oscillation moves through a phase angle of 2π, the angular frequency is given by $2\pi f$.

(b)(i) loss in gravitational potential energy by mass $= m\, g\, \Delta h$
= (0.400)(9.81)(0.200)= 0.785 J

(b)(ii) At equilibrium, $mg = ke$

Elastic potential energy gained by spring = $\frac{1}{2} ke^2 = \frac{1}{2} (mg)(e)$

= ½ (9.81)(0.400)(0.200) = 0.392 J

(c) **Hint:** *i) The question mention that "A mass of 0.400 kg is attached to the spring and gently lowered until equilibrium is reached".*

ii) Up till this part, the spring-mass system is not in oscillation yet!

An external (variable) upward force is required when the spring mass is stretched gently downwards towards its equilibrium point. Thus some gravitational potential energy is lost due to the negative work done by the external force, while the remainder is converted to elastic potential energy.

(d)(i) $F_{net} = ke' - mg = k(2e) - mg = 2mg - mg = mg$

= (0.400) (9.81) = 3.92 N

(ii) By Newton's 2nd Law, $F_{net} = ma$

$a = F_{net} / m$

SHM equation: $a = -\omega^2 x$

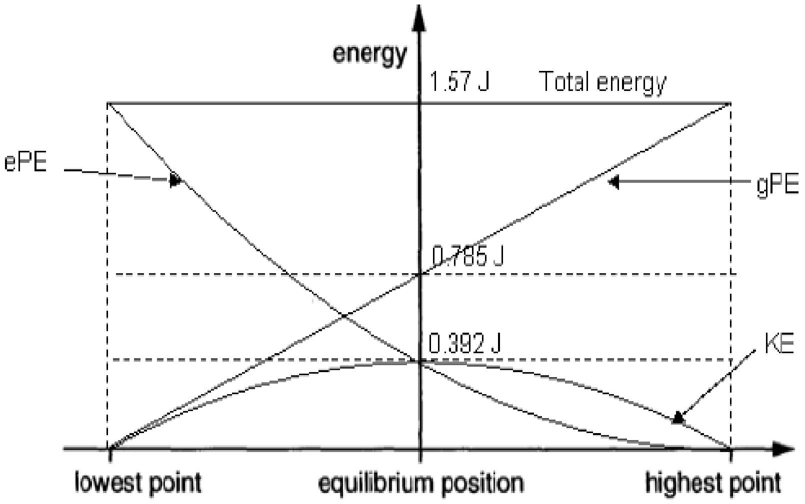
$\omega = \sqrt{(a / -x)} = \sqrt{(F_{net} / -mx)}$

$= \sqrt{[(0.400)(9.81)/(-0.400)(-0.200)]} = 7.00 \text{ rad s}^{-1}$

(iii) maximum speed of mass = $\omega x_o = 7.00(0.200) = 1.40 \text{ m s}^{-1}$

(e) & (f)

	gravitational potential energy/J	elastic potential energy/J	kinetic energy/J	total energy/J
lowest point	0	1.57	0	1.57
equilibrium position	0.785	0.392	0.392	1.57
highest point	1.57	0	0	1.57



D13 If the natural frequency is within the range of frequencies produced by the loudspeaker, resonance can occur for this frequency when the loudspeaker is being used. This results in a rapid amplification of the sound that will cause discomfort to the ear. Further, the timbre of sound produced by the loudspeaker will also experience distortion as sound waves of this particular frequency are greatly amplified relative to other frequencies.

D14 **(a)** Hydrostatic pressure is proportional to the depth of fluid. When an object displaces a fluid, there is a pressure difference between the lower and upper surface and this pressure difference results in upthrust.

(b) Since the tube is floating, it is displacing its own weight of fluid. (Principle of flotation)

OR

The force on the bottom of the tube due to fluid pressure is equal to the weight of the tube.

$m_{tube}g = m_{fluid}\, g$

$m = V\rho = Ah\rho$

(c) *Extension: Can you derive the expression (of a vs x) given?*

(i) Since *A, h, ρ* and *m* are all constants, the magnitude of the acceleration is proportional to the displacement. The negative sign also indicates that the direction of acceleration is opposite to that of displacement. The equation is of the form *a* = - *kx* where *k* is a constant which describes simple harmonic motion.

(ii) Comparing with defining equation:

$$\omega^2 = \frac{A\rho g}{m} \Rightarrow 4\pi^2 f^2 = \frac{A\rho g}{m}$$

$$\Rightarrow f = \sqrt{\frac{A\rho g}{4\pi^2 m}} = \sqrt{\frac{(4.2\times10^{-4})(1.0\times10^{3})(9.81)}{4\pi^2(0.032)}} = 1.8 \text{ Hz (2 s.f.)}$$

(d) (i)1

$$f = \frac{1}{T} = \frac{1}{1.5 \div 3} = 2.0 \text{ Hz}$$

(i)2 From (c)(ii):

$$\rho = \frac{4\pi^2 f^2 m}{Ag} = \frac{4\pi^2(2.0)(0.032)}{(4.2\times10^{-4})9.81} = 1226 = 1200 \text{ kg m}^{-3} \text{ (2 s.f.)}$$

(ii)1 The fluid is viscous and exerts a force on the tube which opposes its motion. It removes energy from the oscillating tube.

There is also friction along the sides of the tube (skin friction). This results in a constant loss of energy and hence the amplitude decreases with time.

(ii)2 Read from Figure, Amplitudes when t = 0 s and t = 1.0 s.

Loss in energy = $E_0 - E_1$

$$= \frac{1}{2}m\omega^2 x_0^2 - \frac{1}{2}m\omega^2 x_1^2$$

$$= \frac{1}{2}(0.032)(4\pi^2 2^2)(0.0150^2 - 0.0085^2) = 3.86\times10^{-4} \text{ J}$$

D15 (a)(i) A forced oscillation is an oscillatory system driven into oscillation by a periodic force.

(b)(i) 1. Maximum linear speed $v_{max} = \omega x_0 = 2\pi f x_0$

At resonance, x_0= 1.60 cm, *f* = 12.0 Hz

$v_{max} = 2\pi f x_0$

$= 2\pi (12.0)(1.60 \times 10^{-2})$

$= 1.21$ m s^{-1}

2. Maximum acceleration $a_0 = |-\omega^2 x_0| = 4\pi^2 (12.0)^2(1.60 \times 10^{-2}) = 91.0$ m s^{-2}

3. resonance.

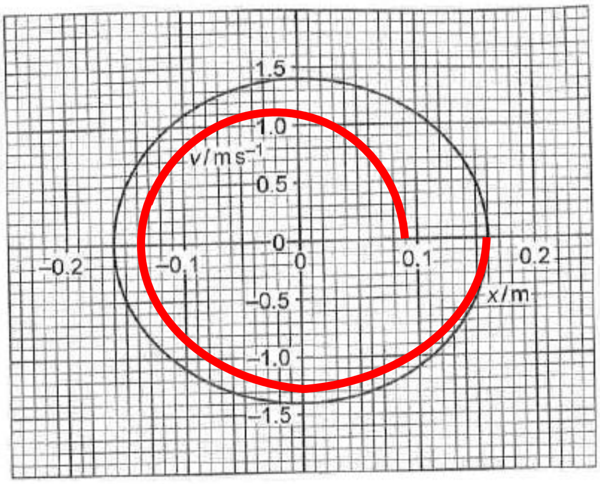
(ii) Time interval = *T* / 4 = 1/12.0 (1/4) = 0.0208 s. Velocity and acceleration are $\pi/2$ out of phase with each other.

D16 (a) Maximum acceleration $a = \frac{v_0^2}{x_0} = \frac{1.4^2}{0.16} = 12.3\ m\ s^{-2}$

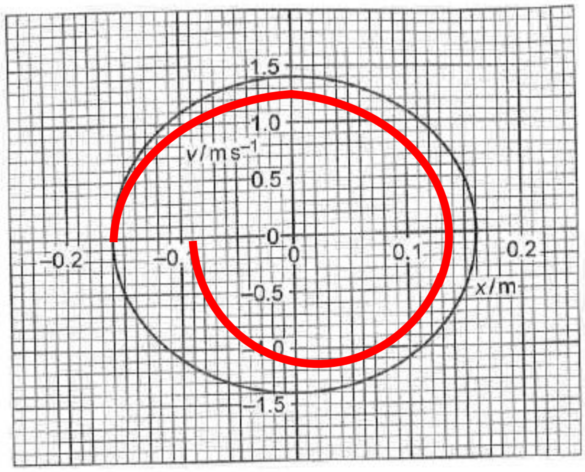
(b) When the light piece of card is added, the oscillations will be damped. The velocity and displacement will decrease gradually as mechanical energy is lost to the system due to the work done against air resistance.

There are two possible solutions to this question.

Solution 1:



Solution 2:



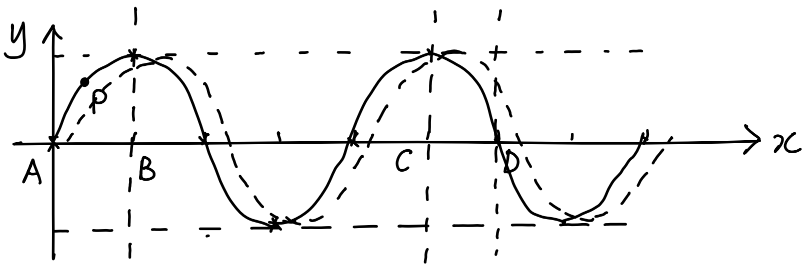
Solution 1 assumes that **downward is positive**, hence when the mass is displaced downwards, it has a positive x displacement of 0.16. When the mass moves **upwards**, **its velocity will be negative**. Hence, as the oscillation progresses, the red line is drawn in a clockwise direction, with energy being lost continuously.

Solution 2 assumes that downward is negative, and hence its initial displacement is negative. However when the mass moves upwards, its velocity will be positive. This way, the red line is also drawn in a clockwise direction.

Topic 9: Wave Motion
Suggested Solutions to Discussion Questions

D1 **Answer: B**

For visualization, a quick sketch is sometimes useful in many Physics Problems - it forms part of problem strategy and it allows better understanding of questions.



Sketch in the wave profile at the next instant of time.

P is displaced upwards but at the point of moving downwards. ⇒ Point P is somewhere between AB (not inclusive of AB, A has zero displacement and B is instantaneously at rest.)

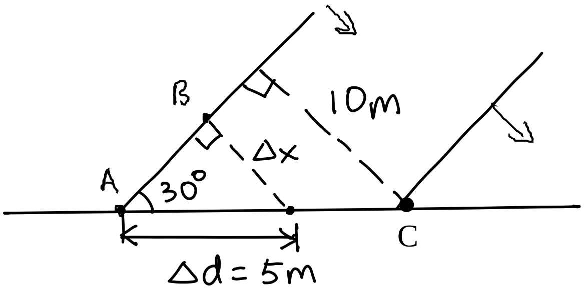
Consider the 2 extreme points A and B, since point Q is a distance $\frac{5}{4}\lambda$ away, point Q must lie between C and D (not inclusive of CD). Therefore, point Q has positive (upwards displacement) and is moving upwards.

D2 **Answer: C**

Note that A and B are in phase as they are along the same wavefront. Phase difference between A and C is the same as that between B and C.

$\Delta x = \Delta d \sin 30^\circ = 2.5$ m

Therefore, 5 m apart along the wall ≡ 2.5 m along the direction of wave travel.



2.5 m corresponds to ¼ cycle ≡ $\pi/2 = 90^\circ$ or $\Delta\phi = \frac{\Delta x}{\lambda} \times 360^\circ = \frac{2.5}{10} \times 360^\circ = 90^\circ$

D3 **Answer: A**

You have learnt in Waves:

$$\frac{\Delta\theta}{2\pi} = \frac{\Delta t}{T} = \frac{\Delta x}{\lambda}$$

Realise that the displacement-distance graph corresponds to $y = y_0 \sin\theta$

Let $y = y_o \sin\theta$

For particle P, $0 = y_o \sin\theta_P$ where θ_P is the phase angle of particle at P
$\therefore$ the phase angle θ_P of particle at P is 180°

For particle Q, $-\frac{1}{2} y_o = y_o \sin\theta_Q$ where θ_Q is the phase angle of particle at Q
$\therefore$ $\theta_P = 210°$
Thus the phase angle between P and Q = 30°

A shorter alternative method: Since the phase difference is equivalent to the phase angle of Q from P, we can set up the equation starting from R: $y = -y_0 \sin\theta$.

D4 **Answer: B**

The air molecules are performing simple harmonic motion and hence the maximum speed is proportional to the amplitude of the oscillation ($v_{max} = \omega x_0$).

Also intensity is proportional to the square of the amplitude. Hence an increase in intensity will result in an increase in the maximum speed.

The speed of the wave is given by the product of the frequency and its wavelength. Since none of these factors are changes, the speed of the wave remains the same.

D5 **Answer: D**

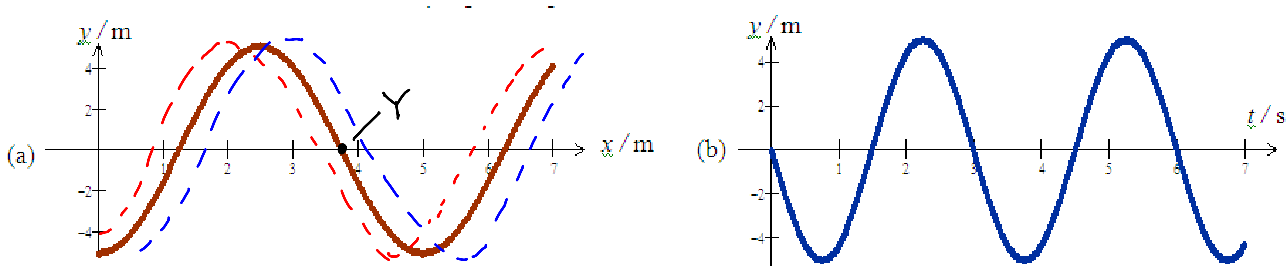
From Graph 1 (displacement-time), period $T = t_3 - t_1$
Or $t_2 - t_1 = T / 2$

From Graph 2 (displacement-distance), wavelength $\lambda = x_3 - x_1$
Or $x_3 - x_2 = \lambda/2$

Since speed of wave $v = f\lambda = \lambda / T = \dfrac{x_3 - x_1}{t_3 - t_1}$

Or $v = \dfrac{x_3 - x_2}{t_2 - t_1}$

D6a Sketch in the wave profile in (a) at the next instant of time:



If the wave progresses to the right (blue dotted line above), point Y should next move upwards. This does not tally with the graph in (b).

If the wave progresses to the left (red dotted line above), point Y should next move downwards. This tallies with the graph in (b) and hence the wave is progressing towards the left in the negative *x* direction.

D6b $\lambda = c/f = 3.00 \times 10^8 / 5.0 \times 10^{14} = 6.0 \times 10^{-7}$ m = 0.60 μm

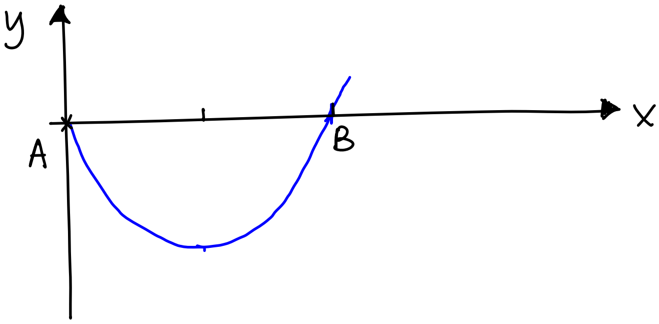
$$\Delta\phi = \frac{\Delta x}{\lambda} \times 2\pi = \frac{1.5}{0.6} \times 2\pi = 5\pi \text{ rad} = \pi \text{ rad}$$

D7 J91/III/2 (modified)

(a) Every single particle on the wave is oscillating horizontally in SHM about their equilibrium position, along the direction of energy propagation.

Since the wave is travelling from left to right, the compressions and rarefactions move from left to right.

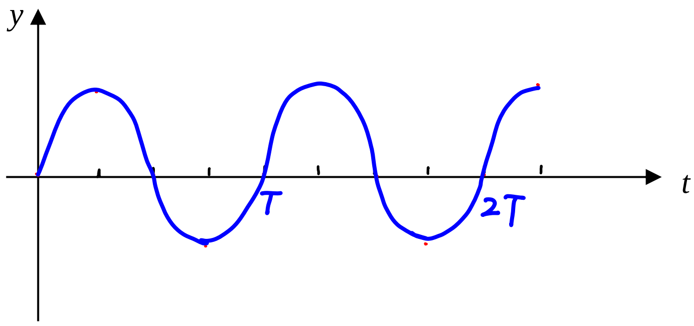
(b) Taking rightwards to be positive, the displacement time graph for the points along A to B is given by



Supposedly, a "full-scale" diagram is provided, so the wavelength can be measured directly from the diagram. The wavelength is the distance between 2 consecutive rarefactions or 2 consecutive compressions.

In many graphical or diagrammatic questions, students are expected to use lengths that span more than half the diagram to make estimations, else they may be penalized in the A-levels by the examiners.

(c) [Look at the displacement of the particle left of A, since the wave is progressing from left to right, the next displacement of A will that of this particle, which is positive.]
Furthermore, as this particle is moving in SHM, therefore its displacement should vary sinusoidally with time.



D8(a) (i) Amplitude = 5.0 μm (read to ½ smallest square precision)

(ii) The wave between $t = 0$ and $t = 2.0$ s corresponds to exactly 1.5 λ

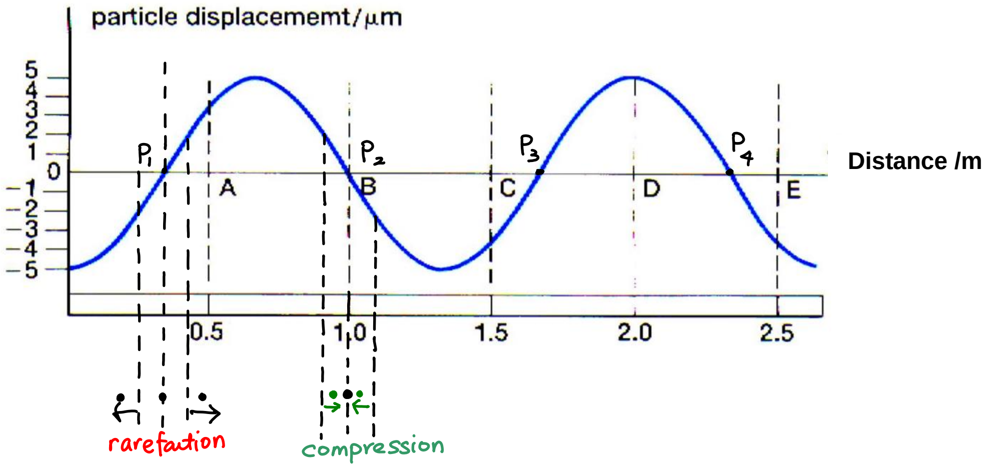
$\frac{3}{2}\lambda$ = 2.00 m $\Rightarrow$ λ = 1.33 m

(iii) $f = \dfrac{v}{\lambda} = \dfrac{(340 \text{ m/s})}{(1.33 \text{ m})} = 255 \text{ Hz}$

(iv) Phase difference between the vibration of the particle at A and B is given by

$$\Delta\phi = \frac{\Delta x}{\lambda} \times 2\pi = \frac{0.50 \text{ m}}{1.33 \text{ m}} \times 2\pi = 2.36 \text{ rad}$$

(b)

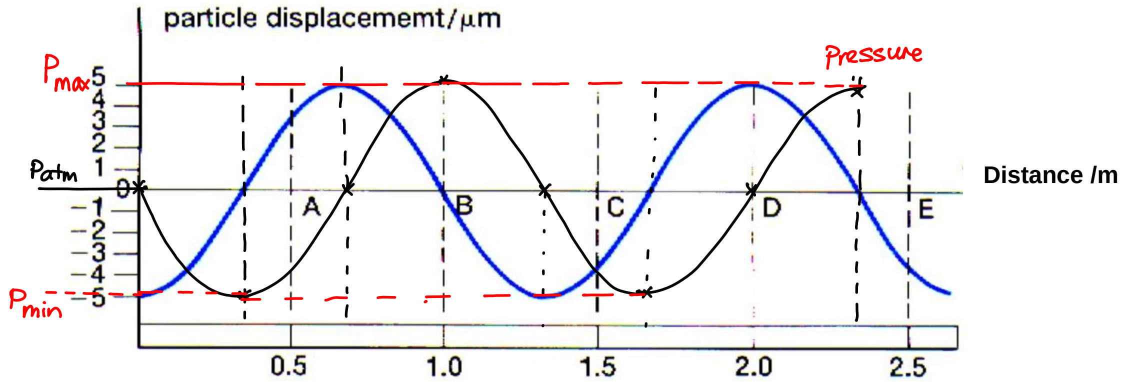


We know that the rarefactions and compressions occur when displacement $y = 0$. Therefore, we sketch the displacement of the particles neighbouring points of the particles at $y = 0$. (See above).

From the sketch above, we see that

- P_1 ,P_3 are points of rarefaction.
- P_2, P_4 are points of compression.

(c) The pressure fluctuates about the atmospheric pressure. The points of rarefaction are points of minimum pressure and the points of compression are points of maximum pressure.



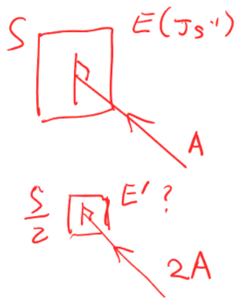
D9 **Answer: B**

Note that

1. intensity of plane wave at any distance remains the same, as the power transmission is unidirectional.
2. *E* stands for energy per unit time intercepted or power received,

Power received = intensity x intercepting area

i.e. $E = IS = k\,A^2S$ where *A* is amplitude of wave, k is a constant.



$\frac{E'}{E} = \frac{k(2A)^2(S/2)}{k(A)^2 S} = \frac{4}{2}$. Thus $E' = 2\,E$

D10 Answer: C

Intensity $I = \dfrac{\text{Power of Source } P}{\text{Surface Area } S} \Rightarrow \dfrac{I_1}{I_2} = \dfrac{S_2}{S_1}$

By similar shape ratio: $\dfrac{Area_1}{Area_2} = \left(\dfrac{length_1}{length_2}\right)^2$

$$\frac{I}{I_2} = \frac{S_1}{S_2} = \frac{16^2}{2.0^2} = 64$$

$$\therefore I_2 = \frac{1}{64} I$$

But $I \propto A^2$ where A is the amplitude

Thus $\dfrac{I_1}{I_2} = \dfrac{A_1^{\ 2}}{A_2^{\ 2}} \Rightarrow \dfrac{I}{I_2} = \dfrac{A^2}{A_2^{\ 2}} \Rightarrow 64 = \dfrac{A^2}{A_2^{\ 2}}$

$$\therefore A_2 = \frac{1}{8} A$$

D11 Answer: D

Note that this is not the usual case of energy spreading in 3D. This is a 2D case. The question says the energy is spread over the circumference ($2\pi r$).

Thus Intensity $I \alpha 1/r$. Going from 150 mm to 1200 mm, the radius has become 8 times. (So the circumference over which the power is spread has also become 8 times.) So intensity becomes 1/8.

Since $I \alpha A^2$ where A is the amplitude, the amplitude is reduced by a factor of $\sqrt{8}$. Thus the amplitude becomes $(1/\sqrt{8})2.0 = 0.71$ mm.

D12 Answer: B

Frequency of wave will not change because it depends on source and crossing a boundary does not affect it.
Speed = frequency x wavelength, hence wavelength is halved when speed is halved.
By intensity $\propto$ squared of amplitude, new intensity will be ¼ of original one.

D13 J88/III/8

Intensity of the wave reaching the target = $\dfrac{P}{4\pi d^2}$

Therefore, energy of the wave received by the target per unit time : $E = \dfrac{P}{4\pi d^2} \times S$

Amount of power reflected by the target, $P_r = \dfrac{kPS}{4\pi d^2}$

Therefore, the mean intensity of the reflected pulse when it is received back at the transmitter:

$$I_r = \frac{P_r}{4\pi d^2} = \frac{kPS}{16\pi^2 d^4}$$

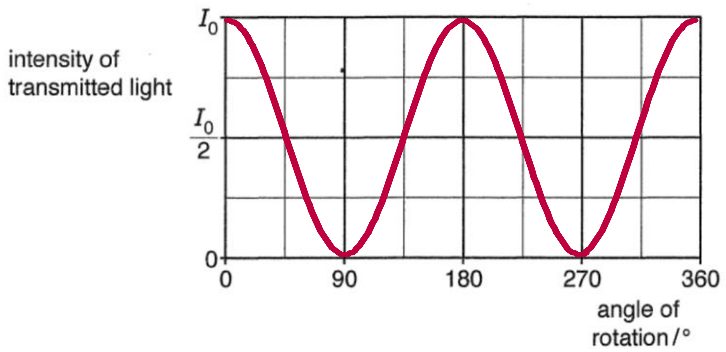
(i) Energy in each pulse = 2 MW × 3 μs = $(2 \times 10^6\ \text{W})(3 \times 10^{-6}\ \text{s}) = 6\ \text{J}$

(ii) Mean intensity of the emitted pulse, $I = \dfrac{P}{S} = \dfrac{2\times10^6\ \text{W}}{4\pi(50\times10^3\ \text{m})^2} = 6.37\times10^{-5}\ \text{W m}^{-2}$

(iii)
$$I_r = \frac{PkS}{16\pi^2 d^4} = \frac{(2\times10^6 W)(1)}{16\pi^2(50\times10^3\ \text{m})^4} = 2.03 \times 10^{-15}\ \text{W m}^{-2}$$

If the pulses were emitted in almost parallel beams, then the energy will not spread out over spherical wavefronts. For parallel beam, the area of the wavefronts do not change significantly as it propagates from the source, therefore the net energy reaching the target will be larger and the overall mean intensity of the reflected pulse received back will also increase.

D14

(a)(i)	A **transverse wave** is a wave in which the points of disturbance oscillate about their equilibrium positions ***perpendicular* to the direction of wave travel / energy propagation**. Example : electromagnetic wave.
	Many student forgot to give an example. Instead of "motion" the "oscillations" should be emphasized instead.
(a)(ii)	A wave is said to be plane polarised when its vibrations are restricted to only one direction in the plane normal to the direction of energy transfer.
	Students need to refer to the direction of energy transfer in their answers.
(b)(i)	Intensity after the first filter, $I_1 = I_o \cos^2 30^\circ = 0.75\, I_o$ Intensity after the second filter, $I_T = I_1 \cos^2(60^\circ - 30^\circ) = 0.75^2\, I_o = 0.563\, I_o$
	Many students did not take note of the fact that the angle between the transmitted wave after the 1st filter is at 30° to the transmission axis of the second filter
(b)(ii)	$\dfrac{A_T}{A_o} = \sqrt{\dfrac{I_T}{I_o}} = \sqrt{(0.75^2)} = 0.75$
(b)(iii)	

	Initially the transmission axis is vertical -> maximum transmission At $\theta = 45^o$, $I = I_o \cos^2 45^o = I_o / 2$
	Most students could get the maximum and minimum values right. But intermediate values were not precisely drawn.
(c)(i)	A sound wave is a longitudinal wave and so its vibrations are parallel to the direction of energy transfer and propagation of the wave. Hence, it is not possible to restrict its vibration to only one angle perpendicular to the direction of energy transfer.
(c)(ii)	$T = \frac{1}{740}$ s 6.8 cm - $\frac{1}{740}$ s 1 cm - 0.198 $\times 10^{-3}$ s = 0.20 ms Ans: 0.20 ms cm^{-1}

Learning Outcomes	Discussion Question
(a) show an understanding of and use the terms displacement, amplitude, period, frequency, phase difference, wavelength and speed.	D2, D3, D8
(b) deduce, from the definitions of speed, frequency and wavelength, the equation $v = f\lambda$.	
(c) recall and use the equation $v = f\lambda$.	D5, D8
(d) show an understanding that energy is transferred due to a progressive wave.	
(e) recall and use the relationship, intensity $\propto$ (amplitude)^2.	D4, D9, D10, D11, D12,D14
(f) show an understanding of and apply the concept that a wave from a point source and travelling without loss of energy obeys an inverse square law to solve problems.	D10,D12,D13
(g) analyse and interpret graphical representations of transverse and longitudinal waves.	D1, D5, D6, D7, D8
(h) show an understanding that polarisation is a phenomenon associated with transverse waves.	D14
(i) recall and use Malus' law (intensity $\propto \cos^2\theta$) to calculate the amplitude and intensity of a plane polarised electromagnetic wave after transmission through a polarising filter.	D14
(j) *determine the frequency of sound using a calibrated oscilloscope.	D14
(k) *determine the wavelength of sound using stationary waves.	

Suggested Solutions

Tutorial 10A: Superposition & Interference

Discussion Questions:

D1

This question tests students on the **Principle of Superposition**

Ans: (C)

The string forms a straight line only for the instant when the two pulses completely overlap.

D2

This question tests students on **waves** and **interference**.

(a)

(i) Given displacement vs time graph, the period T of waves can be determined from graph.

$$f = \frac{1}{T} = \frac{1}{5.0\text{x}10^{-3}} = 200 \text{ Hz}$$

Questions on interference are commonly paired with questions on wave motion!

(ii) $I \propto A^2$

$$\frac{I_2}{I} \propto \frac{(3A/2)^2}{A^2}$$

$$I_2 = \frac{9}{4} I = 2.25I$$

(b) 1. Waves must be of the same kind

2. The waves must overlap, i.e. they must be at the same place at the same time.

(c) **(i)**

1. For minimum intensity, Destructive interference:

phase difference $= (2n+1)\pi$, where n = 0, 1, 2, …

2. For maximum intensity, Constructive interference:

phase difference $= 2n\pi$, where n = 0, 1, 2, …

(ii) $$A_{\text{min}} = (\frac{3}{2}A - A) = \frac{A}{2} \Rightarrow I_{\text{min}} = \frac{I}{4} = 0.25I$$

$$A_{\text{max}} = (\frac{3}{2}A + A) = \frac{5A}{2} \Rightarrow I_{\text{min}} = \frac{25I}{4} = 6.25I$$

A common mistake is adding up the intensities of both waves. This is incorrect: Principle of Superposition tells us that we sum up the displacements, and not the intensities.

D3 **(i)** The number of wavelengths between source A and point P

n_{AP} = 923.7/30 = 30.79

(ii) By applying Pythagoras' Theorem,

the distance BP = $\sqrt{810^2 + 584^2}$ = 998.58 mm

The number of wavelengths between source B and point P

n_{BP} = 998.58/30 = 33.29 ≈ 33.3 (shown)

(iii) 1. The path difference BP - AP = (33.29 – 30.79)λ = 2.5 λ

Since the sources A and B are in phase, destructive interference occurs at P ⇒ intensity = 0 or minimum since they meet in antiphase.

2. Starting with zero intensity at P, detector will next detect a high intensity (2nd order), zero intensity, high intensity (1st order), zero intensity and finally ending with a high intensity (0th order) at O.

> This is a popular question that requires you to deduce the nature of interference at a certain point in space. The strategy is to find the path difference (as a multiple of wavelength) to deduce the phase difference between the two waves at this point, while being cognizant of the phase difference between the sources. See Lecture Example 10.4.1.

D4

From any position of destructive interference to the next position of destructive interference, the path difference (path A – path B) changes by 2 x 0.10 = 0.20 m.

Thus, the wavelength λ = 0.20 m.

> Due to the geometry of the setup, the distance through path A increases by twice the distance moved by the tube. Consecutive positions of minimum intensity corresponds to path difference of one wavelength.

Hence, the speed of sound $v = f\lambda$ = (1.7 x 10^3) (0.20) = 340 m s^{-1}.

> Another question that tests on the idea of path difference. This time, you should also recognize that given frequency and wavelength, you can calculate speed of the wave, which is the same for both paths, since they are passing through the same medium.

D5 **(a)** Since the path difference (XP – XQ) is zero and destructive interference (zero intensity) occurs at X, the signals from P and Q must be emitted in anti-phase, i.e. phase difference of π rad.

(b) Along XY, ship is always equidistant from the 2 sources, so the path difference is zero. Since the two sources are in anti-phase and the path difference is zero, the two signals always arrive π radians out of phase at the ship, and thus destructive interference occurs.

(c) Ship moved from a minimum at Y to the adjacent maximum (somewhere between Y and Q)

This means that the path difference between the signals from P and Q has changed by $\lambda/2$

Recall the conditions for constructive and destructive interference.

The distance moved is therefore $\lambda/4$ = 300/4 = 75 m.

A more rigorous solution to see how the distance moved is given by $\lambda / 4$,

Say the ship has moved a distance x to the right, from Y towards Q, when it reaches the adjacent maximum.

At this point,

Path length for the signal from P is $PY + x$

Path length for the signal from Q is $QY - x$

Path difference

$$\begin{aligned} &= (PY + x) - (QY - x) \\ &= (PY - QY) + 2x \\ &= 0 + 2x \\ &= 2x \end{aligned}$$

Since we have established that path difference = $\frac{\lambda}{2}$ at this point,

$$2x = \frac{\lambda}{2}$$

$$\therefore x = \frac{\lambda}{4}$$

(d) The ship detects a dip every time it moves a distance of $\frac{\lambda}{2}$. Thus, the number of dips in intensity that will be passed is $\frac{QY}{(\lambda / 2)} = \frac{30 \times 10^3}{150} = 200$

Suggested Solutions

Tutorial 10B Young's Double Slits and Diffraction Grating

Discussion Questions on Young's Double Slits:

D1

This question is about **conditions for destructive interference**.

Problem-solving Strategy: Some questions to ask yourself: 1. What are the conditions for destructive interference? 2. What is the path difference? 3. What is the phase difference at the sources?

Since both waves originate from the same source, they are in phase at the sources.
Path difference = $(l_1 + l_3) - (l_2 + l_4)$

Recalling the conditions for destructive interference for sources in phase, we must have $(l_1 + l_3) - (l_2 + l_4) = (2m+1)\lambda / 2$ as the only possible solution.

Ans: (D)

D2

This question is on **Young's Double Slit**.

(a) Read off the fringe separation from Fig. 6.2: Δy = 2.2 x 10^{-3} m.

$$\Delta y = \frac{\lambda L}{d}$$

$$2.2\times10^{-3} = \frac{\lambda(2.8)}{0.75\times10^{-3}}$$

$$\lambda = 589 nm$$

(b) (i)

Q is the first-order minimum (destructive interference), so $\Delta\phi = \pi$ radians

(i.e. the waves from each slit meet in anti-phase at Q).

(b) (ii)

$\Delta\phi = \dfrac{2.8}{2.2} \times 2\pi = 8.0$ rad

Problem-Solving Approach: Some questions to ask yourself:

What is the phase difference between the waves at

1. Point P?
2. Point Q?
3. A point between Q and R, located 2.2mm from P?

Recall the phase difference formula and think through why the phase difference between the two waves overlapping at R must be **proportional** to the distance from the principal axis along the screen.

(c)

Light undergoes diffraction after passing through each of the double slits. The diffraction envelope peaks at the centre (point P) and tapers off towards both sides.

See Page 22 of your Lecture Notes. We will introduce diffraction more formally in Section 10.7.

D3

(b) (i)

Waves from two sources can overlap and interfere. According to Principle of Superposition, the resultant displacement at any point at any instant is the vector sum of the individual displacements caused by each wave.

At positions where the two waves arrive in phase, constructive interference occurs, resulting in high-intensity fringes.

At positions where the two waves arrive out of phase, destructive interference occurs, resulting in zero-intensity fringes.

(b) (ii)

1. Two sources are said to be coherent, when they maintain a constant phase difference with respect to each other.

2. Light waves from two independent light sources are not coherent, and the phase difference between them changes rapidly and randomly. Thus, the resultant fringe pattern would also change rapidly – too fast for the human eye to follow. *By using light from a narrow region of a single source and splitting the light into two, we have effectively two coherent sources, which will produce stable interference fringes that can be seen.*

(b) (iii)

12 Δy = 26 $\Rightarrow \Delta y$ = 2.167 mm

$$\Delta y = \frac{\lambda L}{d} \Rightarrow 2.167 \times 10^{-3} = \frac{\lambda(1.20)}{0.38 \times 10^{-3}} \Rightarrow \lambda = 686 \text{ nm}$$

(b) (iv)

This reduces the percentage uncertainty in the measurement of the fringe width.

(b) (v)

Light reaches the eye through two paths: 1) directly; 2) after reflection off the mirror surface.
As the two paths between the slit and the eye have different lengths, there is a path difference, resulting in an interference pattern.

D4 (a)

Amplitude of light from each source increases $\Rightarrow$ Amplitude of light at maxima increases

Bright fringes become brighter, dark fringes remain dark $\Rightarrow$ better contrast.

(b)

Fringe spacing becomes greater (Using $\Delta y = \frac{\lambda L}{d}$).

(c)

Firstly, bright fringes resulting from constructive interference are less bright than before.

Secondly, dark fringes are less dark than before because destructive interference is only partial (not complete). Hence, the fringe pattern will have poorer contrast.

(d)

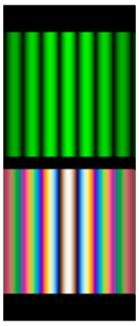
Zeroth order:	central white fringe, reddish on the sides
Higher orders:	for the next few fringes beside the zeroth order, one would get “rainbow” fringes - the visible spectrum lining up in the fringe, starting from blue (or violet) on the edge nearer the principal axis, ending with red on the edge farther away ($\Delta y = \frac{\lambda L}{d} \Rightarrow \Delta y_R > \Delta y_B$ since $\lambda_R > \lambda_B$).

The fringes are likely to overlap at even higher orders, and one will no longer be able to distinguish one fringe from another.

Interference pattern formed by a double slit

Pattern from green incident light.

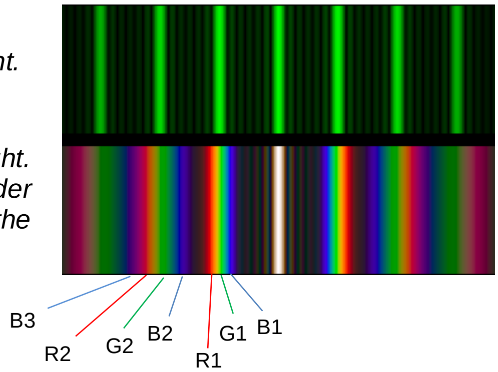
Pattern from white incident light.



Interference pattern formed by a 6-slit grating

Pattern from green incident light.

Pattern from white incident light. The blue end of the 3rd order overlaps with the red end of the 2nd order.

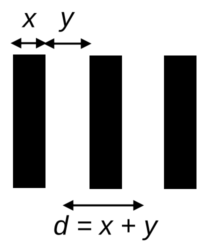


Discussion Questions on Diffraction Grating

D5

This question involves understanding of the quantity '*d*' defined in the diffraction grating equation.

$$\theta_1 = \sin^{-1}\left(\frac{\lambda}{d}\right) = \sin^{-1}\left(\frac{\lambda}{x+y}\right)$$



Ans: (A)

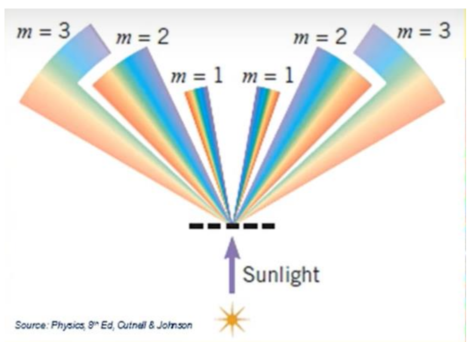
D6 **(a)** Slit separation $d = 1 \times 10^{-3} / 480 = 2.08 \times 10^{-6}$ m.

n	$\theta_n = \sin^{-1}\left(\frac{n\lambda}{d}\right)$		Angular width $\Delta\theta$
	λ = 400 nm (Violet)	λ = 750 nm (Red)	
1	11.1°	21.1°	10.0°
2	22.6°	46.1°	23.5°
*3	35.2°	-	-

* Incomplete 3rd order spectrum, starting at 35.2°.

> **Note:** If your answer is organized and presented as above (in a table), it helps you to see the reason to answer part (b) below.

(b)
- The 2nd order spectrum ends at 46.1°, while the 3rd order spectrum starts at 35.2°. Hence, the 2nd and 3rd order spectra overlap. Thus, only 1st order spectrum is complete and not overlapping with any other spectra.
- The 1st order spectrum is also the brightest. The 2nd and 3rd order spectrum will be quite dim and might not be visible.



D7

This question involves interpretation of physical **measurable lengths** in a diffraction grating setup.

(a) (i)

Using the 1st order spectrum, $\theta_1 = \tan^{-1}\left(\frac{(107.3-37.9)/2}{200}\right) = 9.843^o$.

$$\lambda = \frac{d\sin\theta}{n} = \left(\frac{10^{-3}}{250}\right)\sin(9.843^o) = 6.84\times10^{-9} \text{ m} = 684 \text{ nm.}$$

(a) (ii)

Using the 2nd order spectrum, $\theta_2 = \tan^{-1}\left(\frac{(145.6-0)/2}{200}\right) = 20.0^o$

$$\lambda = \frac{d\sin\theta}{n} = \frac{1}{2}\left(\frac{10^{-3}}{250}\right)\sin(20.0^o) = 6.84\times10^{-9} \text{ m} = 684 \text{ nm.}$$

(b)

$d \sin \theta = n\lambda \Rightarrow \sin\theta = \frac{n\lambda}{d}$.

Since $\lambda_{Blue} < \lambda_{Red} \Rightarrow \theta_{Blue} < \theta_{Red}$ for a given order *n*.

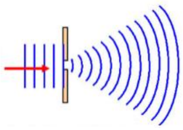
Note:
Only the wavelength of the light has changed. There is no change to the grating, hence *d* is constant.

Thus, the *angular separations* between the blue fringes will be smaller.
The 3rd order fringe and maybe even the 4th order fringe for the blue light can be seen on the screen.
(* if we take λ_B ~400 nm, then for the grating used, θ_3 = 17.5º, θ_4 = 23.6º.)

(c)

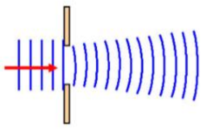
Infra-red radiation is not visible to the naked eye, therefore the spectra cannot be observed directly.
To tackle this, we could use an infra-red detector and move it slowly along the screen.

D8 (ai) a narrow gap



More diffraction of waves when wavelength of wave is comparable to slit size(i.e. waves spread more through the slit into the geometrical shadow).

(aii) a wide gap



Less diffraction of waves (i.e. waves spread less through the slit into the geometrical shadow).

Note: This part is on Diffraction of Waves. Whether the slit width is large or small as compared to the wavelength determines the amount of diffraction.
This will be formally covered in the next section (10.7.1) on diffraction of water waves.

(b) (i)

The slit separation $d = 1 \times 10^{-3} / 500.00 = 2.00 \times 10^{-6}$ m.

N	$\theta_n = \sin^{-1}\left(\frac{n\lambda}{d}\right)$		angular separation, $\Delta\theta$
	λ = 588.99 nm	λ = 589.59 nm	
1	17.13°	17.15°	0.02°
2	36.09°	36.13°	0.04°
3	62.07°	62.18°	0.11°

The maximum order for each wavelength is 3.
Bright fringes (spectral lines) will be located at the angular positions shown in table.
The maximum angular separation is between the 3rd order fringes.

Hence maximum angular separation is $\Delta\theta$ = 0.11° as shown in table.

(b) (ii)

The 3rd order fringes will probably be very dim.

D9 (a)

Diffraction of light is the spreading of light waves into its "geometrical" shadow, after passing through small apertures, or the spreading of light waves round an obstacle.

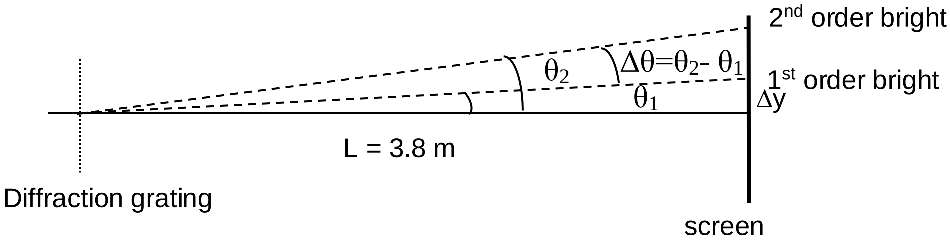
> **Note:** This part is on the Diffraction of Waves.

(b)

The line of spots of light AB is produced by the horizontal nylon threads.

(c)

Using a ruler and the scale provided in Fig. 4.2, the distance between the 2nd order maxima is 4.0 cm. Hence, the fringe separation is 1.0 cm = 1.0 x 10^{-2} m.



$$\theta_1 = \tan^{-1}\left(\frac{\Delta y}{L}\right) = \tan^{-1}\left(\frac{1 \text{ x } 10^{-2}}{3.8}\right) = 2.6 \text{ x } 10^{-3} \text{ rad}.$$

$$\theta_2 = \tan^{-1}\left(\frac{2 \text{ x } 10^{-2}}{3.8}\right) = 5.2 \text{ x } 10^{-3} \text{ rad}$$

$$\Delta\theta = \theta_2 - \theta_1 = 5.2\times10^{-3} - 2.6\times10^{-3} = 2.6\times10^{-3}\text{rad}$$

(d)

Using $d \sin\theta = n\lambda$,

$$d \sin(2.63 \text{ x } 10^{-3} \text{ rad}) = (1)(590 \text{ x } 10^{-9})$$

d = 2.24 x 10^{-4} m = 0.224 mm.

Or

$$d \sin(5.26 \text{ x } 10^{-3} \text{ rad}) = (2)(590 \text{ x } 10^{-9})$$

d = 2.24 x 10^{-4} m = 0.224 mm.

Hence, the number of nylon threads per mm of the mesh is $N = 1/d = 4.5$.

(e)

The threads are organized in a regular square grid.

i.e., the number of threads per mm is the same in both directions.

(Note that d is much larger than the wavelength in this case, and so that angles (as calculated) are very small, and so the spots are equally spaced.)

(f)

Spacing of atoms ~ 10^{-10} m.

$$\sin\theta = \frac{n\lambda}{d} \sim \frac{n\,(590x10^{-9})}{10^{-10}} \sim 10^{3} n$$

This means that, even for n = 1, the 1st order fringe, sin θ is much larger than one. Hence, there is no solution for θ, so no diffraction pattern will be observed on the screen.

X-rays with a wavelength comparable to the spacing of the atoms can be used instead.

Suggested Solutions

Tutorial 10C: Single Slit and Rayleigh's Criterion

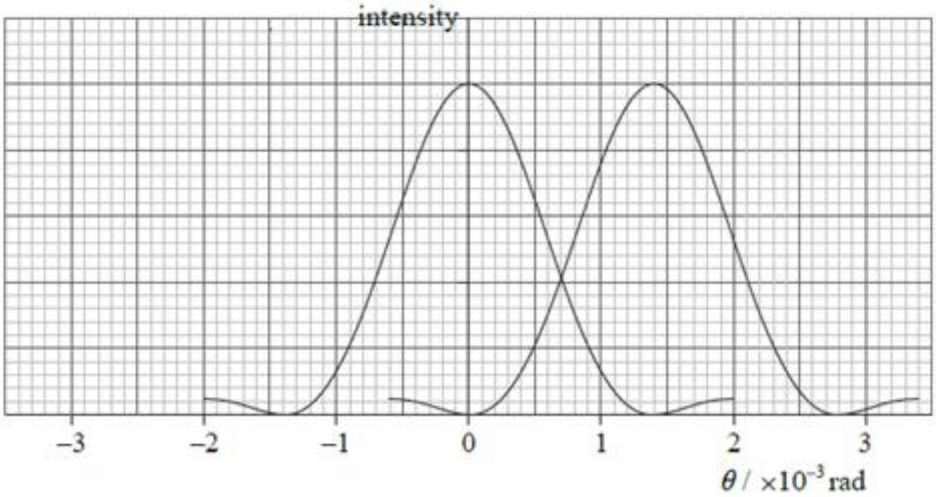
Discussion Questions:

D1

This question involves the application of **Rayleigh's criterion**.

(a) (i) angle of first minimum is 0.0014 rad;
thus $\lambda = b\theta = 0.0014 \times 4.0 \times 10^{-4} = 5.6 \times 10^{-7}$ m; *[2]*

(ii)



as shown above; *[1]*
Accept if second pattern is drawn to the left of the other.

> Note
> For (a)(i), because the angle is so tiny, sin 0.0014 = 0.0014 rad. If the angle is not small, you have to use λ = b sinθ.
> For (a)(ii), according to Rayleigh's criterion, central maximum of one image coincides with the first minimum of the other image. This should be reflected in your drawing.

D2

This question involves the **single slit diffraction formula** $\sin\theta = \frac{\lambda}{b}$ (part a) and **Rayleigh's criterion** (part b).

(a) (i)

Diffraction of light waves is the spreading of light waves (into the geometrical shadows), when passing through slits or around obstacles.

(a) (ii)

$b\sin\theta = \lambda$

Using trigonometry, where *L* is the slit-screen distance and *y* is the distance between C and the first minimum, and since the wavelength λ (10^{-7} m) is so much smaller than slit width *b* (10^{-4} m), the first-order minimum is very small and sin θ = θ = tan θ = *y*/*L*.

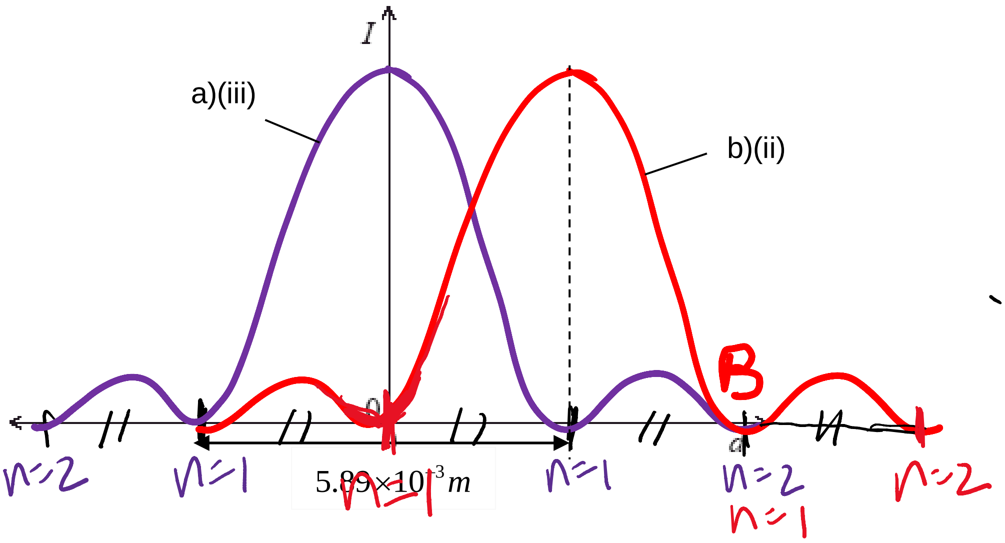
$$b\left(\frac{y}{L}\right) = \lambda$$

$$y = \frac{L\lambda}{b}$$

$$\text{width} = 2y = \frac{2\lambda L}{a} = \frac{2(620\times10^{-9})(1.9)}{0.40\times10^{-3}} = 5.89\times10^{-3}\text{ m}$$

Since the angle is small, you could also use the arc length formula (s = r θ) – the distance along the screen is very well approximated by the arc length.

(a) (iii)



You should label the width of the central maximum as calculated in (a)(ii). As the angles are so small, the distance between the central 0° position and n = 1 minimum is equal to the distance between n = 1 minimum and n = 2 minimum, and if you draw the 3rd order minimum the distance between n = 2 minimum and n = 3 minimum should be the same too.

(b) (i)
Two images are said to be just resolved or distinguishable when the **central maximum of one image** falls on the **first minimum of the other image.**

(b) (ii)
See diagram above.

<u>Marking points:</u> Graph with correct shape, labelled clearly Central maximum of one graph coincide with first minimum of the other graph.

(b) (iii)
The light diffracts after passing through the pupil of the eye.
Each headlight produces a diffraction pattern in the eye.
As the distance from the observer increases, the angular separation between the two headlights decreases, and the two diffraction patterns begin to overlap.
At a distance far enough, the maximum of one pattern coincides with the first minimum of the other and the patterns are just distinguished.
Moving further away still, the patterns get even closer and cannot be distinguished.

D3

This question involves qualitative understanding of the single slit diffraction equation $\sin\theta = \frac{\lambda}{b}$.

Note: The diagram shows the central maximum of a single-slit diffraction pattern. It does not represent the entire diffraction pattern.

(i)

As sin $\theta = \lambda / b$, where θ is the angle to the first minimum of the diffraction pattern, λ is wavelength of light and b is the slit width, when the wavelength of light used increases, θ will increase.

As a result, the central maximum of the observed diffraction pattern will become broader as the light will spread more (i.e. more diffraction) when it passes through the single slit.

(ii)

The amount of spreading (or diffraction) of light is dependent on the wavelength of light.

White light consists of light of different wavelengths.

When white light is incident on a single slit, the different wavelengths of light overlap at the central region (straight through, at 0^o) giving white light. But as different wavelengths have different first order minimum angles, with the longest wavelength red light diffracting the most, the edges are coloured.

D4	**HCI 2019 Prelim P3 Q9(part)** This question involves **both single and double slits.**		
	(i)	An interference pattern of clear/stable maximas and minimas is seen on the screen.	[1]
		Marker's Comments: Nil.	
	(ii)	$\Delta\phi = \frac{1.4}{1.6}\times 2\pi$ $= 5.5 \text{ rad}$	[1] [1]
		Marker's Comments: For this double-slit set-up the spacings between maximas (or minimas) are constant, and for each step-up in order the phase difference between the two waves is 2π. It is simply a matter of proportion.	
	(iii)	Reading off both the 3rd order bright fringes from Fig 9.5, $x = \frac{14.8-5.2}{6} = 1.6 \text{ mm}$ (recall fringe separation formula for double slits: $\Delta y = \frac{\lambda L}{d}$) $a = \frac{\lambda D}{x} = \frac{650\times10^{-9}\times1.5}{1.6\times10^{-3}} = 0.61 \text{ mm}$	[1] [1]
		Marker's Comments: Generally well done.	
	(iv)	Distance of 6th order from principle axis = 1.6 × 6 = 9.6 mm $\theta = \tan^{-1}\left(\frac{0.0096}{1.5}\right) = 0.0064 \text{ rad}$ (recall single slit diffraction pattern formula: $\sin\theta = \frac{\lambda}{b}$, where b = slit width) $b = \frac{\lambda}{\sin\theta} = \frac{650\times10^{-9}}{\sin(0.0064 \text{ rad})} = 0.10 \text{ mm}$	[1] [1] [1]
		Marker's Comments: You could also have used a sin θ = 6λ (for the double slits) and b sin θ = λ (for the single slit diffraction envelope) and simply equated 6λ/a = λ/b → b = a/6.	

Suggested Solutions

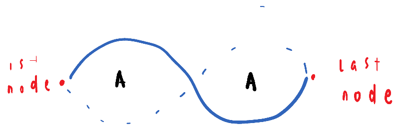
Tutorial 10D: Stationary Waves

Discussion Questions on Stationary Waves

D1

This question involves quantitative calculation by **inferring the wavelength** of **a standing wave**.

By drawing, we can see that if we start from one node, then move through 2 antinodes (with a node in between) to the next node, we would have moved through 1 wavelength.



Hence every 2 antinodes that we move through before the next node will give us 1 wavelength.

$v = f\lambda = (2500)(1.900 / 10) = 475 \text{ m s}^{-1}$

Ans: (D)

Problem-Solving: Some questions to ask yourself:

1. How do you know that the question is about stationary waves?
2. What information hinted about the formation of stationary waves?
3. What does the microphone detect: **displacement** nodes & antinodes, or **pressure** nodes & antinodes?

D2

This question involves quantitative calculations on **resonant frequency**.

Resonance occurs when the driver frequency of the wind matches the natural frequencies of the bridge.

For the fundamental frequency, the span of the bridge would be one loop or half a wavelength.

$$L = \frac{\lambda}{2} = \frac{v}{2f} = \frac{vT}{2} = \frac{400 \times 5}{2} = 1000 \text{ m}$$

Ans: (B)

Problem-Solving Strategy:

If you cannot see why $L = \frac{\lambda}{2}$ in this case, sketching a diagram would be very useful.

Problem-Solving: Some questions to ask yourself:

1. Why are there displacement nodes on the two ends of the bridge?
2. Is there an external driver? What is the frequency of the external driving force?
3. What is one natural frequency of the suspension bridge?
4. Why is there greatest danger when the bridge resonates at its fundamental frequency?

D3

since v $\propto$ √(tension) as given in question, (or refer to page 51 for speed of a wave on a string)
f λ $\propto$ √(mass of hanging load)

The initial case is in fundamental mode or the first harmonic. And final case can be in nth harmonic.
Recall that for stationary waves on strings, frequency of nth harmonic = $f_n = nf_1$
For the 1st harmonic,

$$f_1\lambda_1 = k\sqrt{m_1} \quad \Rightarrow \quad 15(0.90\times 2) = k\sqrt{400}$$

Hence,

$$\frac{f_n\lambda_n}{f_1\lambda_1} = \frac{\sqrt{m_n}}{\sqrt{m_1}}$$

$$\frac{f_n(2\times 0.90/n)}{15(2\times 0.90)} = \frac{\sqrt{m_n}}{\sqrt{400}}$$

$$\frac{f_n}{15n} = \frac{\sqrt{m_n}}{\sqrt{400}}$$

By trial and error of each option in question,

$$\frac{90}{15n} = \frac{\sqrt{900}}{\sqrt{400}} \quad \Rightarrow \quad n = 4$$

n can be 4 ; i.e. f = 90 Hz

<u>OR (an adaptation of the above):</u>

Consider the fundamental mode for each of the masses in the option. With a wavelength of 1.80 m, what is the fundamental frequency for each of the masses? We know that $v \propto \sqrt{T} \propto \sqrt{m}$, and with wavelength always 1.80 m, $\sqrt{m} \propto f_0$

mass / g	f_0 / Hz	Given option / f_0	Conclusion
400	15	-	-
500	$\sqrt{\frac{500}{400}}\times 15 = 16.8$	30/16.8 = 1.8	Non-integer multiple. Not possible.
600	$\sqrt{\frac{600}{400}}\times 15 = 18.4$	40/18.4 = 2.2	Non-integer multiple. Not possible.
800	$\sqrt{\frac{800}{400}}\times 15 = 21.2$	60/21.2 = 2.8	Non-integer multiple. Not possible.
900	$\sqrt{\frac{900}{400}}\times 15 = 22.5$	90/22.5 = 4	Integer multiple! 4th harmonic!

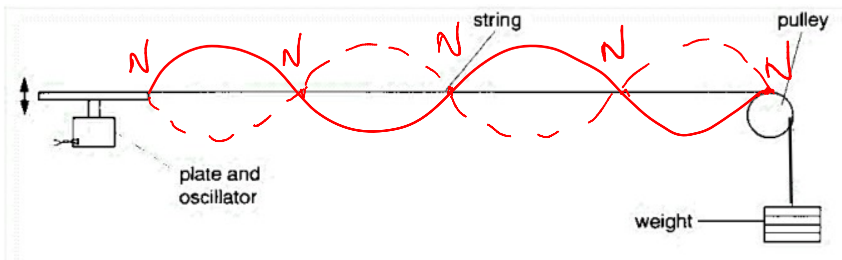
Another OR:

The speed v is proportional to the square root of the tension T, hence $v = k\,T^{1/2}$
The length is fixed at 90 cm, so λ = (1.80 m)/n, where n = 1, 2, 3, …
When m = 400 g, $T = mg$ = (0.400)(9.81) = 39.2 N
$v = f\lambda$ = (15)(1.8) = 27 m s^{-1}, so k = 13.6 m s^{-1} $N^{-1/2}$
Option A is wrong: for m = 500 g, v = 30 m s^{-1}, so λ = (30/30) = 1.0 m
Option B is wrong: for m = 600 g, v = 33 m s^{-1}, so λ = (33/40) = 0.83 m
Option C is wrong: for m = 800 g and f = 60 Hz, v = 38 m s^{-1}, so λ = (38/60) = 0.64 m
Option D is correct: for m = 900 g and f = 90 Hz, v = 40 m s^{-1}, so λ = (40/90) = 0.45 m = (1.8 m)/4

Ans: (D)

D4

(a) (i) & (ii)



Split the length of the string equally into 4 parts (4 loops). Must be accurately drawn.

(iii)

The stationary wave is observed only when the driver frequency of the oscillating plate f matches the natural frequencies of the string (resonance).

Given that both ends of the string must be displacement nodes, the possible wavelengths are 2L, L, 2L/3, L/2 etc. Since the wave speed v is fixed (since tension and mass per unit length of string are not changed), resonant frequencies of the string are $\frac{v}{2L}, \frac{v}{L}, \frac{v}{2L/3}, \frac{v}{L/2}$ etc. To observe a stationary wave the string needs to be driven at these frequencies.

(b)

The speed of the wave in the string is dependent on the tension in the string and the mass per unit length of the string.

Using strings of different materials and/or diameters gives rise to different wave speeds and, for a given length, different resonant frequencies. Hence, using different materials and/or diameters

D5

Problem-Solving: Some questions to ponder:
1. What is the external driver? Can the frequency of the external driver be changed? 2. Does the speed of sound wave change? 3. Is the end correction negligible? How do you know?

Referring to Self-Review Question S4, we note that for consecutive resonant lengths, the difference is just one loop/segment, or half a wavelength.

Sketching the wave profiles for the stationary waves corresponding to the first two resonances allows us to find that $\lambda/2$ = (0.46 – 0.14) = 0.32 m.

Hence, the wavelength λ = 2 x (0.46-0.14) = 0.64 m.

Ans: (D)

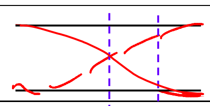
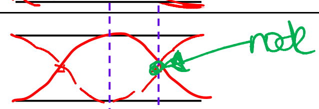
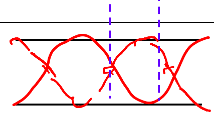
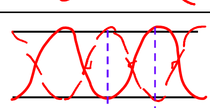
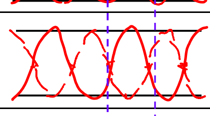
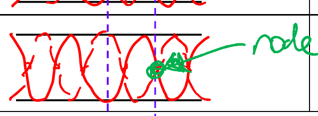
Note: In this question, we deduce from the air columns that end corrections cannot be ignored. When end corrections are negligible, the second resonant length should be 3 times the first, meaning we should be given 0.14 x 3 = 0.42 m, instead of 0.46 m.

Are you able to work out that the end correction for this system is 0.02 m?

D6

Problem-Solving: Some questions to ponder:
- What are the boundary conditions to draw the various harmonics for string, open pipes and closed pipe at one end?

The length of the tube, L = 0.68 m
The first node is located at 0.17 m. This corresponds to ¼ L.
When frequency is increased, wavelength shortens.
Draw the various harmonics where the resonant frequency occurs.

Displacement-wave profile	Harmonic	Is it a node at L/4?	Frequency (refer to page 54 in lecture notes for derivation)
	1st	No	$f_1 = \frac{v}{2L}$
	2nd	**Yes**	$f_2 = 2f_1 = \frac{v}{L} = \frac{340}{0.68} = 500\,Hz$
	3rd	No	-
	4th	No	-
	5th	No	-
	6th	**Yes**	$f_6 = 6f_1 = 6(\frac{500}{2}) = 1500\,Hz$

More efficiently: You could deduce that the next time the microphone encounters a node would be when there are 0.75 wavelengths within that 0.17 m. Hence the new wavelength would be 0.17/0.75 = 0.226667 m, and the frequency is 340/0.22666667 = 1500 Hz.

Ans: (C)

D7

Problem-Solving: Some questions to ponder:

- Where are the displacement nodes and antinodes?
- Where are the pressure nodes and antinodes?

The length of the arrows given is the diagram are an ***indication of the amplitudes of the displacement variations*** at those positions. We observe a displacement node at 0 cm and at 40 cm. This implies that half a wavelength $\lambda/2$ = 40 cm = 0.40 m.

$v = f\lambda = 440(0.40 \times 2) = 352 \text{ m s}^{-1}$

Ans: (D)

Recall: Boundary Conditions for resonant vibrations in pipe

At the closed end of a pipe, there is a displacement node.

At the open end of a pipe, there is a displacement antinode.

D8

(a) (i)

In a *transverse wave*, the **direction of oscillation/vibration** of disturbance is **perpendicular** to the **direction of propagation of energy.**

(ii)

P*olarisation* is a phenomenon whereby the **oscillations/vibrations** in a transverse wave are **restricted to only one direction** in the plane **normal to the direction of propagation of energy.**

Note: For part (a), the question says: "By reference to the direction of propagation of energy…"

(b) (i)

In a stationary wave, there are points where the **amplitude of vibration is maximum** (**displacement antinodes**) and points where the **amplitude is zero** (**displacement nodes**).

At the **displacement antinodes**, the oscillations have **maximum amplitude**, and the **sand is pushed sideways** by the sound wave.

At **the displacement nodes**, there are no oscillations (or **amplitude is zero**), and the heaps of sand **settle** here, creating heaps of sand at regular intervals.

Note: Question asks "By reference to properties of stationary wave,." so there is no need to include the formation of the stationary wave itself.

(ii)

The distance between adjacent heaps of sand is half a wavelength. Hence, there are 2.5 wavelengths in 39.0 cm.

Hence, speed of sound in the tube,

$$v = f\lambda = 2140\left(\frac{0.39}{2.5}\right) = 334 \text{ m s}^{-1}$$

(c)

A stationary wave is formed when 2 identical waves of the **same amplitude, frequency and speed but travelling in opposite directions superpose with each other**.

In this set-up, **the incident and reflected (off the ends of the tube) waves superpose** to form the stationary wave.

The speed calculated is the **speed of the incident and reflected progressive waves** (or the speed of the sound wave from the source.)

Note: Question asks "By reference to the formation of a stationary wave", so this must be explicitly included in your answer.

2023 Thermal Physics Tutorial 11A Suggested Solutions

Tutorial 11A

<u>Discussion Questions</u>

D1. Answer: B

0 K = - 273 ^oC and a change of 1 K = a change of 1 ^oC

OR

from definition, $T_C = T_K - 273$, plotting the graph of T_C against T_K will yield a straight line of gradient 1 and y-intercept of – 273.

Hence the y-intercept is - 273 ^oC and gradient = 1

D2. Answer: D

0 K = - 273 ^oC. A change of 1 K = a change of 1°C.

OR

from definition, $T_K = T_C + 273$, plotting the graph of T_K against T_C will yield a straight line of gradient 1 and y-intercept of 273.

D3. **(i)** Two bodies are in thermal equilibrium if they have the same temperature.

(ii) Two bodies are in thermal equilibrium if there is no net transfer of thermal energy between them.

D4. Answer: D

Heat transfer always happens from a higher to a lower temperature.

Object X absorbs more than it emits. Hence X has a net gain of thermal energy, implying initial temperature of X is less than 30 ^oC.

Similarly, since Y has zero net flow of thermal energy, we can conclude that Y was at thermal equilibrium with water at 30 ^oC and Z has a net loss of thermal energy, implying it was at a higher temperature than water at greater than 30 ^oC.

D5. Answer: C

$P_1 = m_1 L_V + h$ ------ (1)

$P_2 = m_2 L_V + h$ ------ (2)

(1) – (2): $P_1 - P_2 = (m_1 - m_2) L_V$

$L_V = (P_1 - P_2) / (m_1 - m_2)$

D6. Answer: D

Thermal energy supplied = Heat required to raise temp from 30°C to 100°C + Heat required to convert water to steam

Total heat required, $Q = mc\Delta\theta + mL_v$

$= (5.00)(4190)(100 - 30.0) + (5.00)(2260 \times 10^3)$

$= 1.28 \times 10^7$ J

D7. Answer: B

In the question, it is mentioned that "There is no heat transfer or work done on the ideal gas", hence from the first law of thermodynamics, this implies that no change in the internal energy of the gas, $\Delta U = Q + W = 0$.

Recall that for an ideal gas, the internal energy of the gas is the sum of the microscopic kinetic energy due to the random motion of the individual gas particles (and the microscopic potential energy is defined to be zero as there is no intermolecular forces of attraction in an ideal gas). The potential energy and the kinetic energy values mentioned in the question refer to the corresponding energies of the gas as a whole, **macroscopically**, and they do not contribute to internal energy.

Hence the internal energy of the gas at point X remained the same as before.

D8. (i) The ice is not in thermal equilibrium with its surroundings, as it is at a lower temperature than its surroundings, hence it is absorbing heat from its surroundings.

(ii) By conservation of energy,

Power supplied by heater, P + rate of thermal energy gained from the atmosphere, R

= rate of gain of latent heat of fusion

$$P + R = \frac{\Delta m . L_f}{\Delta t}$$

$$R = \frac{\Delta m . L_f}{\Delta t} - IV = \frac{(114.0 - 32.4)(330)}{(5.0 \times 60)} - (6.3)(12.0) = 14.16 = 14.2\ W$$

D9. (b)(ii) There is heat loss to the surroundings (see equation in the next part).

(b)(iii) $Q = Mc(\Delta\theta) + ht$, where h is the rate of heat loss to the surroundings.

Expressing equation on a "per unit time" basis, where $P = Q/t$ and $m = M/t$,

$$P = mc(\Delta\theta) + h$$

$$m = \frac{P}{c(\Delta\theta)} - \frac{h}{c(\Delta\theta)}$$

Gradient = (3.50 - 1.30) x 10^{-3} / (70.0 – 35.0) = 2.20 x 10^{-3} /35.0

`

= 6.2857 x 10^{-5}

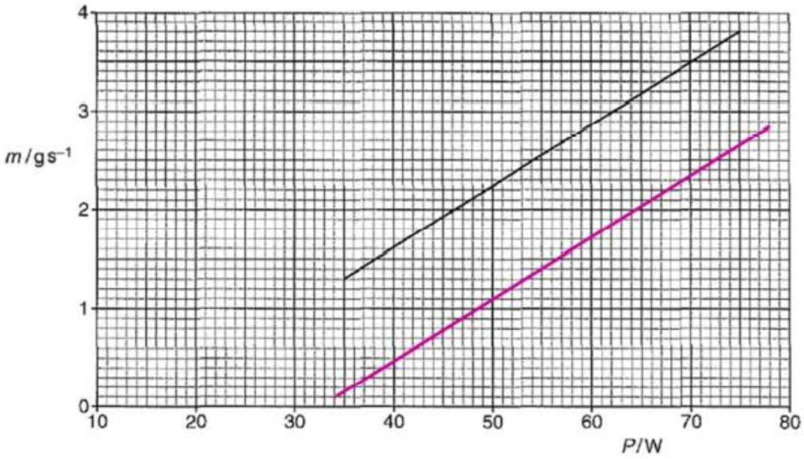
1/c (38.8 – 35.0) = 6.2857 x 10^{-5}

c = 4190 J kg^{-1} K^{-1}

(c) Same substance (water) is still being used, c will be the same.

Difference in inlet and outlet temperatures $\Delta\theta$ are the same. Hence gradient will be the same.

Apparatus is at a higher temperature now. Assuming that the environment is at the same temperature, rate of heat loss h will be higher. Vertical intercept will be "more negative".



Thermal Physics Tutorial 11B Discussion Questions Suggested Solutions

D1. Answer: D

$$\frac{N_X}{N_Y} = \frac{n_X}{n_Y}$$

When the two vessels are connected by a narrow tube, the molecules will flow until the pressures in the two vessels are the same.

Using the equation of state, $pV = nRT \Rightarrow n_X R T_X / V_X = n_Y R T_Y / V_Y$

$$\Rightarrow n_X / n_Y = (T_Y / T_X)(V_X / V_Y)$$

D2. Answer: B

Assume V is constant.

$$p = \frac{nRT}{V} \Rightarrow p \propto T \Rightarrow p_2 = \frac{T_2}{T_1} p_1 = \frac{(273+50)}{(273+25)}(200+100) = 325 \text{ kPa}$$

Hence, the new reading on the pressure gauge is 325 – 100 = 225 kPa

$$\frac{\Delta P_{gauge}}{P_{gauge}} = \frac{25}{200} = 12.5\%$$

D3. Answer: D

$pV = nRT \rightarrow 1/p = (nRT)^{-1} V$

The gradient of the graph is $1/(nRT)$. Since both n and T are doubled, the gradient will be ¼ of the original.

D4. Answer: B

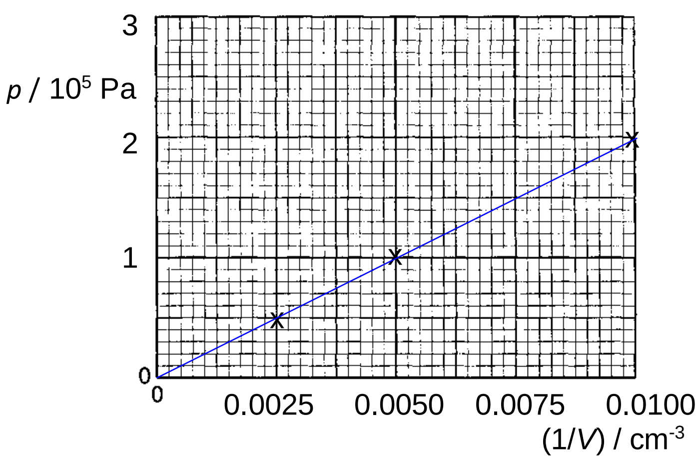
Valve releases gas to keep pressure constant. Of course volume is also constant. $n = \frac{pV}{RT}$

$$\frac{m_f}{m_i} = \frac{n_f}{n_i} = \frac{pV}{R(30+273)} \Big/ \frac{pV}{R(10+273)} = \frac{283}{303}$$

$$m_f = \frac{283}{303} \times 15 = 14 \text{ kg}$$

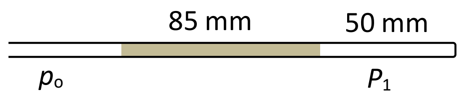
D5. (i) $p = nRT/V$. Plot a graph of p versus $(1/V)$.

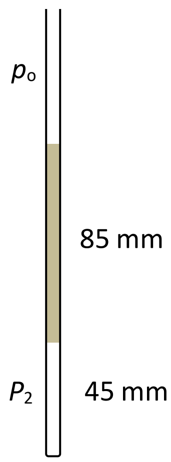
p / 10^5 Pa	V / cm^3	$(1/V)$ / cm^{-3}
0.5	400	0.0025
1.0	200	0.0050
2.0	100	0.010



(ii) A **straight-line** graph passing **through the origin** shows that p is inversely proportional to V.

D6.





When horizontal, $p_1 = p_o$ (1)

When vertical, $p_2 = p_o + \rho_{Hg} hg$ (2)

Assuming trapped air is ideal gas and at constant temperature (i.e. pV is constant),

$p_1(50) = p_2(45)$ (3)

Solving gives p_o = 1.0 x 10^5 Pa

D7. (a) In general, the internal energy is the sum of kinetic and potential energies associated with the molecules of a system.
Since there are no intermolecular forces in an ideal gas, the internal energy of an ideal gas is just the sum of microscopic kinetic energy of the molecules in the gas due to the random motion of the molecules.

(b) (i) Mass of air = (number of moles) x (molar mass)

$$= \frac{PV}{RT} \times M = \frac{(1.0 \times 10^5)(0.075)(0.030)}{(8.31)(273.15 + 25)} = 0.0908 = 0.091 \text{ kg}$$

(b) (ii) Density = mass/volume. Since molar mass and volume are constant, the density is proportional to the number of moles *n*. The number of moles is in turn inversely proportional to the thermodynamic temperature, since pressure and volume are constant.

$$\frac{\rho_{25}}{\rho_{200}} = \frac{n_{25}}{n_{200}} = \frac{T_{200}}{T_{25}} = \frac{273.15 + 200}{273.15 + 25} = 1.59$$

D8. (a) (i) momentum of a molecule = mv (to the right)

(ii) Since the collision is elastic,

change in momentum = final momentum – initial momentum = $-mv - mv = -2mv$

i.e. 2 mv (to the left)

(iii) time taken = $\dfrac{\text{total distance travelled before next collision}}{\text{speed of molecule}} = \dfrac{2l}{v}$

(iv) N (frequency of collision for one molecule) $= N\left(\dfrac{v}{2l}\right) = \dfrac{Nv}{2l}$

(v) Average force = $(2mv)\left(\dfrac{Nv}{2l}\right) = \dfrac{Nmv^2}{l}$

(b) Pressure = (average force) / (area) = $\left(\dfrac{Nmv^2}{l}\right) / l^2 = Mv^2 / V$

(c) Firstly, the molecules are not all moving at the same speed but a range of speeds and so, an average value, $<v^2>$ is more appropriate.

Secondly, they are not all moving horizontally but in random directions with no preference for x, y or z. By Pythagoras' theorem in 3D, $<v^2> = <v_x^2> + <v_y^2> + <v_z^2>$ and having no preferred direction means that the mean square speed in the horizontal, or x-direction $<v_x^2> = \frac{1}{3}<v^2>$.

[$<v_y^2>$ and $<v_z^2>$ are also $\frac{1}{3}<v^2>$.]

The equation in (b) should be modified to $p = \dfrac{M\left(\frac{1}{3}<v^2>\right)}{V}$.

D9. (a) The gas consists of a large number of atoms moving randomly in all directions at different speeds. Hence, the vector sum of all the velocities is zero.

(b) $pV = NkT$
$(3.6 \times 10^5)(4.2 \times 10^{-3}) = N(1.38 \times 10^{-23})(273.15 + 70)$
$N = 3.193 \times 10^{23}$ (good to show that you have verified with a calculator)
Hence, the sample of gas has 3.2×10^{23} atoms (shown).

(c) Volume of the gas atoms = $N \times 4/3\pi r^3 = (3.2 \times 10^{23}) \times (4/3\pi(2 \times 10^{-10}/2)^3) = 1 \times 10^{-6}$ m^3

(d) The total volume of the gas atoms in (c) is negligible compared to the volume of the container containing the gas (4.2×10^{-3} m^3).

D10. (a) The summation of microscopic kinetic energy due to random motion of the molecules in the ideal gas.

(b) (i) $V_1/T_1 = V_2/T_2$

$(3.2 \times 10^{-3}) / (273.15 + 12) = (3.6 \times 10^{-3}) / (273.15 + \theta)$

$\theta = 47.6$ °C

(ii) $W_{by} = p\Delta V = (1.0 \times 10^5)(3.6 \times 10^{-3} - 3.2 \times 10^{-3}) = 40$ J

(c) (i) $\Delta U = Q_{into} + W_{on} = Q_{into} + (-W_{by}) = 101 - 40 = 61$ J

(ii) $<\Delta KE> = \dfrac{61}{\text{no of molecules}} = \dfrac{61}{\left(\dfrac{1.0\times10^5\times3.2\times10^{-3}}{1.38\times10^{-23}\times(12+273)}\right)} = 7.4\times10^{-22}$ J

Alternatively, $<\Delta KE> = 3/2\, k\Delta T = 3/2\,(1.38 \times 10^{-23})(47.6 - 12) = 7.4 \times 10^{-22}$ J. [**This is valid in your syllabus**, but strictly speaking unless it is a monatomic gas or we are just interested in translational KE (and not other aspects like rotational KE), the factor is not 3/2.]

<u>**Thermal Physics Tutorial 11C Discussion Questions Suggested Solutions**</u>

D1. Answer: B

Keyword: *Cool* – Δ*U* is negative *For ideal gas, how is U related to T?*

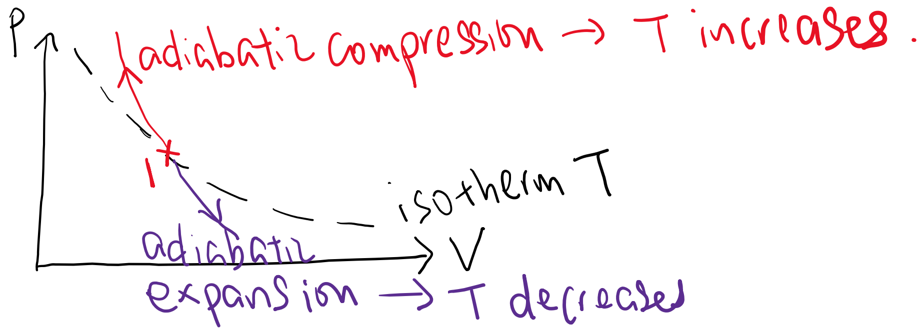
Expand – *w* is negative
Suddenly – *q* is zero (adiabatic)

D2. Answer: E

Recognise that the adiabatic curve is steeper than the isothermal curve since it goes to a different isotherm (see p. 37 Adiabatic process).

Say we start at point 1 on isotherm T below. If we compress the gas adiabatically, W = positive and Q = 0. So ΔU = positive according to the first law. This means that the temperature increases and the compression path goes above isotherm T.

If we expand the gas adiabatically, W = negative and Q = 0. So ΔU = negative according to the first law. This means that temperature decreases and the expansion path goes below isotherm T.



D3. Answer: B

Since pV = nRT, we can use the product pV to determine whether the temperature goes up or down.
At L, pV = (2.0)(0.001) 0.002
At M, pV = (2.0)(0.003) 0.006
At N, pV = (0.8)(0.005) 0.004
So the temperature increases from L to M (linearly since p is constant), then decrease from M to N.

D4. (i) The internal energy of an ideal gas is just the kinetic energy of the molecules, which is directly proportional to the temperature (in Kelvin). The change in internal energy is zero because the gas goes back to its original state or pV-value, and thus temperature.

(pV = nRT is called the equation of state.)

(ii) Consider process acba:

$$\Delta U = Q + W_{on} \Rightarrow 0 = [75 + (Q_{in})_{ba}] + (-12)$$

$$(Q_{in})_{ba} = -63 \text{ J}$$

D5.

(a)(i) ΔU is the increase in the internal energy, Q is the heat supplied to the system and W is the work done on the system.

(a)(ii)

	Solids which *contracts* on melting	Solids which *expands* on melting
ΔU	+	+
Q	+	+
W	+	–

(b)(i) Since the kinetic energies of molecules depends on the temperature, there is no change in kinetic energy of the molecules when ice at 0 °C changes to water at 0 °C .

(b)(ii) Microscopic potential & kinetic energies and the mass of the substance determine the internal energy of 1 kg of substance.

(b)(iii) $\Delta U = Q + W$. But since W is negligible ($W = 0.00009 \times 1.01 \times 10^5 = 9.09$ J), $Q = \Delta U$

Specific latent heat of fusion of ice = $\Delta U = (5.25 - 1.89) \times 10^5 = 3.36 \times 10^5$ J kg^{-1}

(b)(iv) work done by 1 kg of water = $p(\Delta V)$ = $1.01 \times 10^5 (1.67 - 0.00104)$ = 1.69×10^5 J

(b)(v) By $\Delta U = Q + W$, $(26.9 - 5.99) \times 10^5 = L - 1.69 \times 10^5$

$$L = 2.26 \times 10^6 \text{ J kg}^{-1}$$

(b)(vi) The specific latent heat of vaporisation for a given substance is higher than the specific latent heat of fusion, for two reasons:

(1) The required increase in potential energy for boiling is much larger. Transiting to a gaseous state requires a substance to break free from the attractive forces that hold them together in the liquid state. On the other hand, melting a substance requires a smaller amount of energy to overcome the intermolecular forces in the solid state.

(2) Work must be done expanding against the atmospheric pressure during boiling and less so during melting, as the change in volume in melting is insignificant.

Further Comments

Note that in this question, the state of zero potential energy of the molecules is taken to be the solid state, which therefore then means that the potential energy in the liquid and gaseous states are positive as latent heat is supplied for state change.

As the substance is transformed from the ordered solid phase to the disordered liquid phase (melting), latent heat of fusion provides the energy for the molecules to overcome the attractive bonding between them, so that they can slip out of the rigid structure to an arrangement with weaker intermolecular forces.

Since the intermolecular force is weaker in a liquid, the sum of microscopic potential energy is higher (less negative, if we take the conventional notion that the potential energy in a gas is zero) than in a solid. The heat supplied goes only into increasing the potential energy component of the internal energy. The temperature does not rise since the kinetic-energy component remains constant during melting.

As the substance changes from the liquid phase to the gaseous phase (boiling), the molecules are totally liberated, ending up in a state in which there is virtually zero intermolecular force. Latent heat of vaporisation provides the energy for molecules to overcome the attractive bonding and allow them to move around independently as gas molecules. The heat supplied during boiling goes only into increasing the potential energy component of the internal energy to virtually zero (note that in D5 the zero of potential energy is chosen or set to be the solid state), since the intermolecular force is virtually zero in a gas. The temperature does not rise since the kinetic energy component remains constant during boiling.

D6. N08/P3/Q4

(a)(i) The internal energy of a system is the sum of the microscopic kinetic energy due to the random motion of the molecules and the microscopic potential energy due to the intermolecular forces.

$$U = \sum K.E._{microscopic} + \sum P.E._{microscopic}$$

For ideal gas, $\sum P.E._{microscopic} = 0$

(a)(ii) The First Law of Thermodynamics states that the **increase** in internal energy of a system is the sum of the heat **supplied** to the system and the work done **on** the system.

(b)(i) Work done by the gas during the change C →A

= area under curve CA = $(1 \times 10^5)\,[(20-5) \times 10^{-6}]$ = 1.5 J

(b)(ii) From A → B, Using The First Law of Thermodynamics,

$\Delta U = Q + W = 0 + 4.2 = 4.2$ J

From B → C, Since $\Delta V = 0$, Work done on gas, $W = 0$

$\Delta U = Q + W = -8.5 + 0 = -8.5$ J

From C → A, $W = -1.5$ J (refer to (b)(i))

For the whole cycle A→B →C →A, $\Delta U_{ABCA} = 0$

$4.2 + (-8.5) + \Delta U_{CA} = 0$

$\therefore \quad \Delta U_{\text{for } C \rightarrow A} = -4.2 + 8.5 = 4.3$ J

	Q / J	W_{on} / J	ΔU / J
A → B	zero	4.2	**4.2**
B → C	-8.5	**0**	**-8.5**
C → A	**5.8**	**-1.5**	**4.3**

D7. (c)(i) The internal energy is the sum of microscopic kinetic and microscopic potential energies of the molecules. An ideal gas is assumed to have no intermolecular forces of attraction, hence zero microscopic potential energy. The mean kinetic energy of a molecule in an ideal gas is $\frac{3}{2}kT$. With N molecules, the internal energy of the gas in $\frac{3}{2}NkT$. Therefore, the internal energy of an ideal gas is proportional to its thermodynamic temperature.

(c)(ii) 1. By the ideal gas equation, pV = nRT

Initially,

$$n_1R = \frac{p_1V_1}{T_1} = \frac{(1.0\times10^5)(2.0\times10^{-2})}{(25+273)} = 6.7$$

Finally,

$$n_2R = \frac{p_2V_2}{T_2} = \frac{(1.5\times10^5)(2.0\times10^{-2})}{(174+273)} = 6.7$$

Since R is a constant, $n_1 = n_2$, the number of moles (amount of substance) remains constant and thus the mass of gas remains constant.

2. n = 6.7/R = 6.7/8.31 = 0.807 moles
$Q = mc\Delta T$
1220 = (0.807×0.020) c (174-25)
c = 507 J kg^{-1} K^{-1}

(c)(iii) At constant pressure, the gas will expand on heating, and so there is work done *by* the gas. $\Delta U = Q + W_{ON}$. To achieve the same increase in internal energy needed to increase temperature from 25 ºC to 174 ºC, more heat must be supplied to compensate for the negative work done *on* the gas.

D8. (a) $Q = mc\Delta T$ = (5.6)(0.73)(1) = 4.1 J

(b)(i) $\frac{V_1}{T_1} = \frac{V_2}{T_2}$

$V_2 = \frac{281}{280}V_1$

$\Delta V = V_2 - V_1 = \frac{1}{280}V_1 = \frac{4600}{280} = 16\text{ cm}^3$

(b)(ii) Work done by the gas = $p\Delta V$ = (1.0 × 10^5)(16.4 × 10^{-6}) = 1.6 J

(c)(i) The increase in internal energy of a system is the summation of the heat supplied to the system and the work done on the system.

(c)(ii) From (a), we know that for a change in temperature of 1K, ΔU = 4.09 J since there is no work done in a constant volume process.

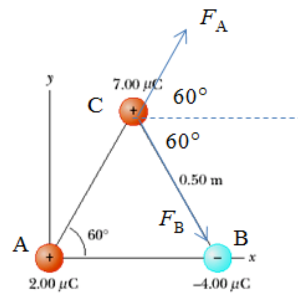
$\Delta U = Q + W$
4.09 = Q + (-1.64)
Q = 5.7 J

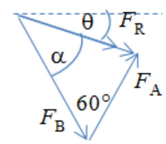
Tutorial 12 Electric Fields

Suggested Solutions to Discussion Questions

Electric Fields and Electric Forces

D1





$$F_A = \frac{1}{4\pi\varepsilon_0}\frac{Q_A Q_C}{r_{AC}^2} = \frac{1}{4\pi\varepsilon_0}\frac{(2.00\times10^{-6})(7.00\times10^{-6})}{0.50^2} = 0.5036 \text{ N}$$

$$F_B = \frac{1}{4\pi\varepsilon_0}\frac{Q_B Q_C}{r_{BC}^2} = \frac{1}{4\pi\varepsilon_0}\frac{(4.00\times10^{-6})(7.00\times10^{-6})}{0.50^2} = 1.007 \text{ N}$$

By the cosine rule,

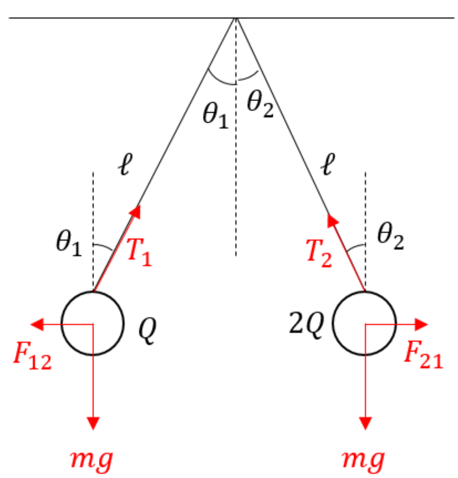
$$F_R = \sqrt{F_A^2 + F_B^2 - 2F_A F_B \cos 60^\circ} = \ldots = 0.872 \text{ N}$$

By the sine rule,

$\frac{\sin\alpha}{F_A} = \frac{\sin 60^\circ}{F_R} \rightarrow \alpha$ = 30.0°, thus θ = 60.0°- 30.0° = 30.0°

Hence, the resultant electric force on the 7.00 μC charge is 0.872 N, 30.0° clockwise from the positive *x* direction.

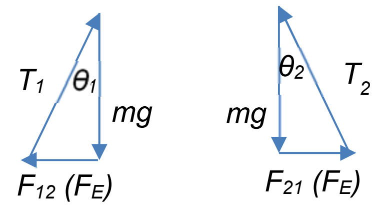
D2 ***Identify the forces acting on each sphere.***



(a) Observe that the system is in equilibrium. Since the horizontal electrical repulsive forces are equal and opposite ($F_{21} = F_{12}$) and the weight (mg) of each mass is the same, the magnitudes of the tensions will also be the same.

By symmetry about the vertical, we deduce that $\theta_1 = \theta_2$

(b) By sketching a vector triangle of the forces acting on each mass, we obtain $\tan\theta_1 = \dfrac{F_E}{mg}$.



Since θ_1 and θ_2 are assumed small, $\tan\theta_1 \approx \sin\theta_1$

Based on geometry, $\sin\theta_1 = \dfrac{r/2}{l} = \dfrac{r}{2l} = \dfrac{F_E}{mg}$

$$\frac{r}{2l} = \frac{k_e}{mg}\frac{2Q^2}{r^2}$$

$$r^3 = \frac{4k_eQ^2l}{mg} \quad \Rightarrow \quad r = \left(\frac{4k_eQ^2l}{mg}\right)^{\frac{1}{3}}$$

D3 (a) $F_{\text{A on B}} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q_AQ_B}{r_{AB}^{\;2}} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{(3.2\times10^{-19})(1.6\times10^{-19})}{\left(0.72\times10^{-9}\right)^2} = 8.88\times10^{-10}$ N to the left.

(b) (i) – (iii), (c)

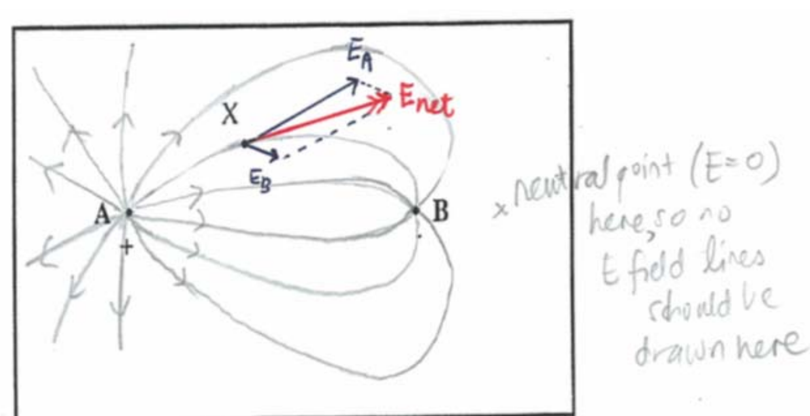
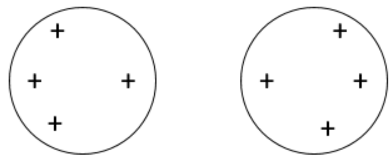


Fig. 7

Number of field lines of A should be double that of B since the magnitude of charge is twice for A as to B.

D4 The equation $F = \frac{Q^2}{4\pi\varepsilon_o d^2}$ describes the force between two *point charges*, or equivalently, for two spherically symmetric charge distributions. For an isolated charged conducting sphere, the electric field is similar to that of a point charge, as the charges are evenly distributed on the surface of the sphere. [1]

However, when two charged conducting spheres are ***relatively close in distance***, mutual polarization occurs and the charges will ***get redistributed non-uniformly on their surfaces*** (The charges repel each other):



Thus charges on the spheres are no longer distributed in a spherically symmetric fashion, and we cannot use $F = \frac{Q^2}{4\pi\varepsilon_o d^2}$.

If the charged spheres are made of insulating materials then the charges are unable to redistribute themselves on the surfaces when the spheres are brought near. Provided that the charges on both the non-conducting spheres are evenly distributed on their surfaces, each individual sphere will behave like a point charge and the force between the charged spheres can be described using $F = \frac{Q^2}{4\pi\varepsilon_o d^2}$, where *d* is the distance between the centres of the spheres (which must be greater than the diameter of each sphere).

[1] A similar argument was used in Chapter 7 (Gravitational Field) about the use of Newton's Law of Gravitation, $F_g = \frac{GMm}{r^2}$, for spherical masses of uniform or spherically symmetric density – namely, that although the formula is for point masses, most planets and stars can be treated as uniform spheres.

D5 **(i)** The gravitational force between masses is attractive, while the electric force between like charges is repulsive. The sign indicates the direction of the force, so they have opposite signs.

(ii) Gravitational force, $F_G = \frac{Gm_1m_2}{r^2} = \frac{(6.67\times10^{-11})(1.67\times10^{-27})^2}{(2\times10^{-15})^2} = 4.65 \times 10^{-35} N$

Electric force, $F_E = \frac{Q_1Q_2}{4\pi\varepsilon_0 r^2} = \frac{(1.6\times10^{-19})^2}{4\pi(8.85\times10^{-12})(2\times10^{-15})^2} = 57.5\ N$

The graphs are not drawn on the same scale, because for any particular separation, the magnitude of the gravitational force is much smaller than the magnitude of the electric force.

(iii) Since the attractive gravitational force is much weaker than the repulsive electric force, there must be another strong attractive force acting on the protons in the nucleus that allow them to stay close together in the nucleus.

Note: In fact, this is what led physicists to conclude the existence of the nuclear strong force between protons and neutrons (called the "strong force" because it had to be very strong). See https://en.wikipedia.org/wiki/Strong_interaction

Electric Potential and Potential Energy

D6 Electric potential energy of the system initially is given by $U_i = \frac{Q_1Q_2}{4\pi\epsilon_0}\left(\frac{1}{10}\right)$

Electric potential energy of the system after the charges are brought closer is given by $U_f = \frac{Q_1Q_2}{4\pi\epsilon_0}\left(\frac{1}{5}\right)$

Hence the change in the electrostatic potential energy of the system is

$$\Delta U = U_f - U_i = \frac{Q_1Q_2}{4\pi\epsilon_0}\left(\frac{1}{5} - \frac{1}{10}\right) = -20 \text{ J}$$

Ans: (A)

Note: *When two unlike charges (electric attractive force between the charges) are brought closer together, there is a DECREASE in the electric potential energy of the system. Analogous to bringing two point masses closer to each other (attractive gravitational force between the masses), there is a decrease in the gravitational potential energy of the system.*

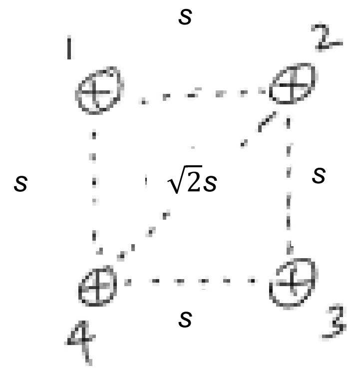
D7 (i) *The four charges are identical (like charges) and thus they repel each other. External force does positive work in assembling them.*

Work required to assemble the system

$$W_{total} = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1Q_2}{s_{12}}+\frac{Q_1Q_3}{s_{13}}+\frac{Q_2Q_3}{s_{23}}+\frac{Q_1Q_4}{s_{14}}+\frac{Q_2Q_4}{s_{24}}+\frac{Q_3Q_4}{s_{34}}\right]$$

$$=\frac{Q^2}{4\pi\varepsilon_0}\left[\frac{1}{s}+\frac{1}{\sqrt{2}s}+\frac{1}{s}+\frac{1}{s}+\frac{1}{\sqrt{2}s}+\frac{1}{s}\right]$$

$$=\frac{Q^2}{4\pi\varepsilon_0 s}\left[1+\frac{1}{\sqrt{2}}+1+1+\frac{1}{\sqrt{2}}+1\right]= 5.41\frac{Q^2}{4\pi\varepsilon_0 s}$$



(ii) Positive work is done on the system since external force exerted to bring the charges in place is in the same direction as the displacement of the charges from infinity.

(iii) The potential energy of the system $= 5.41\frac{Q^2}{4\pi\varepsilon_0 s}$, i.e., the work done by an external force to bring the charges from infinity to this configuration.

(iv) The work done by an external force in bringing one of the charges (say charge 4) to infinity is

$$\Delta U = (U_{12} + U_{13} + U_{23}) - 5.41\frac{Q^2}{4\pi\varepsilon_0 s}$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1Q_2}{r_{12}}+\frac{Q_1Q_3}{r_{13}}+\frac{Q_2Q_3}{r_{23}}\right) - 5.41\frac{Q^2}{4\pi\varepsilon_0 s}$$

$$= \frac{Q^2}{4\pi\varepsilon_0 s}\left[1+\frac{1}{\sqrt{2}}+1\right] - 5.41\frac{Q^2}{4\pi\varepsilon_0 s} = -2.71\frac{Q^2}{4\pi\varepsilon_0 s}.$$

<u>Alternatively</u>,

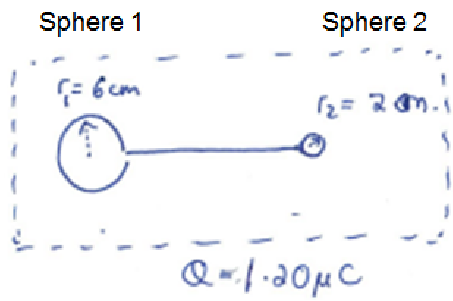
$$\Delta U = 0 - (U_{14} + U_{24} + U_{34}) = -2.71\frac{Q^2}{4\pi\varepsilon_0 s}$$

Note: *This question checks your understanding of electric potential energy.*

D8

(i) **When there is a potential difference, charges will flow.**

At equilibrium, the two conducting spheres will have the same potential at their surfaces.



$$V_1 = V_2$$

$$\frac{1}{4\pi\varepsilon_0}\frac{Q_1}{r_1} = \frac{1}{4\pi\varepsilon_0}\frac{Q_2}{r_2} \text{; simplifying: } \quad \frac{Q_1}{6} = \frac{Q_2}{2} \Rightarrow \frac{Q_1}{Q_2} = \frac{3}{1}$$

$$\Rightarrow \frac{Q_1}{Q_1 + Q_2} = \frac{3}{4} \Rightarrow Q_1 = \frac{3}{4} \times 1.2 = 0.9 \ \mu\text{C}$$

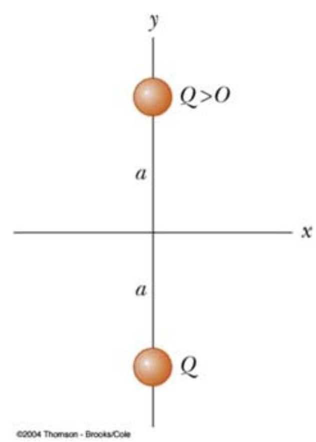
Thus $V_1 = V_2 = \dfrac{1}{4\pi\varepsilon_0}\dfrac{0.9\times10^{-6}}{6.00\times10^{-2}}$ = 1.35 x 10^5 V

(ii) $E_1 = \dfrac{1}{4\pi\varepsilon_0}\dfrac{0.9\times10^{-6}}{\left(6.00\times10^{-2}\right)^2}$ = 2.25 x 10^6 V m^{-1}, directing radially away from sphere 1.

$E_2 = \dfrac{1}{4\pi\varepsilon_0}\dfrac{0.3\times10^{-6}}{\left(2.00\times10^{-2}\right)^2}$ = 6.74 x 10^6 V m^{-1}, directing radially away from sphere 2.

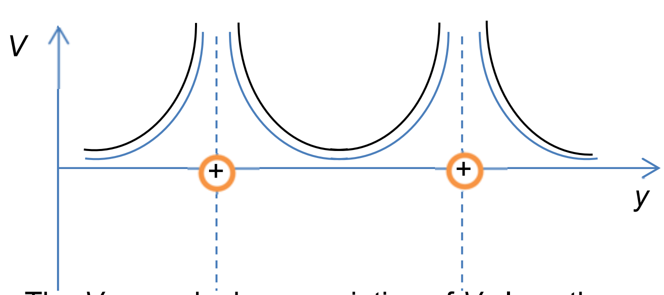
D9

(a) At the origin, $\sum E = 0$**.** At any other point in the region, there is always a non-zero horizontal electric-field component.



The resultant potential at a point in this region is the algebraic sum of potentials at that point due to the individual positive charges.

Thus, the potential is non-zero.



The *V*-*y* graph shows variation of *V* along the *y*-axis.

(b) The resultant electric field strength at any point in the region will be non-zero, as there is always a non-zero vertical electric field component.

The resultant potential at a point in this region is the algebraic sum of the potentials at that point due to the individual charges.

The resultant potential at any point along the *x*-axis is zero, since the charges have the same magnitude and are equidistant from the point.

Note: *Electric potential is a scalar quantity and electric field strength is a vector quantity.*

Relationship between Electric Force and Potential Energy

D10

(a) $E = -\frac{dV}{dr}$. The magnitude of the electric field strength at a point in the electric field is the rate of change of the electric potential with respect to distance at that point. The electric field always acts in the direction of decreasing electric potential, as indicated by the negative sign.

(b) Using the trapezium rule to approximate the area under the graph, we can estimate the electric potential difference by
$\Delta V_{AB} = \frac{1}{2}(0.01)(15 + 10.75 + 10.75 + 8.5 + 8.5 + 7.5 + 7.5 + 7 + 7 + 7.75 + 7.75 + 10)$ kV
$\Delta V_{AB} = 540\text{V}$
Since the electron accelerates in the direction AB, the electric field acts in the opposite direction i.e. BA. Electric potential at A is lower than the electric potential at B.

(c) The kinetic energy gained by the electron (or the **loss** in potential energy → initial PE minus final PE) is given by

$$KE = q(V_A - V_B) = (-1.6 \times 10^{-19})(-540)$$

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}(9.11 \times 10^{-31})v^2$$

Hence the speed of the electron is given by $v = 1.38 \times 10^{7} ms^{-1}$

(d) (i) The electron has maximum acceleration when the electric force that acts on it is the greatest. This occurs when the electric field strength is the largest (since $F = qE$), ie at A. Hence $d = 0$.

(ii) The acceleration decreases to a minimum value when the electron is 4.0 cm from A and increases thereafter until it reaches B.

D11 ***Note:*** *the dotted lines in the figure shows equipotential lines.*

(i) $\Delta V_{XY} = 100 - 0 = 100$ V

(ii) By conservation of energy, loss in KE of e = gain in PE of e

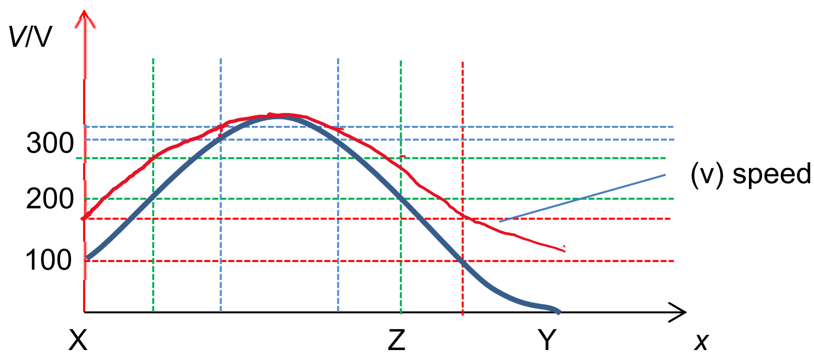
$= U_Y - U_x$

$= -e(V_Y - V_X)$

$= -1.60 \times 10^{-19} (-100) = 1.60 \times 10^{-17}$ J

(iii) Electric field strength at Z $\approx \left|\frac{\Delta V}{\Delta x}\right| = \frac{300-100}{0.013}$ = 15400 V m^{-1} (in the direction from XY).

(iv) Note that the variation in kinetic energy will exactly follow the variation in potential (see part ii). The initial speed has to be non-zero and positive, while the final speed has to be smaller than the initial speed, yet still larger than zero (if the electron moves beyond Y).



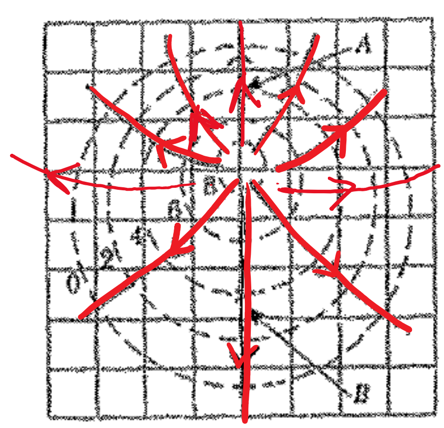
(v) When the electron moves from 100 V to 300 V, it accelerates, thus speed increases. As it moves from 300 V to 100 V, it decelerates and speed decreases.

Note that by conservation of energy, since ΔKE = 0 when $q\Delta V = 0$, the kinetic energy of the electron is the same at the same potential, thus **the speed of the electron should be the same at the same potential.** [*Just in case, the speed graph is on a different vertical axis from the potential graph.*]

D12

(i) Magnitude of E at **A** is larger than at **B**.

Applying $E = -\frac{\Delta V}{\Delta x}$, the potential gradient at **A** is greater than that at **B** as the 2-V and 6-V equipotential lines are spaced closer around **A** than around **B**.



(ii) $E \approx -\frac{\Delta V}{\Delta x} = -\frac{(2-6)\text{V}}{2\text{ cm}}$ = 2 V cm^{-1};

Field strength is 2 V cm^{-1} vertically downward.

(iii) Note the rules for drawing field lines. Hence, they should be perpendicular to the equipotential lines and be extended to the edge of the drawing.

D13

ai) work done by the electrical force from A to B = $(5.00 \times 10^{-7})(20.0 \times 10^{-3}) = 1.00 \times 10^{-8}$ J

ii) work done by the electrical force from A to C = 0 J (since $F \perp d$)

iii) work done by the electrical force from A to D = 1.00×10^{-8} J (since $V_D = V_B$)

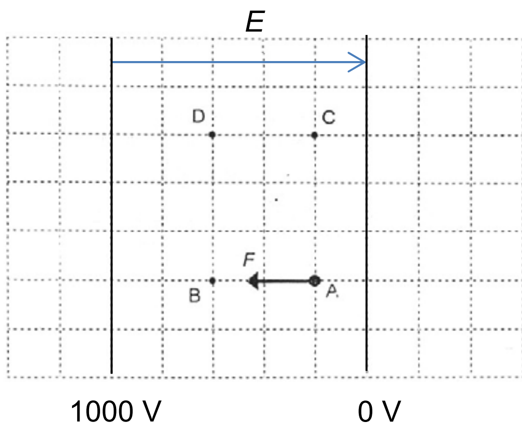
b) ΔW_{AB} by external agent = $q\Delta V_{AB} = q(V_B - V_A) = -1.00 \times 10^{-8}$ J

$V_B = \Delta W_{AB}/q + V_A = (-1.00 \times 10^{-8})/(-2.50 \times 10^{-11}) + 200 = 600$ V

$V_C = 200$ V

$V_D = 600$ V

(c) & (d)



D14

ai) The electric field $E = \frac{\Delta V}{\Delta d} = \frac{0-(-4.2)}{30 \times 10^{-3}} =$ 140 V m^{-1} upwards.

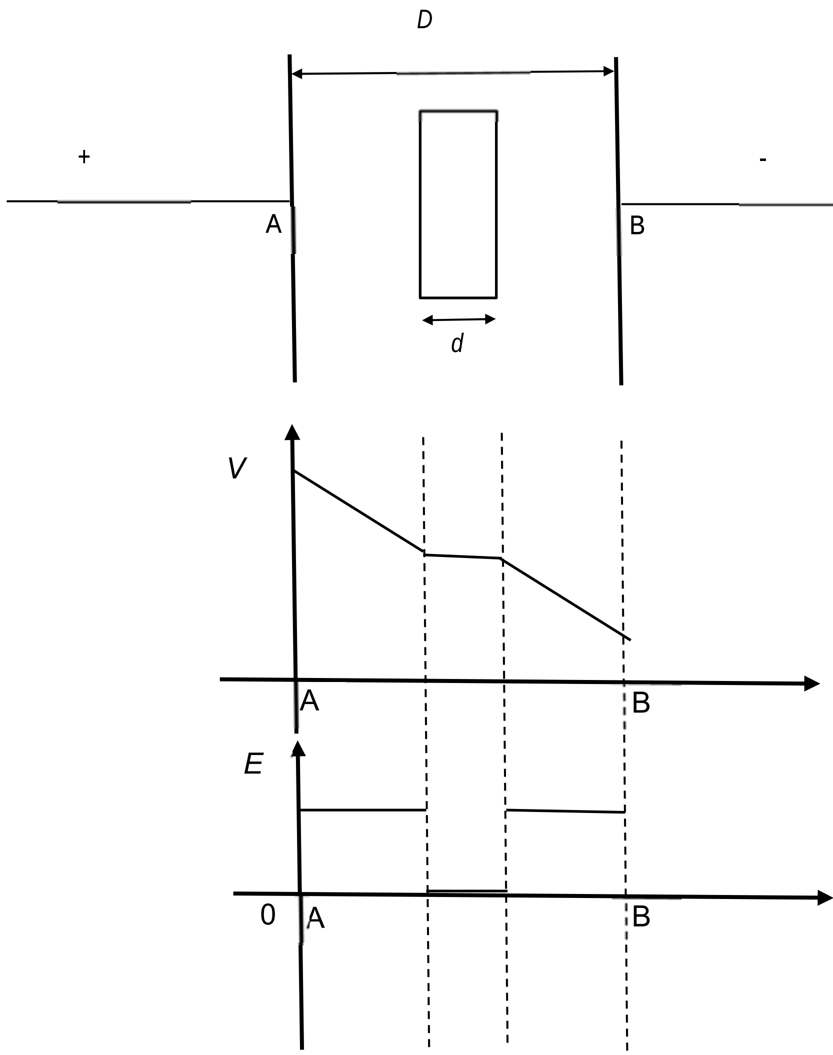
ii) By the principle of conservation of energy, gain in EPE = loss in KE

$-e\,[-4.2 - 0] = \frac{1}{2} m_e v^2 - 0$

$v = 1.2 \times 10^{6}$ m s^{-1}

b) As the potential difference is still the same, based on the above working using the principle of conservation of energy, the initial speed required to reach plate A remains unchanged.

D15 (Cambridge, J87/III/6)



Similar to Qn for GCE A Level, Nov 2020, 9749 (H2 PH) / P2, Qn 3.

Tutorial 13A: Current of Electricity Suggested Solutions

Discussion Questions

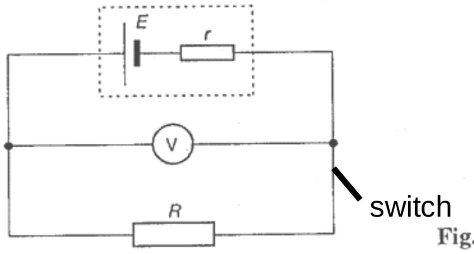
D1	**H1 2013 P2 Q8 (part) :** *On understanding and application of I = nAvq*	
(c)	**(i)**	The direction of electron flow is from the negative to the positive terminal.
	(ii)	The rate of electron flow in all 3 components is the same. Since the components are all connected in series, same current flows through all 3 components. It then follows that rate of electron flow is proportional to current. Rate of electron flow × electronic charge = current ***Learning Points**: This is due to the conservation of charge. For example, if N electrons leave the negative terminal copper wire every sec, then N electrons must enter the filament lamp. Otherwise infinite charge accumulation will occur either in the filament or the copper wire.* *The copper wire and the filament are connected in series, thus the same current through them.*
	(iii)	Since all the electrons flowing in the copper wire are to get through the fine filament of the lamp at the same electron flow, they must move faster in the filament. ***Learning Points:** This is because the current is the same along the wire, and the drift velocity v of the electrons must be inversely proportional to the cross-sectional area of the conductor perpendicular to the current, assuming the charge carrier densities n (i.e. no. of charge carrier per unit volume) of the metallic conductors are approximately the same (since I = nAvq).*
(d)	Since the mobile electrons are moving with a higher average speed in the filament of the lamp, they will collide at higher speeds with the lattice ions of the filament and impart more kinetic energy to the ions. As a result, lattice ions oscillate with greater amplitude and the temperature increases. ***Learning Points:** Link to concept covered in Thermal Physics.* • *the mean kinetic energy of a molecule of an ideal gas is proportional to the thermodynamic temperature.* • *internal energy is determined by the state of the system and that it can be expressed as the sum of a random distribution of kinetic and potential energies associated with the molecules of a system.* • *relate a rise in temperature of a body to an increase in its internal energy.*	

D2	**H2 Phy/2018/P2/Q5** *On understanding and application of I = nAvq*	
	(a)	No. of moles in 1 m^3 of copper $= \dfrac{8960 \text{ kg}}{0.0635 \text{ kg mol}^{-1}} = 141000 \text{ mol}$ Number density of charge carriers (i.e. mobile electrons) in copper = $141000 \times 6.02 \times 10^{23} = 8.49 \times 10^{28} \text{ m}^{-3}$ (Shown)

	(b)	Using $P = I^2R$ Current through tungsten wire, $I = \sqrt{\frac{P}{R}} = \sqrt{\frac{5.0}{30}} = 0.4082$ A Applying $I = Anve$ Average drift velocity of the conduction electrons in tungsten, $v = \frac{I}{Ane}$ $= \frac{0.4082}{\frac{\pi}{4}(0.36\times10^{-3})^2(3.4\times10^{28})(1.60\times10^{-19})}$ $= 7.37\times10^{-4}\ \text{m s}^{-1}$
	(c)	With the length of the wire doubled and diameter kept constant, the resistance of the wire ($R = \rho L/A$) will also double. Since the potential difference is kept constant ($V = IR$), the current will be halved as the resistance has doubled. The *number density of the wire* will remain the same since it is of the same material and both wires are at the same temperature. The cross-sectional area is also the same since the diameter is kept constant. The charge carriers will still be electrons. Hence with the current halved, the drift velocity ($v = I / nqA$) will also be halved. ***Note:*** *Question tells you quantitatively how length and diameter has changed, so you need to provide more detail as well – not just saying 'resistance increases' etc.*

D3	**C** Resistance is defined as the ratio of the ***potential difference across the component*** to the ***current through the component***. Hence $R_Y - R_X = \frac{V_Y}{I_Y} - \frac{V_X}{I_X}$ **A common misconception is to equate resistance to the gradient of the V-I graph (what does the V and I of the V-I graph represent?).* *This is only true for ohmic resistors where the ratio of V to I is a constant and equivalent to the gradient. It is a special case!*

D4	**J89/I/14** *On power dissipation in a resistor, application of $P = IV = I^2R = V^2/R$*	
	C	The current through the internal resistance, r is the same as the external load, R. Using $P = I^2R$ (Since I is constant, P is proportional to R) $\frac{P_{lamp}}{P_{total}} = \frac{R_{lamp}}{R_{total}} = \frac{R}{R+r}$

D5	**J96/III/4 b)**	
	distinguish between electromotive force (e.m.f.) and potential difference (p.d.) using energy considerations. *understanding of the effects of the internal resistance of a source of e.m.f. on the terminal potential difference.*	
(a)	E.m.f. is the work done per unit charge to convert other forms of energy to electrical energy but p.d. is the work done per unit charge in converting electrical energy to other forms of energy.	
(b)	(i)	We can measure the e.m.f. when there is no current drawn from the cell and the terminal p.d. when current is drawn from the cell. Hence the switch must be placed where the current can be switched on and off, while the voltmeter is connected across the cell all the time. The switch can be placed anywhere in the lower branch.
		 Fig. 10
	(ii)	1. open. (so that no current flows out of the battery, and there will be no potential drop across the internal resistance *r* of the battery. Then e.m.f. = voltmeter reading)
		2. closed. Voltmeter reads the terminal p.d.
	(iii)	The voltmeter is measuring the terminal p.d. across the cell which is the same as the p.d. across the load *R*. Consider the single-loop circuit without the voltmeter: Then $E = IR + Ir$ (based on Conservation of energy), but $V = IR$, hence $E = V + Ir$ ** See Pg 21 of Lecture Notes*

D6	**N2017/I/21**	
	recall and solve problems using the equation $R = \frac{\rho l}{A}$.	
	A	$\frac{\Delta R}{\Delta l} = \frac{\rho}{A}$ *Same materials means same resistivity.* *The gradient* $\frac{\Delta R}{\Delta l}$ *of each section of the wire of same diameter is inversely proportional to its cross-section area A.* *Gradient steepens when the diameter decreases.*

D7	**N2000/III/4** *recall and solve problems using the equation* $R = \frac{\rho l}{A}$.

Estimated area of skin in contact with the wire = surface area of cylindrical wire

$= \pi D \times H = \pi(0.4)(9) = 11.3\ \text{cm}^2$ (slight estimate, but OK since skin is only 1mm thin)

current $= \frac{V}{R} = \frac{50 \times 11.3}{300 \times 10^3} = 1.88$ mA = 2 mA. $R = \frac{\rho(1\,\text{mm})}{11.3\,\text{cm}^2}$

Hint: *How will the current flow from wire through hand to the ground? To understand what A to use for* $R = \frac{\rho l}{A}$.

[As this is an estimate question, the answer should be given to 1 s.f. only.]

D8	**J02/I/32** *On power dissipation in a component, application of* $P = IV = I^2R = V^2/R$
	A The ratings on the lamp are for the condition when the lamp is switched on, i.e., at its operating temperature. Hence $R_{room} = \frac{R_{operating}}{16} = \frac{V^2}{16P} = \frac{240^2}{16 \times 100} = 36\ \Omega$

D8 *On power dissipated by a component not operating at rated voltage.*

For the kettle, $P = \frac{V^2}{R}$

Since R is constant, $P \propto V^2$

Hence, $\frac{P'}{P} = \left(\frac{V'}{V}\right)^2$

$P' = \left(\frac{220}{240}\right)^2 \times 2.0 = 1.68$ kW

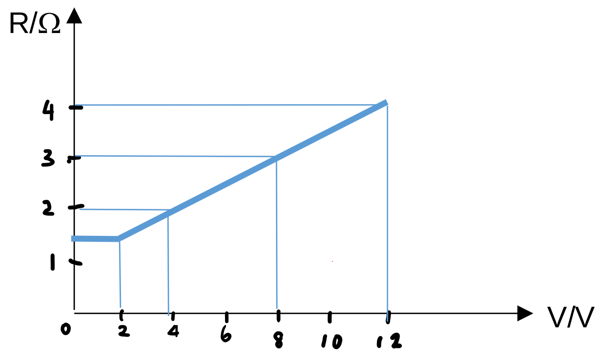
Learning Point:

There are 3 equations for power dissipated by a component. Which one is most suitable here?

D10	**N92/I/14** *On power supplied and power dissipation in a circuit, application of* $P = IV = I^2R = V^2/R$
	E Question is similar to S7, with factory as the output load. $P_{factory} = P_{generator} - P_{cableloss} = P - \left(\frac{P}{V}\right)^2(R)$ ***Learning Point:*** *In a closed loop circuit, there should be conservation of energy.* *In practice, it is usually not possible to measure the p.d. across transmission cables. Thus, the power dissipated in the transmission cable is computed using* $P = I^2R$.

D11	**N01/III/4c** *Determining resistance from IV characteristics graph.*	
	(i) 1. $R = \frac{V}{I} = \frac{12}{3.0} = 4.0\ \Omega$ 2. $P = IV = 12 \times 3.0 = 36$ W	
	(ii) $R = \frac{V}{I} = \frac{6}{2.4} = 2.5\ \Omega$	
	(iii)	

V/V	R/ Ω
2	2/1.4 = 1.4
4	4/2.05 = 2.0
6	2.5
8	8/2.7 = 3.0
12	4.0



* *When a graph is given, you should read off and plot. Resistance is constant between V = 0 and V = 2 V; after that, the data suggest a straight line.*

Learning Point:

The resistance, R of the component at a particular p.d., V_1 applied across it can be determined by either:

1. *reading the coordinates (V_1 , I_1) from the IV characteristics graph and finding R using V_1 divide by I_1.*
2. *Taking the reciprocal of the gradient of the straight line joining (V_1 , I_1) to the origin.*

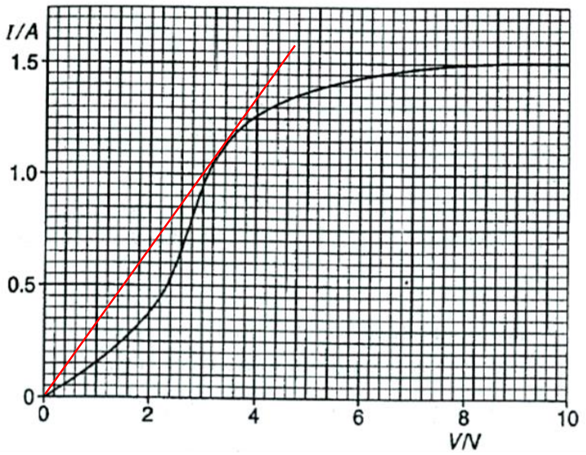
Common mistake – Taking the gradient of V-I graph as resistance – this is incorrect.

D12 **N07/III/3**

Determining resistance from IV characteristics graph.

(a)(i) $R = \frac{V}{I} = \frac{6.0}{1.425} = 4.2\ \Omega$

(a)(ii) Minimum resistance would correspond to the minimum ratio of potential difference to current. Instead of using trial and error, use a ruler and pivot about the origin. Rotate the ruler anti-clockwise to the furthest point where the ruler touches the curve.



Minimum resistance
= 3.4/1.15 = 2.95 Ω

(b) (i) $V_{component} = V_{5\Omega} = 0.85 \times 5.0 = 4.25$ V

(ii) Total current = 0.85 + 1.325 = 2.18 A (read the current for component C from the graph for when V = 4.25 V)

(iii) $E = V_C + I_{tot}r = 4.25 + 2.18(0.80) = 5.99$ V

(iv) Energy = power x time = IVt = 1.325 x 4.25 x 20 x 60 = 6760 J

* *Discrepancy of answers are due to reading off the graph. A range of answers will be accepted.*

Learning Point:

The resistance, R of the component at a particular p.d., V_1 applied across it can be determined by either:

1. *reading the coordinates (V_1 , I_1) from the IV characteristics graph and finding R using V_1 divide by I_1.*
2. *Taking the reciprocal of the gradient of the straight line joining (V_1 , I_1) to the origin.*

Common mistake – Taking the gradient of V-I graph as resistance – this is incorrect.

D13	**N11/III/8** *Show an understanding of the effects of the internal resistance of a source of e.m.f. on the terminal potential difference and output power.*	
(d) (i)	1 Ω, 9 Ω	
(ii)	Efficiency (for R = 1 Ω) $= \frac{P \text{ across R}}{P \text{ supplied by battery}} \times 100\%$ $= \frac{9 \text{ watt}}{IV} \times 100\%$ $= \frac{9}{\left(\frac{12}{1+3}\right)12} \times 100\%$ $= 25\%$	
	Efficiency (for R = 9 Ω) $= \frac{P \text{ across R}}{P \text{ supplied by battery}} \times 100\%$ $= \frac{9 \text{ watt}}{IV} \times 100\%$ $= \frac{9}{\left(\frac{12}{9+3}\right)12} \times 100\%$ $= 75\%$	
(iii)	At max power = 12W, efficiency is $= \frac{P \text{ across R}}{P \text{ supplied by battery}} \times 100\%$ $= \frac{12 \text{ watt}}{IV} \times 100\%$ $= \frac{12}{\left(\frac{12}{3+3}\right)12} \times 100\%$ $= 50\%$ ***Learning Points:*** *What did you infer / learn from the values computed in (ii) and (iii) above?* *From (ii), though both the values of R dissipate the same power of 9.0 W, the efficiency is different for each R value. The higher R value has a higher efficiency.* *From (iii), at max. power (when R = 3 Ω), the efficiency is not max but 50%.*	

(e) The battery should have internal resistance *r* smaller than fixed resistance *R*. From (d), the larger *R* compared to *r* when *R* > *r*, the higher the efficiency of electrical power transferred from the battery to the fixed resistance load *R*.

Learning Points:

Also note that maximum power is dissipated in the external load resistor R when R = r (i.e. external load resistance = internal resistance of cell).

D14

FYI: Extra extension question (not A Level question). Related to the concept in question D13. The mathematics needed for this question not required for A Level H2 Physics.

Consider this circuit:

(a) $P_{output} = IE - I^2 r$

To determine max power w.r.t. current:

$$\frac{dP}{dI} = E - 2Ir = 0 \Rightarrow I = \frac{1}{2}\frac{E}{r}$$

If the circuit is shorted, then the current in the circuit would be *E/r*. Hence maximum power of the cell occurs when the current is half of the current in the shorted circuit.

(b) Power in resistor *R*,

$$P_R = I^2 R = \frac{E^2}{(R+r)^2} R$$

To determine max power w.r.t. resistance:

$$\frac{dP}{dR} = \frac{E^2}{(R+r)^2}(1) + R\left(\frac{-2E^2}{(R+r)^3}\right) = 0$$

$$\Rightarrow \frac{E^2}{(R+r)^2}\left[1 - \frac{2R}{R+r}\right] = 0$$

$$\Rightarrow \frac{E^2}{(R+r)^2}\left(\frac{r-R}{R+r}\right) = 0$$

$$\Rightarrow R = r$$

Actually, with answers in part (c), you can straightaway deduce that P is max when R = r.

Also, maximum power when *R*= *r* gives $P_{max} = \frac{E^2}{(r+r)^2} r = \frac{E^2}{4r}$

Learning Points:

Maximum power is dissipated in the external load resistor R when R = r (i.e. external load resistance = internal resistance of cell).

D15	**N22/III/2** *recall the First Law of Thermodynamics and apply it to an electrical system*
(a)	$R = \frac{\rho l}{A}$, thus $\rho = \left(\frac{R}{l}\right)A = \left(\frac{R}{l}\right)\pi\left(\frac{d}{2}\right)^2$ $= (1.73)\pi\left(\frac{1.02\times10^{-3}}{2}\right)^2 = 1.41\times10^{-6}\ \Omega\ \text{m}$
	Examiner's comments: *Common mistakes: errors in the calculation of area; not include an appropriate power of ten on the answer line; rounded answer to two significant figures.*
(b)	(see table below) Thought Process: Since positive work is done by the electric field to accelerate the electrons in both cases, w is positive. Since the temperature of the wire is always greater than the surroundings, there is always heat loss from the wire to the surroundings (i.e. $q < 0$). Finally, since there is no change in the temperature of the system, there is no change in internal energy (i.e. $\Delta U = 0$).
	Examiner's comments: *Application of the first law of thermodynamics to an electrical system is challenging.* *Most student realised that as temperature increases the internal energy increases and when the temperature is constant there is no change in internal energy.* *However, many did not realise that the positive work is done on the system (electrons in the circuit) by the d.c. power supply, and that thermal energy is released by the metal wire to the surroundings as its temperature rises and when it is at constant temperature (there is always a temperature difference ΔT between the wire and the surroundings).*
(c)	(see table below)
	Examiner's comments: *Common mistake: Mixing up of +/- signs for q and w.*

Table for (b):

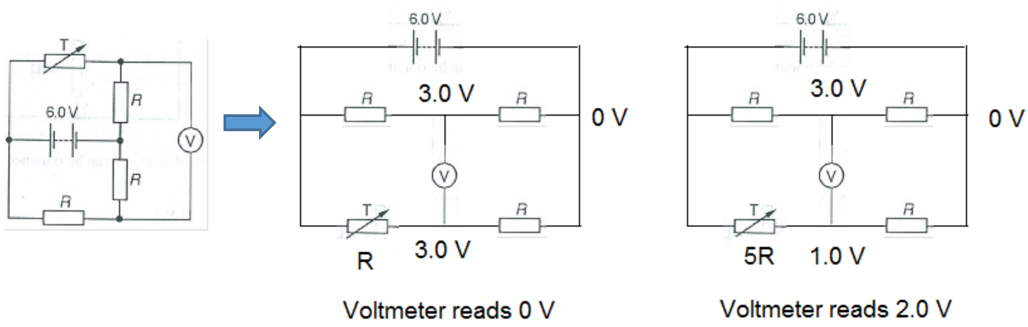
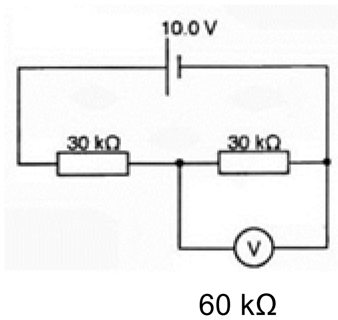
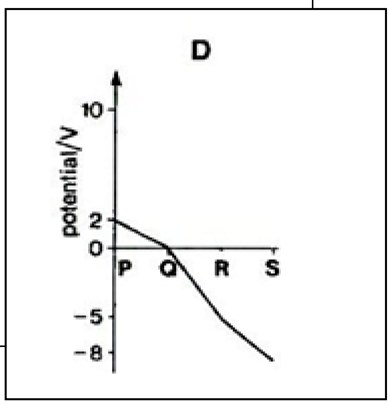
time after being switched on/ s	ΔU	q	w
0–59	positive	negative	positive
60–100	zero	negative	positive

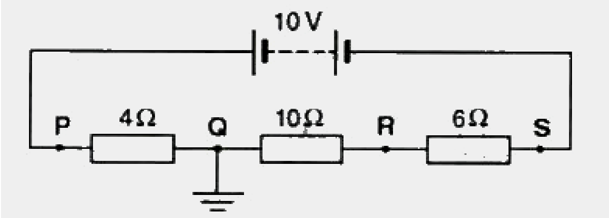
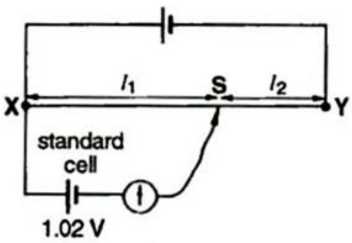
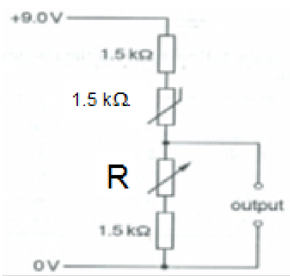
Table for (c):

ΔU / J	q / J	w / J
0	-1.10×10^5	$IVt = (12)(230)(40)$ $= 1.10 \times 10^5$

Tutorial 13B DC Circuits: Suggested Solutions

Discussion Questions

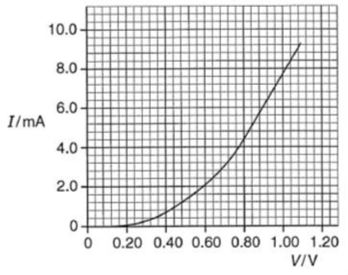
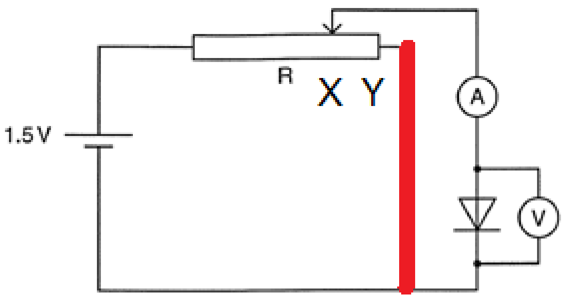
D1	**B**	*On applying Potential Divider Principle and recognising resistors in series / parallel.* *Redraw circuit to allow you to see more clearly the configuration of resistors.*  When resistance of T set to R, Voltmeter reads 0 V. When resistance of T set to 5R, Voltmeter reads 2 V The change in the reading of the high resistance voltmeter is thus 2 V
D2	**A**	*On applying Potential Divider Principle and determining equivalent resistance of resistors in parallel.* By using potential divider method, $V = \frac{(\frac{1}{60}+\frac{1}{30})^{-1}}{(\frac{1}{60}+\frac{1}{30})^{-1}+30} x10.0 = 4.0V$  ***Note:*** *You might want to read up Pg 31 of Lecture Notes, Appendix A, on two different arrangements of voltmeter and ammeter to find an accurate resistance of an unknown resistor.*
D3		**J95/I/15** *Distinguishing between potential at a point and potential difference across a resistor.* *On applying Potential Divider Principle.* Potential at Q = 0 V (as point Q is earthed or grounded) By applying potential divider principle, Potential Difference between P and Q is 2 V, between Q and R is 5 V and between R and S is 3 V. Since current flows in the anti-clockwise direction, potential at R and S must be less than that at Q and be negative. Potential of P is +2 V, Potential of R is -5 V and Potential of S is -8 V. 

		
D4	**C**	**N11/I/26:** *On Potential Divider circuit and Power output* When S is closed, the effective resistance across the parallel network containing R_3 is less than the resistance of R_3. Thus p.d. across R_1 increases and p.d. across R_3 decreases. Current from the battery increases as the effective resistance *R* of the circuit decreases. Since battery power output *P* is given by I^2 R, P increases.
D5	**E**	**N88/I/17:** *On Potentiometer circuit*  Since given that when sliding contact is at S, there is no current in galvanometer, P.d. across wire XS is proportional to the length XS. (basically, you should realise that the potentiometer circuit will measure the e.m.f. across the standard cell) $$\frac{l_1}{l_1+l_2}V_{xy} = 1.02$$ $$V_{xy} = 1.02\frac{l_1+l_2}{l_1}$$
D6	**B**	**N06/I/19:** *On applying Potential Divider Principle to Potential Divider Circuits.* Circuits A, C and D will provide a potential difference between 3.0 and 6.0 V.
D7	**C**	**N08/I/28:** *On applying Potential Divider Principle to Potential Divider Circuits. Recognise symbols for thermistor and variable resistor.* Resistance of thermistor R_T at 0730 is 1.5 kΩ. By potential divider principle, $$6.0 = \frac{R+1.5}{R+1.5+1.5+1.5} \times 9.0 + 0$$ *Resistance of variable resistor,* R_V = 4.5 kΩ  V = 6.0 V

D8	**B**	**J81/II/23:** *On Potentiometer circuit* Let *L* be the length of the resistance wire and *E* be the potential difference across the entire resistance wire When X is connected to Y, potential drop across length *l* of the wire is $V_l = \frac{400}{L}E = V_{100}$................(1) [Note: $V_{100} = \frac{100}{100+R}E_1$, where E_1 is the e.m.f. of cell in series with 100 Ω and *R*.] When X is connected to Z, potential drop across length *l* of the wire is $V_l = \frac{588}{L}E = V_{100+R}$..............(2) [Note: $V_{100+R} = E_1$] Divide (1) by (2), $\frac{400}{588} = \frac{V_{100}}{V_{100+R}} = \frac{100}{100+R}$ (Since the potential drop across the two resistors follow the same ratio as that of their resistance) $R = 47$

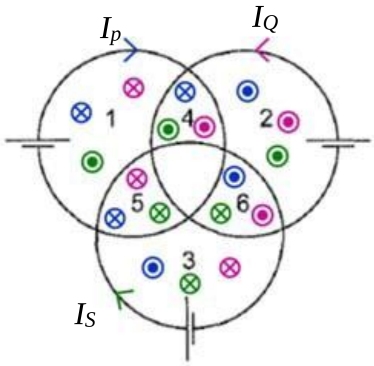
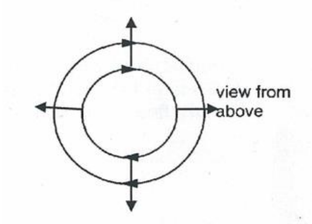
D9	**N02/II/6:** *On applying* $R = \frac{\rho L}{A}$ *and use of potentiometer.*
(a)	(i) $R = \frac{\rho l}{A} = \frac{1.1x10^{-6}(1.20)}{\pi(0.55x10^{-3})^2}$ =1.389 = 1.4 $\wedge$ (ii) potential difference (p.d.) per unit length of XY = $\frac{\frac{1.4}{1.4+0.70}\times 3.0}{1.2}$ $= 1.67\ V\ m^{-1}$
(b)	(i) If contact J is **not connected** to end Y, from answer in (a) (ii), V_{xy} = (1.67)(1.20) = 2.0 V and it is greater than 1.5 V (e.m.f of cell C). Hence, when contact J is connected to end Y, the direction of current through C is (i.e. from positive to negative terminals through cell C)

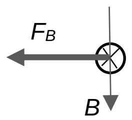
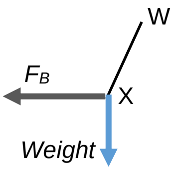
	(ii) this part of qn is about contact J on wire XY such that there is no current through the Cell C. $$1.5 = \frac{l_{xy}}{1.20} x 2.0$$ l_{xy} = 0.90 m Hence, the **contact J should be placed 0.90 m to the right of X**. (iii) Add a resistor in series with XY in the driver Circuit. Use a thicker resistance wire of the same material (i.e. resistance of wire XY decreases). *Note that both ways above will cause p.d. across wire XY to decrease.* *Why do you want the balance length to be longer?*	

D10 **(a)**	**N12/II/3 :** *IV characteristics of Diode*  **Fig. 3.1** The variation with potential difference V of current I for a semiconductor diode is shown in Fig. 3.1. (i) 0 – 0.18 V there is no current flowing. 0.18 V is the operating voltage as starting from this potential difference across the diode, current starts to flow. As voltage increases, current also increases. Resistance of the diode (ratio of V to I) decreases as the voltage increases. (ii) Resistance of diode at 0.80 V, R = 0.80 / (4.4×10^{-3}) = 181.8 = 180 Ω *How do you determine the resistance of diode from the IV characteristics graph?*
(b)	From (a)(ii), the resistance of the diode is 180 Ω at 0.8 V. This would imply a current in the circuit of I = 0.8 / 180 = 0.0044 A. To obtain this, the variable resistor would have to be set to R = (1.5 – 0.8) / 0.0044 = 160 Ω. This value is beyond the range of the variable resistor.
(c)	(i)  (ii) The resistor R is working as a potential divider. The potential difference across diode will be the same as the potential difference across section XY of the resistor R. The shorter the length of XY, the smaller the potential difference across diode. With this setup, the whole range of potential difference over the diode, from 0 to 1.5 V, can be obtained. (i.e. resistor XY will be parallel to the diode)

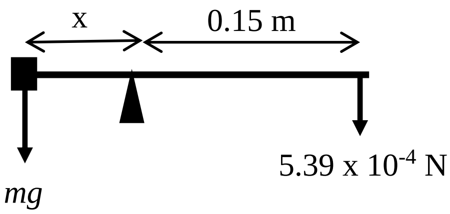
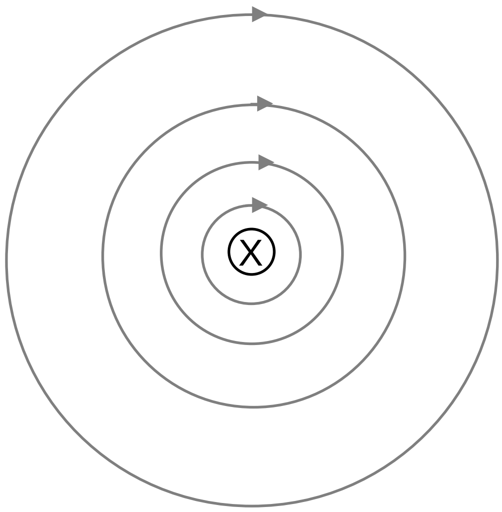
Suggested Solutions for Discussion Questions

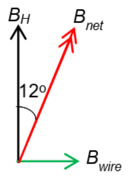
Tutorial 14A: Magnetic Field and Magnetic Force

D1	**B**	Concept: Applying RHGR for magnetic field produced by current-carrying coil and vector sum them Apply the "right-hand grip rule" for each coil to determine direction of magnetic field created by the current in each coil in areas 1 – 6. The magnetic field due to each current I_P, I_Q and I_S (colour coded) are as shown. Coil P(blue), coil Q(pink) and coil S(green). Areas 2 and 5 show the magnetic fields due to the three coils are in the same direction and hence reinforce each other. 
D2	**D**	Concept: Applying magnetic force equation When θ = 0°, maximum magnetic force. When θ = 90°, zero magnetic force $F_B = BIL\sin\theta$, where θ is the angle between B and I. Since θ is not the angle between B and I, the magnetic force should be $F_B = BIL\cos\theta$, where θ is the angle between B and I. Hence $F_B \propto \cos\theta$
D3	**D**	Concept: Applying 'LIKE currents attract and UNLIKE currents repel' Apply "LIKE currents attract". Since all the forces acting on the smaller coil are of equal magnitude, acts radially outwards and pass through one point (at the centre of the small coil), the net force on small coil is zero. 

D4	**C**	Concept: Applying RHGR for magnetic field produced by current-carrying coil twice and vector sum the magnetic flux density Suppose that the current in X and Y are both clockwise. Then their magnetic fluxes are in the same direction at Q and O, but opposite at P. Suppose that the current in X and Y are one clockwise and the other anticlockwise. Then their magnetic fluxes are in the same direction at P, but opposite at Q and O. Since coil Y decreases the flux density at O, we can conclude that it is the second scenario and then determine the corresponding directions of the magnetic field due to each coil and vector sum them.
D5	**E**	Concept: Applying FLHR and 3-D visualisation The forces acting on the vertical sides of the coil = *nBIL* (since the current in the vertical wires are always perpendicular to the horizontal magnetic field).
D6	**D**	Concept: Applying FLHR, understanding current balance and moments about a pivot $F_B = BIL = (3.6 \times 10^{-2})(0.093)I = 3.348 \times 10^{-3}\ I$ Moment about pivot = $F_B d$ $7.4 \times 10^{-3} = (3.348 \times 10^{-3}\ I)(0.23)$ $I = 9.6$ A
D7	**A**	Concept: Applying FLHR and equilibrium of forces  The current flows from X to Y and the magnetic field where XY is, is directed downwards. Using FLHR, we can deduce that the magnetic force on XY is towards the left.  The magnetic force results in a clockwise rotation about the pivots WZ. It initially swings left wards but once it does, weight which acts vertically downwards will now have an anti-clockwise moment about the axis WZ. The wire will settle into a new equilibrium position where there is no net moment by the magnetic force and the weight.
D8		Concept: Applying magnetic force equation, FLHR and N3L
	(a)	The magnetic force due to the 2.0 A is (144.6 – 142.0) g of force = 2.6 g Since the magnetic force *F* = *BIL* Thus for the same *B* and *L* for the magnetic force experienced would be $\frac{3.0}{2.0} \times 2.6 = 3.9$ g of force Since the current is reversed in direction, reading will decrease to (142.0 – 3.9) = 138.1 g
	(b)	With a current flowing in XY direction, the reading on the balance increased. This means that the wire is exerting a downward magnetic force on the horseshoe

		magnet. By Newton's 3rd Law, the horseshoe magnet must be exerting an upward magnetic force on the wire. Applying Fleming's left-hand rule, with the current in the direction XY and magnetic force upward, we are informed that the magnetic field in the horseshoe magnet is towards the back of the balance. (North pole is closer to the front of the top pan balance.)
D9	**(a)**	Concept: Applying RHGR, FLHR and N3L By right hand grip rule, the current in wire Y produces a magnetic field at wire X acting into the paper. Using Fleming's left-hand rule, wire X experiences a rightwards magnetic force F directed towards wire Y. By Newton's 3rd law of motion, an equal and opposite force is exerted on wire Y by wire X. Hence both wires experience an attractive force. *Comments: Answers like 'each current produces a magnetic field and the field will be distorted hence attracting each other', 'like currents attract', 'due to electrostatic attraction' are not acceptable.*
	(b)	Concept: Applying magnetic force equation and magnetic field equation due to a wire (given in formula list) Force on the wire X, $F_x = B_Y I_X L_X$ Force per unit length on wire X, $\frac{F_X}{L_X} = B_Y I_X = \frac{\mu_o I_Y}{2\pi d} I_X = \frac{(4\pi\times10^{-7})(8.4)(5.5)}{2\pi(0.12)}$ $= 7.7\times10^{-5}\ \text{N m}^{-1}$ Direction : horizontally to the right (towards Y) *Recall the result of "LIKE currents attract, UNLIKE currents repel"*
D10	**(a)**	**(i)** Concept: Applying FLHR and understanding current balance BC: magnetic force on BC must be acting in the vertical direction (perpendicular to the magnetic field). (It is probably downward instead of upward so that when the current balance is in operation, counterweights could be placed and adjusted conveniently along DQ or AP as described in (c)(iii)).
		(ii) Concept: Applying RHGR, FLHR PB: No magnetic force because current along PB is parallel to the magnetic field within the solenoid.
	(b)	Concept: Applying concept of series connections in circuit and RHGR To reverse direction of the force, only change **ONE** of the following: 1. Current direction in frame PBCQ – by exchanging the connections P and Q 2. Magnetic field in the solenoid – by exchanging the connections to the two ends of the solenoid so that the current flow in the solenoid is in the opposite direction.

		Note: Frame PBCQ and solenoid are connected in series. *If current direction is reversed, both currents in frame PBCA and solenoid will be reversed and there is no change in the direction of the magnetic force on BC.* *Answers like 'current reversed' or 'battery connections interchanged' will not work.*	
	(c)	**(i)**	Concept: Applying magnetic field due to a solenoid equation (given in formula list) $B = \mu_o n I \; = (4\pi \times 10^{-7})(700)(3.5) \; = 3.08 \times 10^{-3}\ T$
		(ii)	Concept: Applying magnetic force equation Force on BC $= B I L \; = (3.08 \times 10^{-3})(3.5)(0.05) \; = 5.39 \times 10^{-4}$ N
		(iii)	Concept: Applying principle of moments By principle of moments: At equilibrium,  $m g x = (5.39 \times 10^{-4})(0.15)$ $(0.10 \times 10^{-3})(9.81) x = (5.39 \times 10^{-4})(0.15)$ $x = 8.24$ cm
D11	**(a)**	Concept: Knowledge of how to draw magnetic field patterns due to a current-carrying conductor (please bring compass for exams) Concentric circles with increasing spacing between them as distance increases. Direction of field lines are clockwise.  ***Examiner Comments:*** Students should use a compass to draw the field pattern. A minimum of 4 field lines is needed to illustrate the increasing separation. Many students forgot the direction of the field lines.	

	(b)	Concept: Applying magnetic field due to a current-carrying conductor or wire (given in formula list) $B=\dfrac{4\pi\times10^{-7}(8.5)}{2\pi(0.19)}=8.9\times10^{-6}$ T	
	(c)	**(i)**	Concept: Applying vector sum of magnetic flux density  $\tan 12^o=\dfrac{B_{wire}}{B_H}$ $\Rightarrow B_H=\dfrac{8.9\times10^{-6}}{\tan 12^o}=4.2\times10^{-5}$ T
		(ii)	Concept: Applying vector sum of magnetic flux density To obtain zero magnetic flux density, the direction of the magnetic flux density due to current must be opposite to that of the Earth. Using the right hand grip rule, this will be on the right of the current. To have the same magnitude as the Earth's magnetic flux density, the position should be nearer than 19 cm. ***Examiner Comments:*** Students should not use "cancel" or "balance" in their explanation.

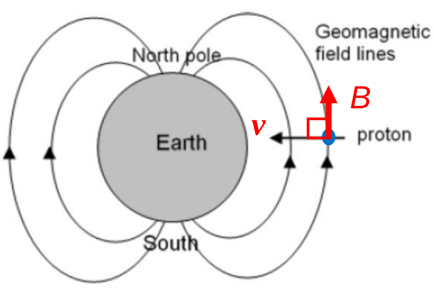
Suggested Solutions for Discussion Qns

Tutorial 14B: Moving Charges in Magnetic Field

D1	**C**	Concept: Applying FLHR and derivation for radius of circular path The magnetic force experienced by the particle must be in the centripetal direction. Using the Fleming's left hand rule (thumb in the centripetal direction, index finger out of the paper) at the starting point, we figure that the "current" is in the same direction as the particle's trajectory (because the middle finger points that way). This tells us that the particle must be positively charged. The particle is spiralling inward, meaning the radius of circular motion becomes smaller and smaller. We know that this happens when the speed of the charged particle become slower and slower assuming mass, magnetic flux density and charge remains the same. Similarly, the radius of the circular path can be derived below. Magnetic force provides the centripetal force. $Bqv = \frac{mv^2}{r}$ $r = \frac{mv}{Bq}$

D2	**(a)**	**(i)**	Concept: A charged particle moving perpendicularly to a magnetic field, experiences a centripetal force and moves in a circular motion. The electrons will experience a force perpendicular to the magnetic field, resulting in uniform circular motion. The paths are circular arcs.
		(ii)	Concept: A charged particle moving perpendicular to an electric field, experiences an electric force and moves in a parabolic path. The electrons follow a parabolic path.
	(b)		Concept: resolving a vector. Component of velocity perpendicular to the magnetic field, $v_{\perp}$ is given by, $v_{\perp} = v\sin\theta$ $= (7.0\times10^{6})(0.6)$ $= 4.2\times10^{6}\ \text{m s}^{-1}$ (shown)
	(c)	**(i)**	Application: using the concept magnetic force provides the centripetal force to deduce radius of helical path Magnetic force provides the centripetal force. $qvB = \frac{mv^2}{r}$ $r = \frac{mv_{\perp}}{Bq} = \frac{(9.11\times10^{-31})(4.2\times10^{6})}{(3.14\times10^{-5})(1/6\times10^{-19})}$ $= 0.762\ m$
		(ii)	Concept: applying magnetic force provides centripetal force ($mr\omega^2$) and $\omega = 2\pi/T$

			Magnetic force provides the centripetal force. $Bqv = mr\omega^2$ $T = \frac{2\pi m}{Bq} = \frac{2\pi\left(9.11\times10^{-31}\right)}{\left(3.14\times10^{-5}\right)\left(1.6\times10^{-19}\right)} = 1.14\ \mu\text{s}$
		(iii)	Concept: resolving a vector. The velocity component of the electron parallel to the field, $v_{//} = v\cos\theta$ $= \left(7.0\times10^{6}\right)(0.8)$ $= 5.6\times10^{6}\ \text{m s}^{-1}$
		(iv)	Concept: distance = speed x time. The pitch, $p = (v_{//})t$ $= \left(5.6\times10^{6}\right)\left(1.14\times10^{-6}\right)$ $= 6.38\ \text{m}$

D3	**D**	Concept: Applying FLHR and derivation for radius of circular path $r > d$ in order for the charge to reach the collector. $Bqv = \frac{mv^2}{r}$ $r = \frac{mv}{Bq}$ $v > \frac{dBq}{m}$
D4	**(a)**	Concept: Applying FLHR Using Fleming's Left Hand Rule, the magnetic force on the proton acts into the paper. 
	(b)	Concept: Application of the concept that charged particles move in a circular path when it is travelling perpendicular to the direction of magnetic field

		The protons experience a magnetic force due to the Earth's magnetic field which prevents them from reaching the Earth's surface. Instead, the protons travel in a circular path around the magnetic field lines of the Earth.
	(c)	Concept: Application of the concept that charged particles do not experience magnetic force when it is travelling parallel to the direction of magnetic field At the North and South magnetic poles of the earth, where the protons travel parallel to the magnetic field lines will not experience a magnetic force, and will hit the surface of the Earth without deflection. *Additional: Protons travelling at an angle with the Earth's magnetic field lines will also travel in helical paths along the field lines and hit the surface of the Earth in the regions near the North and the South magnetic poles.*

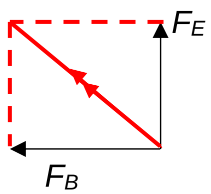
Suggested Solutions for Discussion Questions

Tutorial 14C: Moving Charge in Electric and Magnetic Fields

D1 **A**

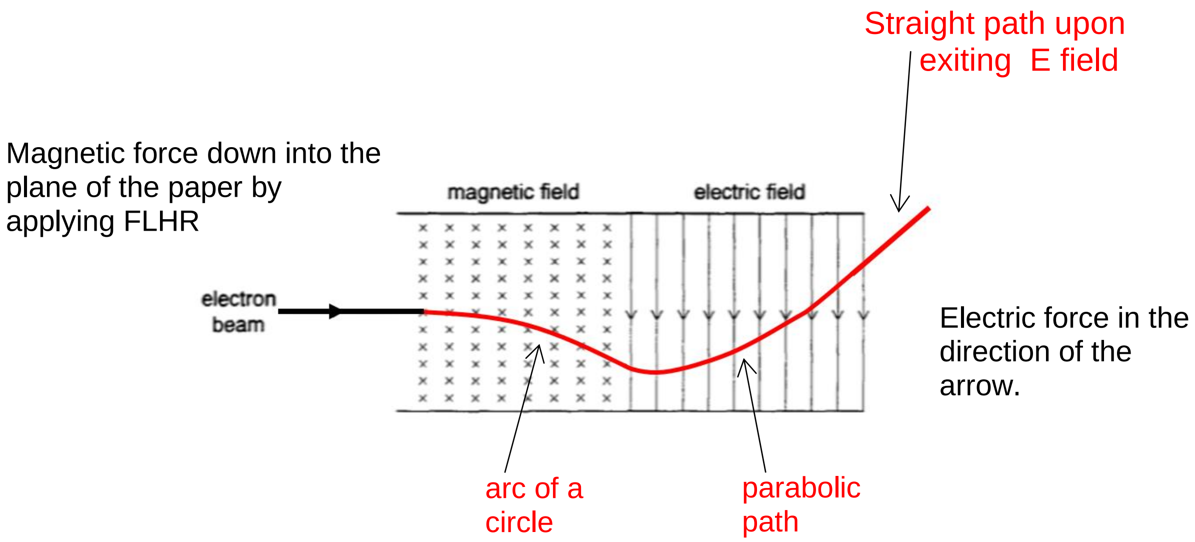
Concept: Applying FLHR, current is the flow of positive charges and electric force direction

The electrons travel out of the paper as you are viewing the front of the screen of the cathode ray oscilloscope, so the current is into the paper. Using Fleming's left-hand rule, the magnetic force on the electrons is to the left. The electric force on the electrons is upward.
Hence, vector sum of the 2 forces, the spot on the screen is upper-left.



D2

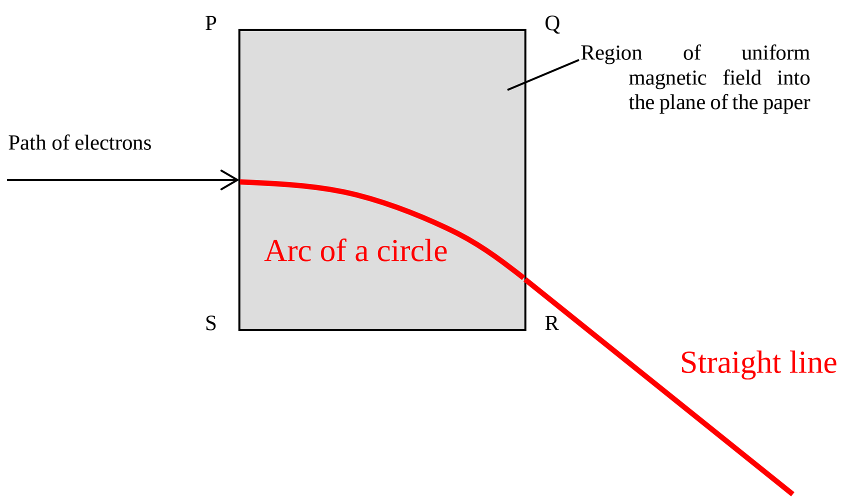
Concept: Applying FLHR and sketches of charged particles' path in electric and magnetic field

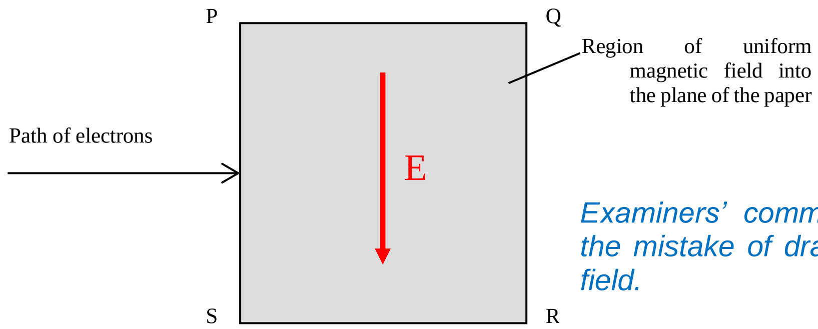


__NOTE:__ Students must know how to distinguish between a circular arc and a parabolic path when sketching and to label them. Most students have problems with drawing the path at the boundary, as the electron moves into the electric field. Despite knowing the direction of the electric force, students are often unable to draw the subsequent motion/deflection.

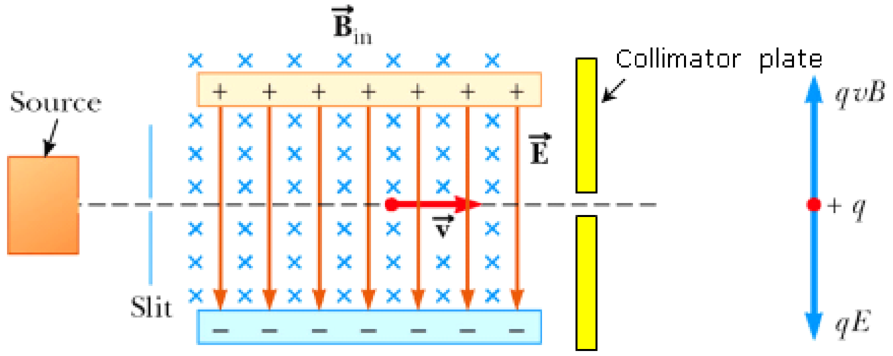
D3 **(a)**

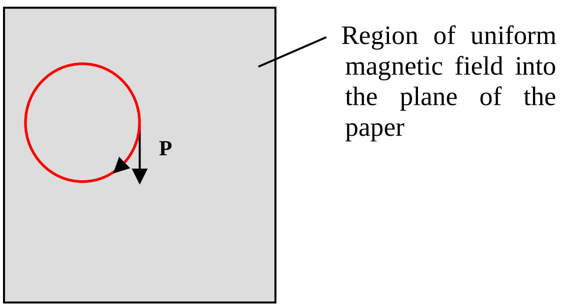
Concept: Applying FLHR and sketch of charged particles' path in magnetic field

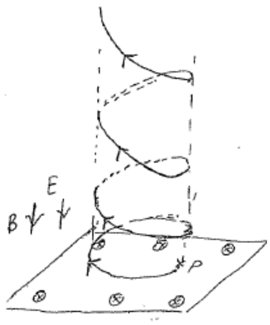


			Comments *– Many provided very rough sketches where the shape in the field could not be distinguished. One should also be careful to ensure that the path after passing through QR is not a curve. Exit path is tangent to the circular arc.*
	(b)		Concept: Applying concept of velocity selector  *Examiners' comments – Some made the mistake of drawing a horizontal E-field.*
	(c)	**(i)**	Concept: Applying magnetic force equation on charged particle The magnetic force is proportional to its velocity. When the velocity doubles, the magnitude of magnetic force is larger but the magnitude of electric force remains a constant. As a result, the net force is not zero. Since the resultant force acts in a downward direction, the electron beam will bend downward. ***Comments*** *– Many did not include reference to the fact that the force due to the electric field would be unchanged. Do not just focus on what changed, but also mention what remains unchanged.*
		(ii)	Concept: Applying magnetic force equation on charged particle and finding the direction of magnetic force and electric force When the charge is +e, the electric force acts downward but the magnetic force acts upward. Since the directions are opposite and the magnitude are the same, the resultant force is zero and the particles will be undeflected. ***Comments*** *– Many realized that the force due to the magnetic field would be reversed, but failed to consider the reversal of the force due to the electric field.*

D4	**(a)**	**(i)**	Concept: Testing definition A region of space where a force is experienced, e.g. a mass in a gravitational field experiences a gravitational force; a charge in an electric field experiences an electric force, a moving charge which has a velocity component perpendicular to a magnetic field experiences a magnetic force. A field force is non-contact in nature.
		(ii)	Concept: Characteristics of magnetic field lines The density of lines reflects the strength of the field. The stronger the field, the closer the lines are together; the weaker the field, the more spaced-out the field lines.
	(b)		Concept: Understanding g, E and B forces
		(i)	mass
		(ii)	***positive*** charge [need to mention positive because the question wants the property of the object that experiences a force in the direction of the field]

			Comments: *Most answers were in terms of charge but relatively few specified positive charge.*
		(iii)	north pole of a magnet
			Comments: *Very few answers were given in terms of the force on a north pole. Most referred to a moving charge, without seeming to realise that the force on a moving charge is not in the direction of the field.*
	(c)		Concept: Applying velocity selector  - Make clear the directions of the *E* and *B* fields. - Stately clearly that the forces due to the two fields are equal in magnitude and opposite in direction. - Derive $v = \frac{E}{B}$.
			Comments: Part of this question was answered poorly. Diagrams were drawn indicating a magnetic field and also parallel plates. Seldom was the direction of the electric field made clear. The directions of both the magnetic field and the electric field should be shown clearly. The direction of motion of the charged particle was frequently assumed from a short arrow drawn sketchily on the diagram. With few exceptions, the relation $v = \frac{E}{B}$ was given. It should be stated clearly that the forces due to the two fields are not only equal in magnitude but also opposite in direction.

D5	**(b)**	**(i)**	Concept: sketch of charged particle in magnetic field  ***Comments:*** *Many students know the path will be circular but are unable to draw the correct circle where the arrow at P is tangent to the circle drawn. Despite being told that the electron does not leave the magnetic field, many still drew the path such that the*

			electron will leave the region. The arrow at P should also be <u>tangential</u> to the initial path of the electron.
		(ii)	<u>Concept: Applying magnetic force equation</u> 1. Force due to magnetic field, *F* $F = qvB$ $= (1.6\times10^{-19})(2.9\times10^{7})(1.5\times10^{-3})$ $= 6.96\times10^{-15}\text{ N}$
			<u>Concept: Applying magnetic force equation to derive radius of circular path</u> 2. The magnetic force provides the centripetal force. $qvB = \frac{mv^2}{r}$ $r = \frac{mv}{qB} = \frac{(9.11\times10^{-31})(2.9\times10^{7})}{(1.6\times10^{-19})(1.5\times10^{-3})} = 0.11\text{ m}$
	(iii)	<u>Concept: understanding helical path of charged particles in B and E fields</u> The electrons will move in a circular helical path upwards with increasing pitch as it experiences an upward vertical acceleration. (Note the difference compared to Tut 14B, D2(c)(iv))  ***Comments**: The difference between a helix (constant radius) and a spiral (changing radius) should be appreciated.*	

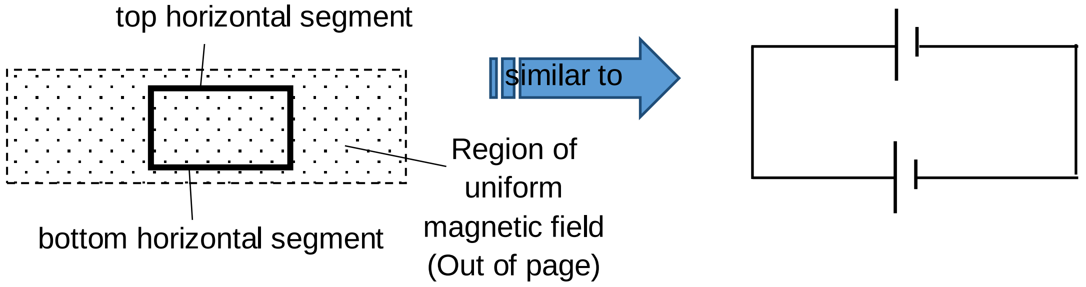
D6	**(a)**	**(i)**	<u>Concept: Applying FLHR</u> Singly ionised atoms are positively charged. Using Fleming's left-hand rule, the magnetic field is pointing out of the page.
		(ii)	<u>Concept: Applying velocity selector concept and electric force direction</u> Since the magnetic force is to the left, the electric force is to the right. Since the ions are positively charged, the E-field must be rightward. Hence, the left plate is at a higher potential (+) than the right plate (-).
	(b)	<u>Concept: Applying velocity selector concept</u> In compartment *C*, for ions to go straight, by Newton's second law, $F_{net} = m\,a$ $\rightarrow$ +ve $F_e - F_B = 0$	

		$qE = qvB$ $v = E/B = (3.06 \times 10^3)/(0.0870) = 3.52 \times 10^4$ m s^{-1} $v_{Ne\text{-}20} = v_{Ne\text{-}22} = 3.52 \times 10^4$ m s^{-1}
	(c)	Concept: Explaining circular path In compartment *D*, each moving ion will experience a magnetic force which is always perpendicular to its velocity or motion and this force provides the centripetal force for its circular motion.
	(d)	Concept: Applying magnetic force equation and derivation of radius of circular motion The magnetic force provides the centripetal force, $F_B = F_c$ $Bqv = mv^2/r$ $r = (mv)/(Bq)$ Since *v*, *B* and *q* are the same for both, a smaller radius implies a smaller mass: Ne-20.
	(e)	Concept: Recognising that twice the radius of circular path equals the separation GH The magnetic force on the ions provides the centripetal force for circular motion, $F_B = Bev = \frac{mv^2}{r}$ $r = \frac{mv}{Be}$ separation = diameter of path II - diameter of path I $= 2(r_2 - r_1)$ $= 2\frac{v}{Be}(m_2 - m_1)$ $= 2[\frac{3.52 \times 10^4}{0.0870(1.60 \times 10^{-19})}](22 - 20)(1.66 \times 10^{-27}) =$ 0.0168 m …………………………………………………………………………………….. *To find mass of an ion:* Eg. Mass of Ne-22 ion ≈ 22 u (where 1 u = 1.66×10^{-27} kg) = $(22 \times 1.66 \times 10^{-27})$ kg = 3.65×10^{-26} kg (Assume: Mass of electrons to be negligible) Alternatively: Mass of 1 mole (= 6.02×10^{23}) of Ne-22 ions = 22 g = 22×10^{-3} kg Hence mass of one Ne-22 ion = $\frac{22 \times 10^{-3}}{6.02 \times 10^{23}}$ = 3.65×10^{-26} kg

Tutorial 15: Electromagnetic Induction
Discussion Questions Suggested Solutions

D1	**Answer: A** From Faraday's Law, the magnitude of e.m.f. induced in the coil is dependent on the rate of change of magnetic flux linkage with the coil. At all times, the magnetic flux linkage with the coil in option B, C and D is zero. The net magnetic flux linkage with the coil in option B is zero because the magnetic field in left side of loop will be of opposite direction to that in right side of loop at any time. Concept: *Induced e.m.f. from rate of change of magnetic flux linkage*
D2	**Answer: C** The axle PQ cuts magnetic flux when axle is at point Y (the motion of PQ is perpendicular to the Earth's magnetic field), there will be induced e.m.f. (i.e. motional e.m.f.). Using FRHR, we can deduce that potential P is higher. At X, the motion of PQ is parallel to the Earth's magnetic field. Thus no e.m.f. is induced. Concept: *Induced e.m.f. from rate of cutting of magnetic flux. Motional e.m.f.*
D3	**Answer: A** By right-hand-grip rule, we can deduce the direction of the magnetic field (due to current *I*) experienced by rod PQ. By FRHR, we can deduce that induced current flows from Q to P. As a source of emf, potential P is higher. Alternatively, by applying FLHR to the charge carriers (i.e. mobile electrons) in conductor PQ, we can determine that there will be a downward magnetic force on the charge carriers. The charge carriers (i.e. mobile electrons) will thus accumulate towards Q. Therefore P will be at higher potential. By the given formula in the question, the strength of magnetic field (due to current *I*) decreases with increasing r. Thus, magnitude of induced e.m.f will drop. Concept: *Induced e.m.f. from rate of cutting of magnetic flux. Motional e.m.f.*
D4	**Answer: B** The loop is moving at a constant speed. Hence, the area of the loop that is in the magnetic field is initially *increasing* at a constant rate (i.e. the magnetic flux linkage *increases* at a constant rate), meaning the induced e.m.f. and current are constant too. When the whole loop is completely inside the magnetic field, the magnetic flux linkage is constant. Hence, for a short period of time, the induced current is zero. As the loop leaves the magnetic field at a constant rate, the magnetic flux linkage *decreases* at a constant rate. Hence, the e.m.f. is now a constant value in the opposite direction.

Another way to view why the induced current is zero when the whole loop is completely inside the magnetic field is to view the horizontal segments (both the bottom and the top) of the loop as rods.



Due to the rate of cutting of magnetic flux lines, there should be an induced e.m.f. in both rods of the same magnitude. (You can also use the fact that there will be an accumulation of charges due to the movement of the rods to arrive at the same conclusion). For the loop, the direction of the induced current due to the induced e.m.f. in the top rod and bottom rod is opposite to one another, effectively making this scenario similar to connecting two identical batteries in series but in opposing directions resulting in zero current flowing throughout the loop. The ‘net’ e.m.f. induced in the loop is hence also zero.

Concept: *Induced e.m.f. from rate of change of magnetic flux linkage.*

D5

Answer: average e.m.f. induced in the coil = zero

There is no change in magnetic flux linkage through the coil as it falls the distance x so the induced e.m.f. is zero at any instance during the fall. Hence the average e.m.f. induced in the coil is zero.

Note as is the case for D4, when the loop is fully in the magnetic field, there is still a separation/accumulation of charges. However, the “net” e.m.f. over the whole loop will be zero. Hence, no current will flow in the loop.

Concept: *Induced e.m.f. from rate of change of magnetic flux linkage.*

D6

Answer: A

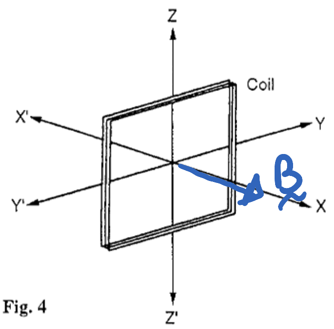
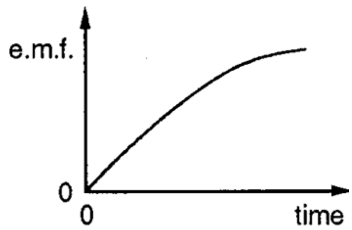


Fig. 4

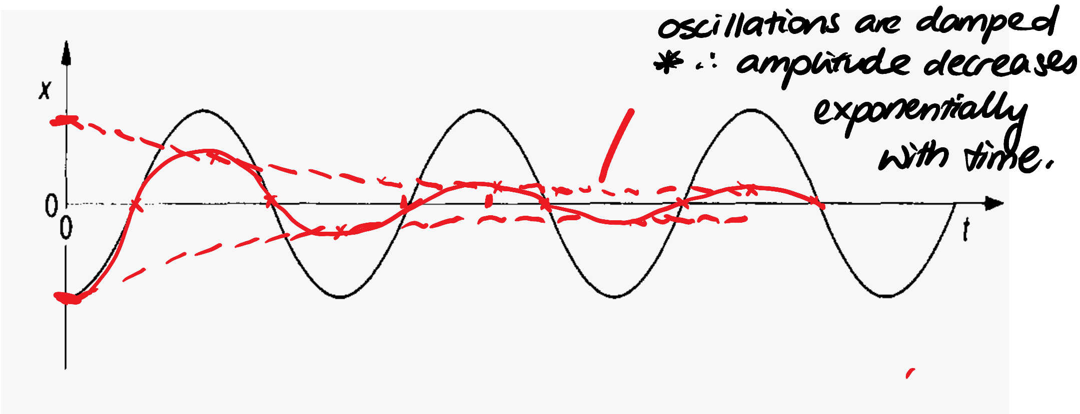


The e.m.f. curve is a part of a sinusoidal graph. Since $\varepsilon = -\frac{d\phi}{dt}$ hence the ϕ must also vary sinusoidally with time.

If the coil rotates about XX', there is no change in flux linkage and hence, no e.m.f. would be induced in the coil. $\Rightarrow$ options B and D are wrong.

	A full rotation of the coil would correspond to a full sine (or cosine) period. Hence, this must be the graph for a quarter period. *See Pg 13-14 of EMI Lecture Notes.* Concept: *Induced e.m.f. from rate of change of magnetic flux linkage. A.C Generator.*
D7	**Answer: B** $\varepsilon = \frac{1}{2} BR^2 \omega = B(\pi R^2) f$ where ω is angular velocity of the disc. e.m.f. = $(3.6 \times 10^{-5})(\pi)(2.3\times10^{-2}/2)^2 (1500/60) = 3.74 \times 10^{-7}$ V Concept: *Induced e.m.f. from rate of cutting of magnetic flux. Motional e.m.f. Faraday's Disc*
D8	**Answer: A** Refer to Appendix One in lecture notes Pg 18. Concept: *Induced e.m.f. from rate of cutting of magnetic flux. Motional e.m.f. Faraday's Disc*
D9	**Answer: A** By Faraday's Law, $\varepsilon = -\frac{d}{dt}(N\phi) = -N\frac{d\phi}{dt}$ When magnet is entering the top opening of solenoid, the peak of induced current or e.m.f. is smaller. The direction of the induced current in the solenoid should produce a South pole at the top of the solenoid. When magnet is leaving the bottom opening of solenoid, the peak of induced current or e.m.f. is bigger. The direction of the induced current should produce a South pole at the bottom of the solenoid. *Why? Hint: consider the speed of magnet when entering the top opening of solenoid and when it is leaving the bottom opening of solenoid.* Concept: *Induced e.m.f. from rate of change of magnetic flux linkage.*
D10	**Answer: E** When the current in the coil is switched on, a magnetic field is set up that passes through the ring (i.e. **increasing** magnetic flux density through the ring). By Lenz's Law, the ring will move away to oppose the change (i.e. ring will move away from the coil to oppose the increasing magnetic flux density). ***Alternatively, a more elaborated explanation:*** When the current in the coil is switched on, the current in coil will generate a magnetic field. The ring will therefore experience an increase in magnetic flux linkage. By Faraday's Law, as there is a rate of change of flux linkage with the aluminium ring, an e.m.f. is induced in the ring. As aluminium is a good electrical conductor and the ring forms a closed electrical path, a current will be present in the ring. By Lenz's Law, the current in the ring should be in such a direction as to generate a magnetic field in opposite direction to that of the magnetic field due to current in the coil (*to oppose the*

	increasing flux linkage), and cause the ring to move away from the coil (to decrease the effect of the increasing B-field it is experiencing) and hence the coil will move towards the East. *Note: aluminium is a good electrical conductor but not a non-ferromagnetic metal.* Concept: *Induced e.m.f. from rate of change of magnetic flux linkage.*	
D11	**Answer: B** This is similar scenario as D10. Copper ring will undergo a damped oscillation. (relates to Topic of Oscillations (SHM)) When the electromagnet is switched on, the copper ring will experience an increasing flux linkage when it approaches the magnet. (The magnetic field around the electromagnet is non-uniform). By Faraday's Law, as there is a rate of change of flux linkage with the copper ring, an e.m.f is induced in the ring. As copper is a good electrical conductor and the ring formed a closed electrical path, a current will be present in the ring. By Lenz's law, the current will flow in such a direction as to induced a magnetic field to oppose the change in magnetic flux linkage (i.e. oppose the increasing flux linkage). The current in the ring will thus be in an anticlockwise direction when viewed from the left of the ring. There is thus a repulsive magnetic force between the right end of the electromagnet and the copper ring which will decelerate the copper ring causing its amplitude to decrease. The copper ring will experience a decreasing flux linkage when it moves away from the coil. By Lenz's Law, the current in the copper ring will be in such a direction as to produce an effect to oppose the change in magnetic flux linkage. Again, the copper ring will decelerate. Thus, the oscillation will become damped, whereby the amplitude of oscillation will decrease continuously until the copper ring stops. *Note: Copper is not a ferromagnetic material and hence it will not be magnetised. If the ring were magnetised, it might actually "stick" to the coil.* Concept: *Induced e.m.f. from rate of change of magnetic flux linkage.*	
D12	(i)	1. When the switch is just closed, an anti-clockwise current will start to flow in loop B. This increasing current in loop B induces an increasing magnetic field. Thus there is a rate of change of flux linkage with coil A and an e.m.f. is thus induced in coil A. As coil A forms a closed electrical path, a current will be present in coil A. By Lenz's law, the current in coil A will be in a ***clockwise direction*** to produce an effect to **oppose the increasing flux linkage** with coil A.
		2. When the switch is closed and the slider is moved from P to Q, the current flowing in loop B increases. This increasing current in loop B induces an increasing magnetic field. Thus there is a rate of change of flux linkage with coil A and an e.m.f. is thus induced in coil A. As coil A forms a closed electrical path, a current will be present in coil A. By Lenz's law, the current in coil A will be in a ***clockwise direction*** to produce an effect to oppose the increasing flux linkage with coil A.
		3. When switch is opened, the current in loop B decreases to zero. This decreasing current in loop B induces a decreasing magnetic field. Thus there is a rate of change of flux

		linkage with coil A and an e.m.f. is thus induced in coil A. As coil A forms a closed electrical path, a current will be present in coil A. By Lenz's law, the current in coil A will be in an **anti-*clockwise direction*** to produce an effect to **oppose the decreasing flux linkage** with coil A.
	(ii)	1. The magnetic flux through loop A will vary when loop A rotates, given by *magnetic flux = (magnetic flux density perpendicular to the plane of area of loop)* time *(area of loop). The magnetic flux can be reduced to* $\Phi = \pm BA\cos(\omega t)$, where B is the effective magnetic flux density through loop A. It is similar to a coil rotating in a uniform magnetic field. Thus, **the magnetic flux w.r.t. time will look similar to one of either positive or negative cosine shape**. 2. By Faraday's law, t*he induced e.m.f.* $= \pm \omega BA\sin(\omega t)$, where B is the effective magnetic flux density through loop A. It is similar to a coil rotating in a uniform magnetic field. Thus, the e.m.f. w.r.t. time will look similar to one of either positive or negative sine shape. As induced current = induced e.m.f. / R, **induced current w.r.t. time will look similar to one of either positive or negative sine shape**. (i.e. an alternating sinusoidal current)
D13	 For lightly damped oscillations it is possible to illustrate the reduction in amplitude over a few cycles, whereas the corresponding increase in the period of the oscillation is too small to indicate. **1.** The ***magnitude*** of the induced emf ε is ***directly proportional*** to the ***<u>rate of change of magnetic flux linkage</u>*** *or* ***<u>rate of cutting of magnetic flux</u>***. **2.** When the switch is closed, a current flows through the electromagnet and the electromagnet will generate a non-uniform magnetic field. The aluminum sheet cuts the magnetic field as it oscillates and thus an e.m.f. is induced in the aluminium sheet. As there are closed electrical paths, eddy currents are set up in the sheet. As the induced eddy currents flow, joule heating takes place. This dissipates the energy of the oscillating sheet such that the oscillations are continuosly dampened and hence reduced in amplitude. Alternatively, as the induced eddy currents flow, by Lenz's law they will flow in such a way as to produce an effect to oppose the change that causes the e.m.f. to be induced in the sheet. Thus,	

there will always be a force opposing the motion of the sheet. This braking force acts to dampen the oscillations.

Concept: *Induced e.m.f. from rate of cutting of magnetic flux. Motional e.m.f*

To demonstrate critical damping:

Connect a variable resistor in series with the electromagnet to vary the current and hence the magnetic flux generated by the electromagnet. Start with the largest possible resistance.

A larger magnetic flux will cause larger induced currents when the aluminimum sheet is set into the initial same amplitude of oscillation (displaced from equilibrium by a same displacement – measure with metre rule). This causes greater joule heating and faster dissipation of the energy of the oscillatory system and hence damping can be increased. The current in the electromagnet can be increased by decreasing the variable resistor till the system to returns to equilibrium without oscillating. The time taken for it to return to equilibrium can be measured with a stopwatch.

Critical damping is demonstrated when the aluminum sheet returns to its equilibrium position, after being displaced, in the shortest time without oscillating.

D14

(a) (i) The magnetic flux density generated by the solenoid is proportional to the current *I* through the solenoid:

$$B = \mu_o n I_s \propto I$$

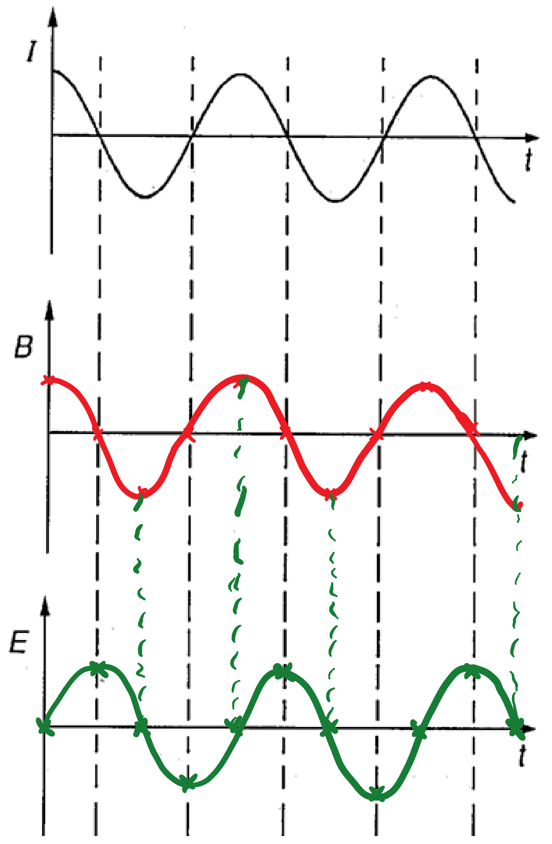
(a)(ii) As the search coil experiences a varying magnetic flux linkage, by Faraday's Law, an e.m.f. will be induced in the coil and it is given by:

$$\varepsilon = -\frac{d}{dt}(NB'A) = -NA\frac{dB'}{dt} \propto -\frac{dB'}{dt}$$

B' is the magnetic flux linkage experienced by the search coil which is a fraction of *B*.

B vs t is a positive cosine graph, therefore ε will follow a positive sine shape.

(b)(i) When a ferrous core is slowly introduced into the solenoid, the core will be magnetized by the external magnetic field of the solenoid. The amplitude of the magnetic field experienced by the search coil increases, but the period with which it varies is still the same. Hence, the amplitude of the induced e.m.f. increases and there is no change in the frequency with which the e.m.f. varies.



(b)(ii) When the frequency is increased, the peak-to-peak change of the magnetic flux linkage experienced by the coil happens in a shorter time interval. In other words, the rate of change of flux linkage increases and thus the amplitude of the induced e.m.f. will increase. The frequency with which the induced e.m.f. varies, is the same as the frequency with which the magnetic field varies and hence the frequency with which the e.m.f. varies, will also increase.

Mathematically, we can also see that from:

	$\Rightarrow \varepsilon = -\frac{d}{dt}(NBA) = -NA\frac{dB}{dt} \propto -\frac{dB}{dt} \propto -\frac{d}{dt}(I_o \cos \omega t) = I_o \omega \sin \omega t$ Frequency increases $\Rightarrow$ ω increases $\Rightarrow$ Amplitude of e.m.f. increases.
D15	**(a)** Increase in current causes an increase in magnetic field by the coil. Soft iron core concentrates the magnetic field resulting in a (large) increase in the magnetic flux linkage through the ring. According to Faraday's law, this change in flux linkage induces an e.m.f. in the ring. Since the ring is a conductor a current flows in it. According to Lenz's law, the current flows in such a direction as to produce a magnetic field to oppose the increase in flux linkage through the ring. The magnetic pole at the bottom of the ring is of the same polarity as that at the top of the coil. A repulsive magnetic force is produced. *Note: aluminium is a good electrical conductor but not a ferromagnetic metal.* Concept: *Induced e.m.f. from rate of change of magnetic flux linkage.* *Current-carrying conductor in an external B-field experiences a force.* **(b)** The ring does not move. Even though there is induced e.m.f. in it, a current does not flow in it due to it being an insulator, and hence there is no repelling pole produced in the ring. *Note: The ring experiences a changing magnetic flux so there should be an induced e.m.f. This is independent of whether a current will actually flow or not.*

<u>Tutorial 16 – Alternating Current</u>

D1	Answer: B What's the same inside each coil is the magnetic flux (prduct of cross-sectional area of core and the magnetict flux density) which is produced by the constant direct current. The shape of the ring is non-uniform. This affects the spacing of magnetic field lines. The smaller the area is, the closer magnetic field lines will be. Hence, the magnetic flux density will be stronger in the smaller coil area, resulting in a constant magnetic flux.
D2	a) $\langle I \rangle = \dfrac{4\,\text{A x } 0.25\text{ ms}}{1\,\text{ms}} = 1\text{ A}$ b) $\sqrt{\langle I^2 \rangle} = \sqrt{\dfrac{4^2\,\text{A}^2 \text{ x } 0.25\text{ ms}}{1\,\text{ms}}} = 2\text{ A}$ c) 2 A DC and 2 A r.m.s. AC produce the same average heating power.
D3	Let's call the "same peak value emf" V_{pk}. The **sinusoidal voltage** has an r.m.s. voltage of $V_{pk}/\sqrt{2}$. Mean power in case (a) would be $P_a = V_{r.m.s.}^2/R = [\,V_{pk}/\sqrt{2}\,]^2/R = V_{pk}^2/2R$ The **square voltage** alternates between V_{pk} and $-V_{pk}$. The power dissipated in case (b) is thus constant at $P_b = V_{pk}^2/R$ Hence $P_a{:}P_b$ = 1:2
D4	Answer: B Initially the power P is delivered by the dc current I_{dc}. So $P = I_{dc}^2 R$ The same power P is delivered by the r.m.s. current $I_{r.m.s.}$ when the resistance is halved. $P = I_{r.m.s.}^{\;2}\,(R/2)$ Equating the two powers gives us $I_{r.m.s.} = \sqrt{2}\, I_{DC}$

D5

Answer: A

The question is about a full-rectifier. In this case, the power is dissipated in the resistor for both directions of alternating current.

$$V_{r.m.s.} = \sqrt{\frac{\frac{(V_0)^2}{2}(\frac{T}{2}) + \frac{(V_0)^2}{2}(\frac{T}{2})}{T}} = \frac{V_0}{\sqrt{2}}$$

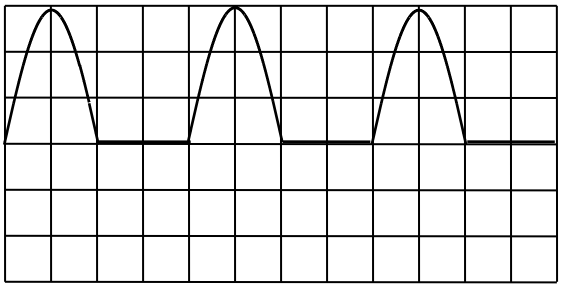
$$I_{r.m.s.} = \frac{V_{r.m.s.}}{R} = \frac{224}{\sqrt{2}}\frac{1}{70.0} = 2.26 \text{ A}$$

$$\langle P \rangle = I^2_{r.m.s.}R = (2.26274)^2 70.0 = 358.4 \text{ W}$$

D6

Answer: A

The peak current is 1 A. The mean value for half a cycle when current is greater than zero is more than 0.5 A. Thus, the mean value for a complete cycle will be less than 0.5 A.



Note: The r.m.s. value is 0.5 A. The mean current is 0.32 A, calculated from $\frac{\int_0^{\frac{T}{2}} \sin(\omega t)}{T}$.

D7

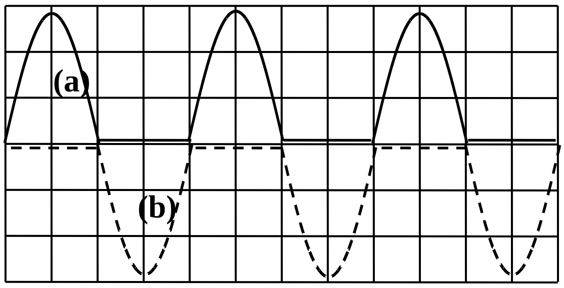
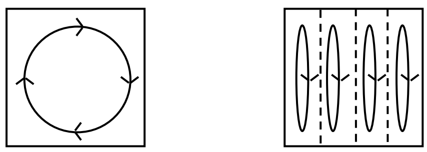
Answer: B

From the figure, the r.m.s value of current in a sinusoidal wave is $I_{r.m.s} = \frac{2}{\sqrt{2}}$ A

When the rotational speed of the a.c. generator is halved, using $\varepsilon = \omega NBA$, the supply peak voltage will also be halved. (The angular speed is proportional to the rotational speed of the a.c. generator.)

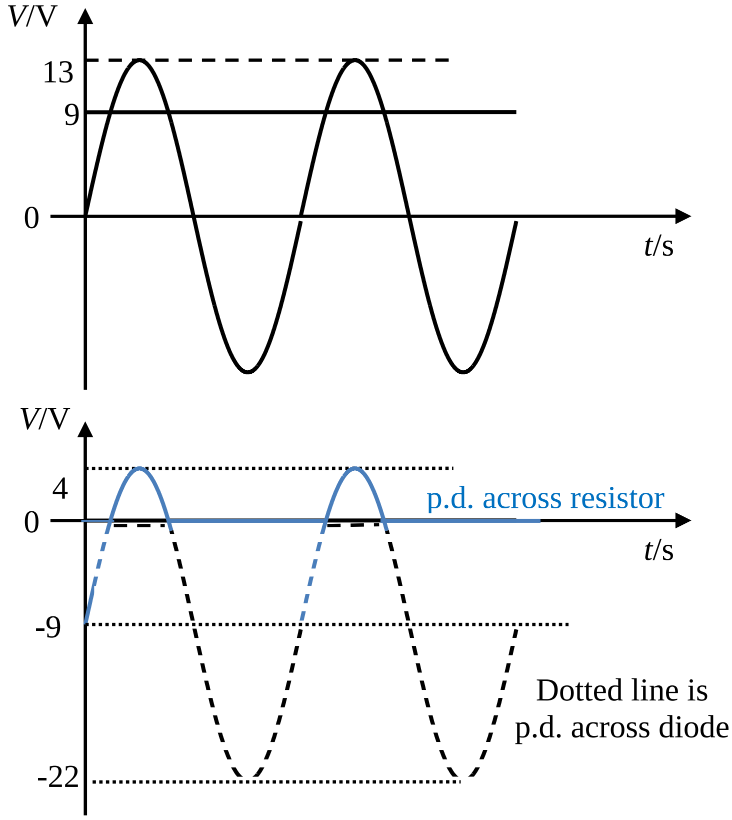
For the same resistor, the peak alternating current and the r.m.s. current would be halved as well. Therefore, the mean power in the resistor would be

$$\langle P \rangle = \left(I_{r.m.s_{new}}\right)^2 R = \left(\frac{1}{\sqrt{2}}\right)^2 20 = 10 \text{ W}$$

D8	$V_{peak} = \sqrt{2} \times 6.0 = 8.5$ V = 2.8 vertical division on screen $T = 1/50 = 20$ ms = 4 horizontal division on screen  (on screen: dimensions of each box is supposed to be 1cm by 1cm)
D9	(a) (i) The alternating current in the primary coil creates a changing magnetic field in the primary coil. Thus, there will be <u>a changing magnetic flux in the iron core</u> that passes through the secondary coil. So there will be changing magnetic flux linkage in the secondary coil. By Faraday's law, an e.m.f. is induced in the secondary coil and is proportional to the rate of change of magnetic flux linkage of the coil. (ii) An ideal transformer is 100% efficient: the input power to the primary coil is equal to the output power from the secondary coil. (iii) The soft iron has low hysteresis loss since it would be able to magnetise and demagnetise quickly, thus not opposing the changing of magnetic flux. Due to the higher permeability of the soft iron, the magnetic flux density achievable in the core would be much higher and the magnetic flux linkage of the primary and secondary coils will be much higher. (b) (i) There is an alternating magnetic flux in the core. By Faraday's law, an e.m.f. is induced within the iron core, which is proportional to the rate of change of magnetic flux linkage of the iron core. Since iron is a good electrical conductor, and the core provides a closed circuit for induced current, thus, this induced current flows in the iron core with a resistance, heating up the iron core. (ii)  no lamination laminated When core is laminated, the core is divided into smaller sections, electrically insulated from each other. The flux linkage in each section is reduced. This leads to a smaller induced e.m.f. ($\varepsilon = -d\Phi/dt$). Also, the narrowing of the path the current takes further reduces the magnitude of current in each section as resistance is increased ($R = \rho l/A$). Lastly, the sum of the power loss in each laminated section will be smaller than that in a solid core.

D10	Answer: D Turns ratio is doubled → secondary voltage is halved → power in *R* is one-quarter of original. This means that power drawn from the supply is one-quarter of original. With the same primary voltage the primary current will be one-quarter of the original.
D11	(a) $I_{r.m.s.} = \langle P\rangle / V_{r.m.s.}$ = 4400/11 = 400 A (b) i. For transformer T, N_s/N_p = 275/11 = 25 For transformer U, N_s/N_p = 1/25 = 0.04 ii. $I_{r.m.s.} = P_{output}/V_{r.m.s.}$ = 4400k / 275k = 16 A (c) Power lost in the transmission cable is I^2R. So current should be minimised to reduce power loss in cable. In order to still transmit the same power, transmitting voltage must be increased, since $P = VI$. *Note: For efficient transmission of electrical energy, voltages must be stepped up before transmission. For safety, voltages must be stepped down upon arrival at the destination. The fact that AC voltages can be stepped up and down efficiently and cheaply using transformers was the main reason AC was chosen over DC.* This was another question that was done well although in *(a)* many candidates introduced √2 unnecessarily and in *(b)* a significant number of candidates had the ratios the wrong way round. Candidates should be told not to leave numbers in the form 275/11.
D12	(a) i. N_s/N_p = 40/1000 = 1/25. V_s = 230/25 = 9.2 V ii. $\sqrt{2} \times 9.2$ = 13.0 V (b) i. Without the diode, the battery will be discharging during times when V_s is less than the battery's e.m.f. (9 V). The diode is there so that current flows into the battery to charge up the battery when V_s > 9 V, but current cannot flow out of the battery when V_s < 9 V. ii. An ideal diode presents zero resistance when forward biased. Without the resistor during the charging process, the current may be too large since the two voltage sources are basically shorted together with tiny total resistance in the circuit. The resistor takes up the difference of potential difference of two voltage sources, preventing excessive large current in the circuit.

(c)



Note: Good to also sketch the potential difference across the diode.

2024 Tutorial 17A : Photoelectric Effect

Discussion Questions Suggested Solution

D1	**D** Time taken for light to travel 1 m = $\dfrac{\text{distance travelled}}{\text{speed}} = \dfrac{1}{c}$ In this time, energy emitted by laser = power × time = $P \times \dfrac{1}{c}$ ∴ energy contained in a one metre long beam = $\dfrac{P}{c}$ ∴ no. of photons within this 1 m laser beam = $\dfrac{\text{energy in beam}}{\text{energy per photon}} = \dfrac{P/c}{hf} = \dfrac{P}{chf}$. Alternative trick if you are really lost, if you see that the options are quantities of different base units. You can also try to use unit analysis to try and reduce the options to make an educated choice. Sometimes you will be lucky and end up with only one option. – Try it in this case, to revise your Measurements ☺
D2	**(a)(i)** If the intensity of light does not change, the no. of photons incident per unit time on the emitter metal is fixed. The probability of each photon emitting an electron is also fixed. Once saturation current is reached, it means that all the electrons that are emitted are collected at the collector, further increase of potential difference *V* does not increase the no. of photoelectrons emitted hence the current will not increase.
	(a)(ii) Photoelectrons are emitted with a range of kinetic energies. When $V < 0$ V, a photoelectron can still reach the collector if its kinetic energy sufficiently large when leaving the emitter. By conservation of energy, electrons with kinetic energies which exceeds eV will be those will reach the collector.
	(a)(iii) Photoelectrons are still emitted but do not have sufficient kinetic energy at the point of emission to reach the collector plate. Instead, they are attracted back to the emitter plate before reaching the collector plate by the electrostatic force they experience due to the electric field setup between the collector and emitter plate.
	(b)(i) Intensity = (Power)/(Area) $\Rightarrow$ $I = \dfrac{P}{A}$ But the power is supplied by the photons each of energy $E = hf = hc/\lambda$, Hence, $I = \dfrac{P}{A} = \dfrac{N_{photons} \times hf}{t \times A}$ This gives the rate at which photons are incident, $\dfrac{N_{photons}}{t} = \dfrac{IA\lambda}{hc} = \dfrac{(8.2\times10^{-3})(2.0\times10^{-4})(365\times10^{-9})}{(6.63\times10^{-34})(3.00\times10^{8})} = 3.01\times10^{12}\ \text{s}^{-1}$

D2 cont

(b)(ii) At saturation current, all the photoelectrons emitted will reach the collector. Hence, we can use that to calculate the required value.

$$I_{saturation} = \frac{N_{photoelectrons}}{t} e \quad \Rightarrow \frac{N_{photoelectrons}}{t} = \frac{I_{saturation}}{e} = \frac{5 \times 10^{-9}\text{ A}}{(1.6 \times 10^{-19}\text{ C})} = 3.125 \times 10^{10}\ \text{s}^{-1}$$

(c) The **no. of photoelectrons liberated per unit time is less than the no. of photons incident on the emitter.** This means that **not all photons will liberate an electron**.

Electrons in the metal are bound to metal by different degrees. Some photons are absorbed by electrons which loses the energy gained through collisions with the lattice atoms before it reaches the surface of the atom and hence do not have sufficient energy to liberate from the metal surface and will not contribute to the current.

Some photons are also not absorbed at all and are reflected from the surface. These photons did not interact with any electrons in the metal.

(Additional note: The ratio of the **no. of photoelectrons emitted per unit time** to **the no. of photons incident per unit time** is know as **quantum yield.** *It represents the probability of a photon liberating an electron from the metal.*)

(d) From the graph, V_s = 1 V.

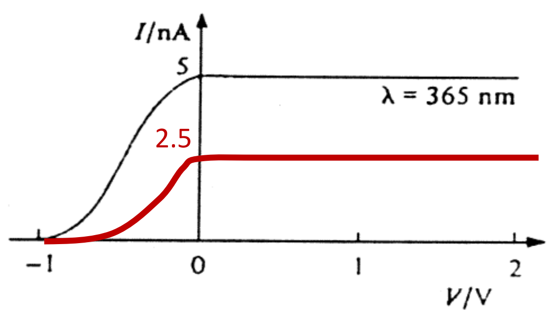
$\Rightarrow$ Maximum kinetic energy $KE_{max} = eV_s = (e)(1\text{ V}) = 1\text{ eV} = 1.60 \times 10^{-19}$ J

(e) From Einstein's photoelectric effect equation, $KE_{\max} = hf - \phi = h\frac{c}{\lambda} - \phi$

$$\phi = h\frac{c}{\lambda} - KE_{\max} = (6.63 \times 10^{-34})\frac{(3.00 \times 10^{8})}{(365 \times 10^{-9})} - (1.60 \times 10^{-19}) = 3.85 \times 10^{-19}\ \text{J} = 2.41\text{ eV}$$

(f) (i) Changing the intensity does not affect the maximum kinetic energy of the photoelectrons and hence the **stopping potential should stay the same.**

However, with half the intensity (and frequency of light source the same) half the photons are incident per unit time. Therefore, the no. of photoelectrons emitted per unit time is also halved. Hence, **the saturation current is halved.**



(f) (ii) With a decrease in wavelength (i.e. an increase in frequency), the energy of each photon is now higher. Hence, if we have the same intensity (and assuming some area of light beam incident on metal) of light on the metal. There will be less number of photons per unit time on the metal.

Assuming further that the <u>probability of a photon liberating an electron</u> (i.e. the quantum yield*) <u>is unchanged</u>, then with less photons incident per unit time on the metal is less, then there will correspondingly be less photoelectrons emitted per unit time. And the <u>saturation current decreases.</u>

(*Many students will assume that the quantum yield will increase if the frequency (hence energy of photon) increases. However, the quantum yield actually depends on many other factors e.g. angle of polarisation etc and can actually decrease. Hence, for H2 Physics, we will assume that the quantum yield stays and same and explicitly state it as an assumption in attempting questions such as this.)

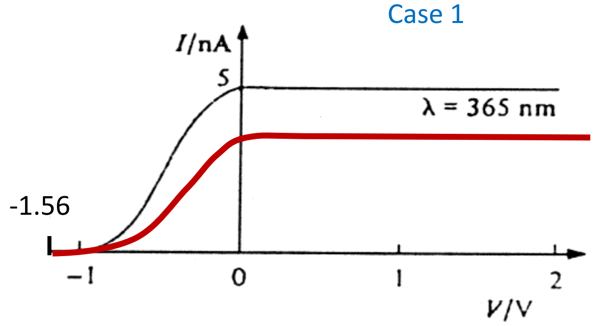
Effect on Stopping potential: With an increase of photon energy, the maximum kinetic energy of the photoelectrons will increase, hence the stopping potential should increase. Since there is a value given for the new wavelength, we can use it to determine the new stopping potential :

$E_{photon} = hc/\lambda = (6.63 \times 10^{-34})(3.00 \times 10^{8}) / (313 \times 10^{-9}) = 6.35 \times 10^{-19}$ J = 3.97 eV

Max K.E of the photoelectrons released, eV_s = 3.97 – 2.41 = 1.56 eV

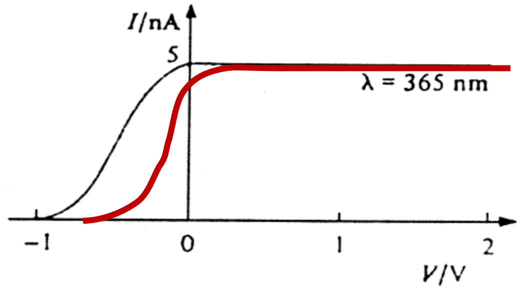
The new stopping potential is 1.56 eV. The graph will meet the V axis at -1.56 V.

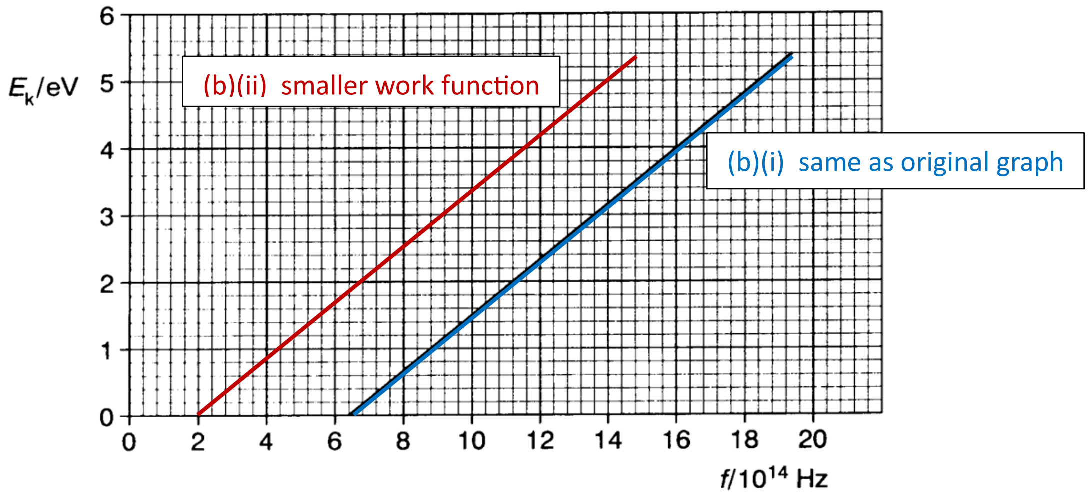
Case 1



(f)(iii) For this question, we are going to assume that the probability of each photon liberating an electron from the metal surface to be constant (i.e. the quantum yield is constant). Hence, in this case since intensity of light did not change and the frequency also did not change, the number of electrons liberated per unit time from the metal is unchanged and the saturation current is the same as before.

From Einstein's photoelectric effect equation : $KE_{max} = eV_s = hf - \phi$, we can also see that with a greater work function, the maximum kinetic energy of the photoelectrons will also reduce as more work is required to liberate the least bounded electrons and hence the stopping potential will reduce too.

	
D3	***Interpreting the graphs:*** *The horizontal axis indicates that the polarity of F is used as a reference. Hence, when V is positive, plate E is positive relative to plate F. When V is negative, plate E is negative relative to plate F; plate F is positive relative to E.* *For λ_1, there is a current when V is positive (i.e., when plate E is positive relative to F). This means* ***electrons were emitted from plate F (due to photoelectric effect)*** *and accelerated to plate E (collector).* *For λ_2, there is a current even when V is negative, which correspond to when F is positive relative to E. This shows* ***plate E is now also emitting electrons (due to photoelectric effect).*** (a) λ_2 is less than λ_1 as photons with wavelength of λ_2 have sufficient energy to overcome the work functions of *both* metals to cause electron emission. (b) ϕ_E is greater than ϕ_F as photons with wavelength of λ_1 could cause electron emission only for metal F. Alternatively, with reference to the second graph, when $V = 0$, the net current is flowing from E to F. This means that there is a net electron flow from F to E. Hence, more electrons coming from F have enough energy to make it across than electrons coming from E, that is, the electrons released from F must have a larger KE_{max}. Thus, $\phi_E > \phi_F$, since the photons falling on E have the same energy as those falling on F.
D4	**(a)(i)** From the graph, threshold frequency = 6.4×10^{14} Hz
	(a)(ii) $E_K = hf - \phi = hf - hf_o = (6.63\times10^{-34})(18.0\times10^{14} - 6.4\times10^{14}) = 7.69\times10^{-19}$ J
	(a)(iii) From Einstein's photoelectric effect equation : $E_K = hf - \phi$ We can see that the gradient of the graph of E_K vs. $f = h$. Using (18.4, 5) and (6.4,0) and working in SI units, we have $$h = \frac{(5-0)\times(1.6\times10^{-19})}{(18.4-6.4)\times10^{14}} = 6.67\times10^{-34} \text{ Js}$$
	(b)(i) When the light intensity is doubled, only the no. of photons incident on the metal per unit time is doubled. Einstein's photoelectric effect equation describes the maximum kinetic energy for the interaction between a photon and the electron which is liberated with the maximum kinetic energy. The energy of each photon (*hf*) is still the same, the work function energy ϕ of the metal is unchanged and hence, the maximum kinetic energy of the photoelectron remains unchanged. The graph is therefore the same as before.
	(b)(ii) If the material is replaced by another one with smaller work function energy, the maximum kinetic energy of photoelectrons will increase as less work is done to liberate the least bounded electrons. The threshold frequency will therefore be reduced.

	However , since the gradient of the graph = *h* which is the Planck constant. The gradient of the graph is unchanged. This new graph is parallel to the original one.
	
	(c) (i) $E_K = hf - \phi \Rightarrow eV_s = hf - \phi \Rightarrow V_s = \frac{h}{e}f - \frac{\phi}{e}$ Gradient = *h/e* ; vertical intercept = -ϕ/*e*
	(c) (ii) $E_K = hf - \phi \Rightarrow E_K = hc(\frac{1}{\lambda}) - \phi$ Gradient = *hc* ; vertical intercept = -ϕ
	(c) (iii) $E_K = hf - \phi \Rightarrow eV_s = hc(\frac{1}{\lambda}) - \phi \Rightarrow V_s = \frac{hc}{e}(\frac{1}{\lambda}) - \frac{\phi}{e}$ Gradient = *hc/e* ; vertical intercept = -ϕ/*e*
D5	**(a)** $KE_{max} = \frac{hc}{\lambda} - \phi = \frac{(6.63\times10^{-34})(3.00\times10^{8})}{490\times10^{-9}} - (2.5\times1.60\times10^{-19}) = 5.92\times10^{-21}$ J $\frac{1}{2}mv_{max}^{2} = 5.92\times10^{-21}$ J $v_{max} = \sqrt{\frac{2(5.92\times10^{-21})}{9.11\times10^{-31}}} = 1.14\times10^{5}$ ms^{-1}
	(b)(i) Blue light has a frequency above europium's threshold frequency, (or the frequency of the blue light is sufficiently high so that the energy of the photons is above the work function energy of europium) thus photoelectrons will be emitted, causing the gold leaf and metal rod to lose their negative charge and hence the repulsion between them reduces and the leaf falls. Red light has a frequency below europium's threshold frequency and no photoelectrons are emitted. Examiners comments: Students must make use of the relevant scientific terminology to gain credit.

(b)(ii) If the frequency of light is higher than the threshold frequency, photoelectrons will be emitted. As a result, the gold leaf and metal rod will be more positively charged and repel each other more.

If the frequency of light is lower than the threshold frequency, photoelectrons will not be emitted. The charge will remain the same and therefore position of the gold leaf and metal rod will not change.

In either scenario, the leaf will not fall.

You may checkout the following links or scan the QR code to understand how a gold-leaf electroscope works.

1) https://www.youtube.com/watch?v=OUAplNaGD-c&list=PLha6P7MqlaccwqEsECXbVz0DT0K5jbDGo&index=17&t=1s
2) https://www.youtube.com/watch?v=l-gwAs2ApPw&list=PLha6P7MqlaccwqEsECXbVz0DT0K5jbDGo&index=24

D6

(a) $E = hf = hc/\lambda$ = (6.63 x 10^{-34}) (3.00 x 10^{8}) / (500 x 10^{-9}) = 3.98 x 10^{-19} J = 2.49 eV

(b) $KE_{max} = hf - \Phi$

$\Phi = hf_0$ = 2.49 – 1.99 = 0.50 eV = 0.50 (1.60 x 10^{-19}) J

Minimum frequency of light f_0 = 1.2 x 10^{14} Hz

(c) Power arrived at each pixel = intensity x area
= (1.0) (20 x 10^{-6})^2 W
= 4.0 x 10^{-10} W

Power = (total energy) / (total time)
= (number of photons per unit time) x (energy per photon)
= (rate of photons incident on each pixel) x (energy per photon)

Rate = (power) / (energy per photon)
= (4.0 x 10^{-10}) / (3.98 x 10^{-19}) = 1.01 x 10^{9} s^{-1}

D7

How do you know this question is about photoelectric effect?

*As light is incident on the sphere, photoelectrons are emitted from the isolated conducting sphere, causing it to **become positively charged**. It will reach a state at which the most energetic photoelectrons can no longer escape from the positively charged surface to infinity (i.e. the photoelectrons have insufficient KE to overcome the electric field due to the positively charged sphere).*

From Einstein's photoelectric equation:

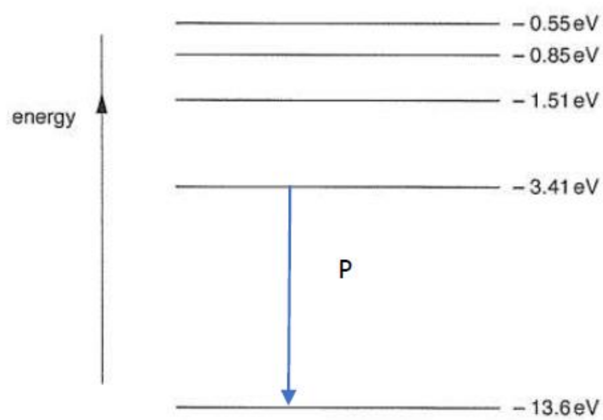
$KE_{max} = hf - \Phi = hc/\lambda - \Phi$ = (6.63 x 10^{-34}) (3.00 x 10^{8}) / (200 x 10^{-9}) – (4.70) (1.60 x 10^{-19})

	= 2.425 x 10^{-19} J The electrons can escape from the sphere when their electric potential energy *EPE* at the surface of the sphere is -2.425 x 10^{-19} J. $EPE = \frac{qQ}{4\pi\varepsilon_o r} = \frac{\left(-1.6\times10^{-19}\right)Q}{4\pi\left(8.85\times10^{-12}\right)\left(5.00\times10^{-2}\right)} = -2.425\times10^{-19}$ J *Q* = 8.43 x 10^{-12} C

2024 Tutorial 17B : Line Spectra & Xray Spectra

Discussion Questions Suggested Solution

D1	**(a)**	A **photon** is a <u>quantum</u> of <u>electromagnetic radiation</u> which energy *E* is given by *E* = *hf* where *h* is the Planck constant and *f* the frequency of the electromagnetic radiation.
	(b)(i)	Visible line emission spectrum appears as a series of discrete coloured lines on a dark background.
	(b)(ii)	A single photon is emitted when an excited atom transits from a higher energy level to a lower energy level. The energy of this photon emitted corresponds to difference in the two energy levels. Since, the energy of each of the emitted photon has an energy *E=hf* where *f is* frequency of the photon and only discrete frequencies are emitted. These discrete frequencies in the line spectra must therefore resulted from transitions between discrete energy levels. **Comment from Examiner (CIE 9702 Nov 2022 P42 Q9)** Many students answered a different question from that which was asked and gave a description of the appearance of an emission spectrum or how an emission spectrum is formed. There was much confusion between concepts of photon energy, energy transitions and energy levels, with many responses lacking accuracy in the way these technical terms were used. It is important that candidates use technical language accurately. Candidates are not able to obtain full credit if they use an inappropriate word that makes the response technically incorrect.
	(c)(i)	Energy of the photon, $E = hf = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34})(3.00\times10^{8})}{400\times10^{-9}}$ = 4.97 x 10^{-16} J = (4.97 x 10^{-16})/(1.60 x 10^{-19}) eV= 3.11 eV Consider electron transitions from a higher energy level too -13.6 eV, The minimum energy of photon emitted will be from -3.14 eV to -13.6 eV = -3.14 –(-13.6) = 10.46 eV > 3.11 eV This is greater than the energy of the most energetic visible light at wavelength of approximately 400 nm. Thus electron transitions to the energy level -13.6 eV result in photons of even smaller wavelengths which do not lie in the visible spectrum.
	(c)(ii) 1.	Minimum energy of photon of visible light is 1.6 eV. Hence, photons lower than 1.6 eV will also not lie in the visible light range (they will likely be in the infra red range). Hence, we need only look at transitions to -3.41 eV. Counting all downward transitions to -3.41 eV, there are 3.
	(c)(ii) 2.	$E_{photon} = hf = h\frac{c}{\lambda}$. Hence, the shortest wavelength correspond to the largest energy change and hence it corresponds to the transition from -0.55 eV to -13.6 eV.

		$\lambda = \frac{hc}{E} = \frac{(6.63\times10^{-34})(3.00\times10^{8})}{(-0.55-(-3.41))\times1.60\times10^{-19}} = 4.35\times10^{-7}$ m
	(d)	A spectral line correspond to a transition between a higher energy level to a lower energy level i.e. a transition between 2 levels. There are 5 energy levels in the diagram, hence there should be ${}^{5}C_{2}$ = 10 transitions. (Alternatively you can name each energy level E_1 …..to E_5 and systematically count the possible transitions.)
	(e)(i)	The longest wavelength correspond to the smallest frequency and hence the smallest difference in the energy level that is still greater than work function energy of 5.6 eV to cause a photoelectric effect. That will be from -3.41 eV to -13.6 eV.  **Fig. 8.1** (not to scale)
	(e)(ii)	Using Einstein's photoelectric effect equation, $KE_{max} = hf - \phi = ((-3.41)-(-13.6)) - 5.6 = 4.59$ eV
	(f)	It suggests that there are also <u>only certain allowed energy levels that the nucleus of an atom can take</u> and these <u>energy levels are discrete</u>, so that only specific photons of frequencies in the gamma ray regions are emitted when an excited nucleus transit from a higher energy level to a lower energy level. Furthermore, the typical wavelength of gamma radiation is 10^{-15}m, the energy of the photons emitted are hence extremely higher and hence difference between the energy levels is very large at approximately $h\frac{c}{\lambda} = (6.63\times10^{-34})\frac{(3.00\times10^{8})}{(10^{-15})(1.6\times10^{-19})}eV \sim 10^{9}$ eV (Giga eV range)
D2	**(a)(i)**	The electrons in the hydrogen atoms jump from a lower to a higher energy level and absorb the photons, from the incident light beam, of energies equivalent to the difference between the two energy. Since, the energy of each photon is given by *hf* where *f* is the frequency of each photon, the reduction of the photons of these frequencies will show up as "dark lines" in the continuous spectra. This spectra is known as the absorption spectra.

(a)(ii) Light is re-emitted in all directions and hence not all will be reemitted in the direction of the beam.

Furthermore, for photons that were absorbed that caused the electron to jump to much higher energy levels, they may not be replaced when the electrons transits back to the lower energy level. The excited electrons may not transit back to original energy level before the jump but go through a few transitions before reaching the original energy level. These re-emitted photons will therefore have less energy than the original absorbed photon and have lower frequencies than the absorbed photon.

(b) Since the photons of light have energies only ranging from 1.60 eV to 2.60 eV. Then photons can only cause the atoms to be excited from -3.40 eV to -0.85 eV and -3.40 eV to -1.51 eV.

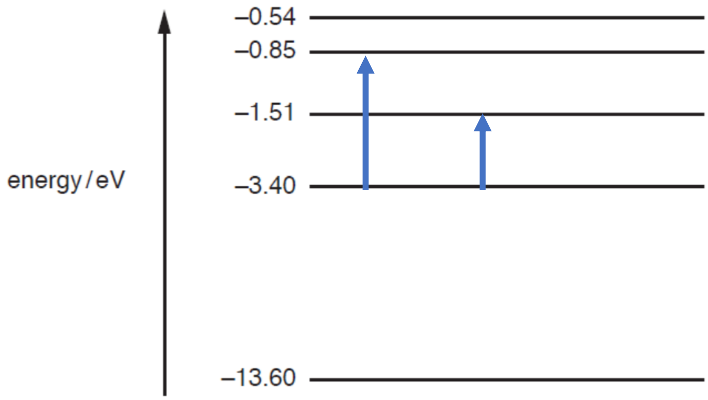


Fig. 10.1 (not to scale)

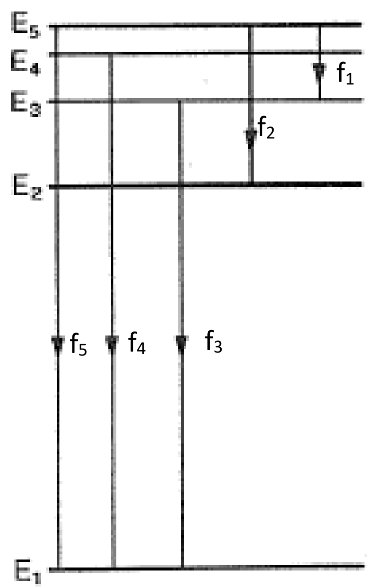
Note : In many questions, they will say the gas is a "cool gas", then you may need to assume that all the atoms are at ground state. This will then mean none of the photons in this beam will be absorbed. However, this question does not mention "cool gas" and furthermore, it is already given in the question that there are dark lines present in the visible range, hence we need not assume that the hydrogen gas atoms were initially at ground state.

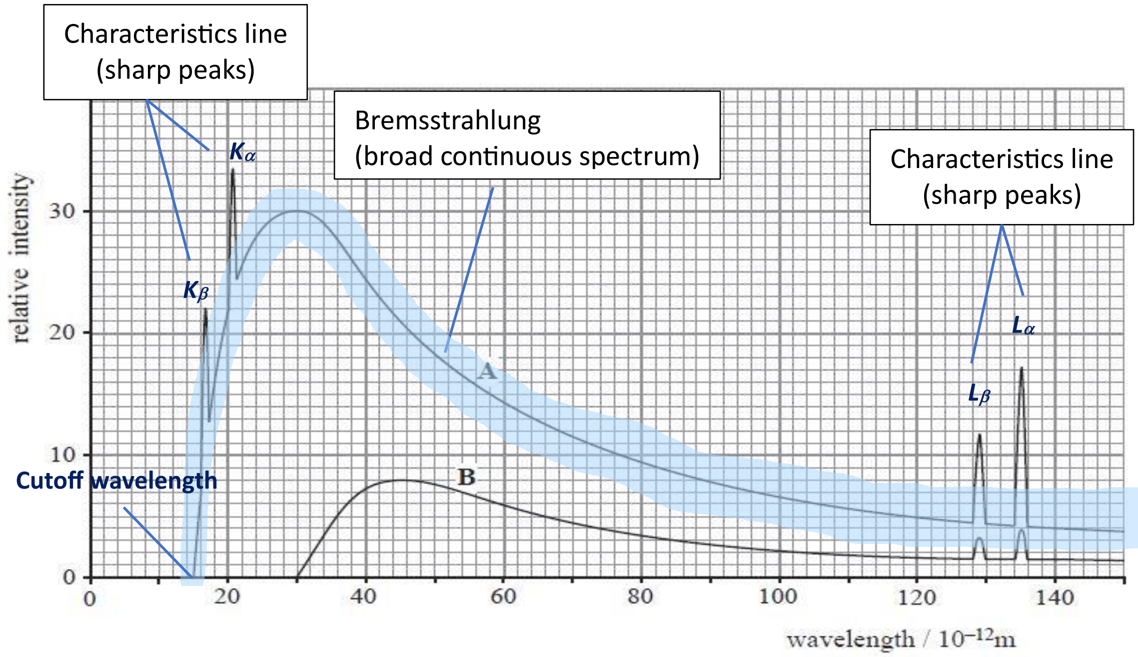
(c) $E_{photon} = hf = h\dfrac{c}{\lambda}$. The shortest wavelength absorbed, implies the greatest frequency as well as the greatest energy of photon that is absorbed. That corresponds to the photon absorbed to cause the electron to jump from -3.40 eV to -0.85 eV.

Hence,

$$E_{photon} = E_H - E_L = h\frac{c}{\lambda}$$

$$\Rightarrow \lambda = \frac{hc}{(E_H - E_L)} = \frac{(6.63\times10^{-34})(3.00\times10^{8})}{((-0.85)-(-3.40))(1.60\times10^{-19})} = 4.88\times10^{-7}\text{ m}$$

D3	**B**		The frequency of the emitted photon is proportional to the energy difference between the energy states. Hence, $f_1 < f_2 < f_3 < f_4 < f_5$. Also the energies of the emitted photon for f_1 and f_2 are closer; hence the lines for the low frequencies are clustered together. Similarly, the energies of the emitted photons for f_3, f_4 and f_5 are closer and we end up with 3 lines of higher frequencies clustered together.
D4	**(a)**	The ionisation energy is the energy required to get an electron from the ground state (n = 1) to an unbound state ($n = \infty$). $\Rightarrow$ Ionisation energy = 0 – (-5.13 eV) = 5.13 eV = 5.13 x (1.60 x 10^{-19}) J = 8.21 x 10^{-19} J	
	(b)	The electron in the atom upon collision is excited from ground state to the energy level n = 4 (3.61 eV), and then it de-excites to the energy level n = 3 (3.19 eV), emitting the 0.42 eV photon. By conservation of energy, the energy of the scattered electron = (3.65 – 3.61) = 0.04 eV	
	(c)	This would mean that the bombarding electrons would have lost 3.65 - 2.41 = 1.24 eV to the electrons in the sodium atoms. However, since the sodium atoms are initially at ground state, it requires at least -3.03-(-5.13) = 2.10 eV for the atom to jump from ground state to the 1st excited state. Hence, it would not have been possible for the scattered electrons to have an energy of 2.41 eV. <u>Alternatively</u> However, since the sodium atoms are initially at ground state, it requires at least -3.03-(-5.13) = 2.10 eV for the atom to jump from ground state to the 1st excited state. The maximum energy the scattered electrons can have will then be 3.65 eV – 2.10 eV = 1.64 eV. 2.41 eV would not have been a possible energy the scattered electron can have since it exceeds the maximum value.	
	(d)	3.15 eV does not correspond to energy difference between the ground state of the atom and any of the higher excited states . Hence, photons of energy 3.15 eV would not be absorbed by the sodium atoms. These photons would pass through without causing any excitation of sodium atoms.	
		N.B. *n* does *not* enumerate the quantum number/shells in this case, as a sodium atom has 11 electrons. *n* = 1 just represents the atom in the ground state. The valence electron is actually in the M shell. **N.B. We are actually ignoring all atomic recoil in this case. But both energy and momentum have to be conserved in all interactions.**	

D5	**(a)**	
	(b)	Characteristic lines are peaks in the X-ray spectrum at certain wavelengths. They are formed when the inner shell electron of the target atom is knocked out by incident electrons and an electron from a higher shell transits to the vacancy created, giving off the excess energy as a photon with an energy equals to the difference the energy level. Since the atom of an element has an unique shell structure, the characteristic lines are dependent on the target material. Both graph A and B have the same characteristic lines at the same wavelengths, this shows that the target material of both cases A and B are the same.
	(c)	Electrons, accelerated through the accelerating voltage, incident on the target interacts with the nuclei of the target atoms decelerate or stops, losing part or all of their kinetic energies as photons with wavelengths in the X-ray range. For the electron that loses all its kinetic energy gained as a single photon, the photon produced will have the largest possible energy. This will correspond to the smallest wavelength of photon possible. This wavelength is known as the cutoff wavelength. ***Common mistake / misconception:*** *Students tends to confuse the production of X-ray radiation with the photoelectric effect. Very few identified the source of X-ray radiation as being the deceleration of electrons. Many also neglected to mention that maximum electron energy is given to a single photon and hence the link to minimum wavelength could not be made. It was common to find that there was no mention of photons.*
	(d)	The electrons accelerated through the accelerating voltage *V* gains a kinetic energy of e*V*. At cutoff wavelength λ_{min}, this kinetic energy gained is lost as a single photon. From the graph, the cutoff wavelength is 15 x 10^{-12} m. By conservation of energy, $eV = h\frac{c}{\lambda_{min}} \Rightarrow h = \frac{eV\lambda_{min}}{c} = \frac{(1.6\times10^{-19})(80\times10^{3})(15\times10^{-12})}{(3.00\times10^{8})} = 6.4\times10^{-34}$ J s

(e)

From the graph of B, it can be seen that the cutoff wavelength is bigger than that of spectrum A. This means that that the most energetic photon that is produced in spectrum B is smaller than spectrum A. Since the most energetic photon is produced from the kinetic energy gained by the electrons from being accelerated by the accelerating voltage. This implies the accelerating voltage must be smaller than 80 kV.

(f)

$$eV = h\frac{c}{\lambda_{\min}} \Rightarrow V = \frac{hc}{e\lambda_{min}} = \frac{(6.63\times10^{-34})(3.00\times10^{8})}{(1.60\times10^{-19})(30\times10^{-12})} = 41.4 \text{ kV}$$

(g)(i)
(ii)

From the graph, you can get the wavelengths for each of the characteristic line. Now, the characteristic line photon is produced when the electron from a higher shell transits down to a lower shell and hence energy of the photon = difference in energy of the energy of the 2 shells i.e.

Hence, $E_H - E_L = h\frac{c}{\lambda}$ and then divide the answer by 1.60 x 10^{-19} to convert to eV, we have

Wavelength of K_β = 17 x 10^{-12} m $\Rightarrow$ Energy difference between M shell to K shell : 73.1 keV

Wavelength of K_α = 21 x 10^{-12} m $\Rightarrow$ Energy difference between L shell to K shell : 59.2 keV

Wavelength of L_β = 129 x 10^{-12} m $\Rightarrow$ Energy difference between N shell to L shell : 9.64 keV

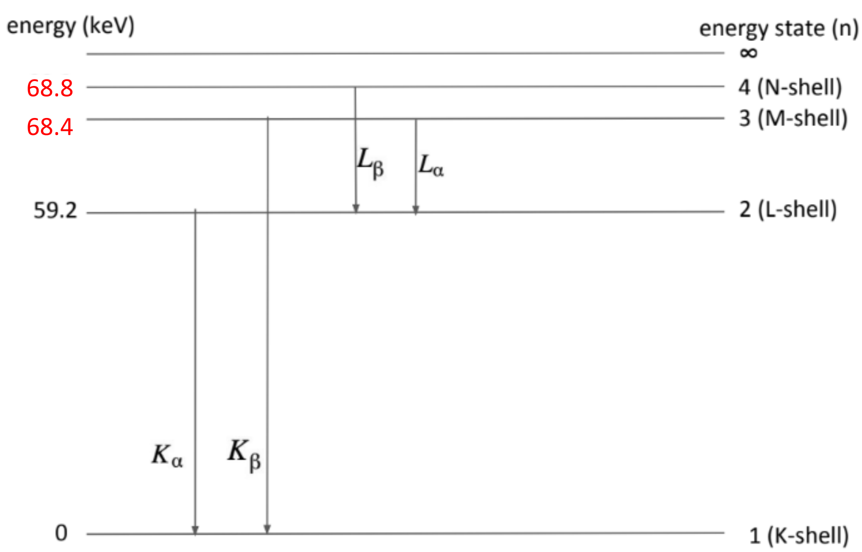
Wavelength of L_α = 135 x 10^{-12} m $\Rightarrow$ Energy difference between M shell to L shell : 9.21 keV

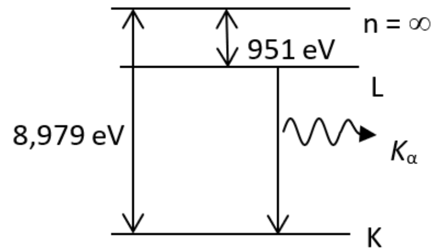
Note there is actually some anomally for this set of data that set by the WJEC examination board.

The diagram below gives one possible answer where:

- the energy of the M-shell is taken to be 59.2 + 9.21 =68. 4 keV
- the energy of the N shell then follows to be 59.2 + 9.64 = 68.8 keV

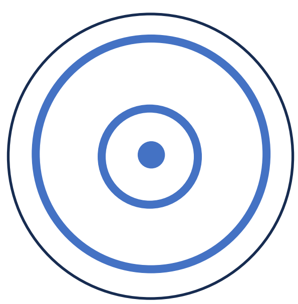
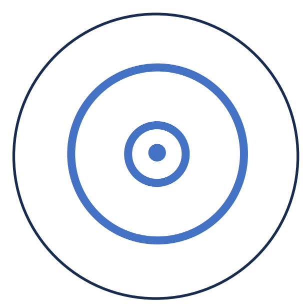
However that will be incoherent with the K- beta line which gives 73.1 keV. Hence, we do note that the examination board has not triangulated their data. – So simply take this as an academic exercise……till we look for a better graph then.



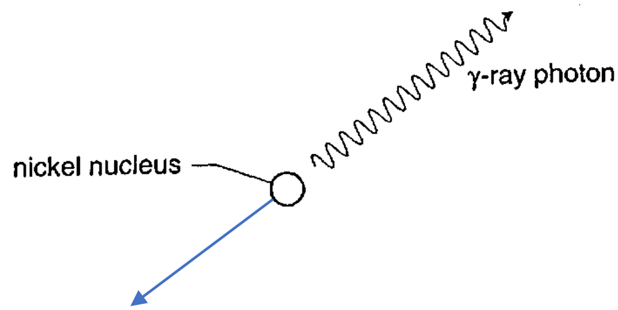
D6	A sketch of an energy diagram will always help in solving question. From the energy level diagram, we can see that : Wavelength of K_α, $E_{photon} = E_{L-shell} - E_{K-shell}$ $\Rightarrow h\frac{c}{\lambda} = E_{L-shell} - E_{K-shell}$ $\Rightarrow \lambda = \frac{hc}{E_{L-shell} - E_{K-shell}} = ... = 1.55 \times 10^{-10}\text{m}$	
	Minimum voltage = 8979 V **In order to produce the K_α transition, the incoming electron will need to create a vacancy in the K-shell first.** The **subsequent de-excitation of an L-shell electron will then give rise to the $K\alpha$ photon.** The minimum voltage is therefore *NOT* 8979 - 951 = 8028 V as the L-shell is occupied, so the K-shell electron cannot be directly excited into the L-shell.	

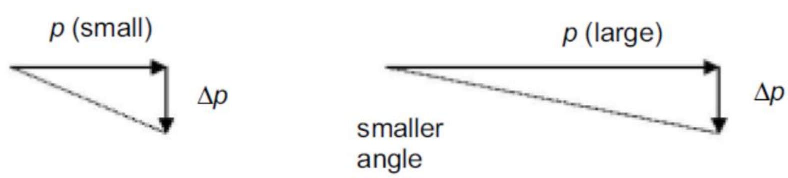
2024 Tutorial 17C : Wave Particle Duality and Uncertainty Principle

Discussion Questions Suggested Solution

D1	**(a)**	Electrons possess wavelike nature and the graphite film have a regular crystal structure. Hence, the electrons will diffract after passing through the film and the electron waves will interfere to form the concentric rings. (At points where the electron waves interfere constructively, the probability of electrons landing there is higher. When it lands, the electrons will cause the screen to fluorescent and the bright rings are the points of constructive interference. Where the electron waves interfere destructively, the probability of electrons landing there is low and correspond to the dark regions between the bright concentric rings.)
		Examiner Comments: Some students mistakenly thought the diffraction occurred at the gap of anode. More serious misconceptions were to consider the rings being produced by production of photons when the electrons collided with graphite or emission of photons from atom's electron energy levels.
	(b)	As the potential difference increases, the kinetic energy of the electrons reaching the anode will increase and hence, the speed/momentum of the electrons will increase. By de-Broglie relationship *p= h/λ*), the de Broglie wavelength of the electrons will decrease. This leads to the diffraction angle of the electrons to become smaller resulting in radii of the concentric rings decreasing.
		 
		Examiner Comments: Only a minority considered momentum and de Broglie wavelength. Use of proper terminology is assessed in qualitative answers and hence the term de Broglie wavelength needs to be explicitly stated. Many wrongly used the equation *hc/λ= eV*. The electrons are not photons and hence would not be moving at the speed of the light. *hf = hc/λ* is energy of a photon. To prevent confusion in equations, students should have a mental visualisation of the context of each equation in Physics.
	(c)	KE gained = EPE lost = *eV* $\frac{p^2}{2m} = KE \quad \Rightarrow \quad p = \sqrt{2m(KE)} = \sqrt{2meV}$ This gives $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV}} = \frac{6.63\times10^{-34}}{\sqrt{2(9.11\times10^{-31})(1.60\times10^{-19})(1.2\times10^{3})}} = 3.54\times10^{-11}$ m

	(d)	The size of the graphite atoms must be comparable to the comparable to the wavelength of the electrons ~ 10^{-11} m. Since the graphite film can diffract the electrons, the atoms in the graphite film must be arranged in an orderly manner and the spacing of the atoms must be also comparable to the size of the wavelength of the electrons.
D2	**(a)**	$E = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34})(3.00\times10^{8})}{(680\times10^{-9})} = 2.93\times10^{-19}$ J
	(b)	Power, $P = IA$ where I is intensity and A is area $\frac{nE}{t} = IA$ where E is the energy of each photon $\frac{n}{t} = \frac{(3100)\left(\pi\times(0.60\times10^{-3})^2\right)}{2.93\times10^{-19}} = 1.20\times10^{16}\ \text{s}^{-1}$
	(c)	Momentum of each photon, $p = \frac{h}{\lambda} = \frac{6.63\times10^{-34}}{680\times10^{-9}} = 9.75\times10^{-28}\ Ns$ Momentum perpendicular to the surface, $p_N = p\cos 52^o = 6.00\times10^{-28}\ Ns$ (towards surface) For photons absorbed, the change momentum of each photon, $\Delta p = 6.00\times10^{-28}\ Ns$ (in direction away from surface). For photons reflected, the change in their momentum for each photon, $\Delta p = 12.00\times10^{-28}\ Ns$ (in direction away from surface). Therefore, force by surface on photons = rate of change in momentum of photons $= 0.45\times1.20\times10^{16}\times6.00\times10^{-28} + 0.55\times1.20\times10^{16}\times12.00\times10^{-28} = 1.12\times10^{-11}\ N$ (upward). By Newton's 3rd law, force by photons on surface $= 1.12\times10^{-11}\ N$ (Downward)
		Examiner's Comment: To get the correct answer, students must consider both the absorbed and reflected photons.

D3	**(a)(i)**	$\lambda = \dfrac{hc}{E} = \dfrac{6.63\times10^{-34}\times3.0\times10^{8}}{1.17\times10^{6}\times1.6\times10^{-19}} = 1.06\times10^{-12}$ m
	(a)(ii)	$p = \dfrac{h}{\lambda} = \dfrac{6.63\times10^{-34}}{1.06\times10^{-12}} = 6.25\times10^{-22}$ Ns
	(b)(i)	By conservation of linear momentum, the momentum of the nickel nucleus will be back to back (180 deg) in opposite direction to travel of the photon. 
		Examiners comments : Students who did not use a ruler were not given credit.
	(b)(ii)	By Principle of Conservation of Momentum, the magnitude of the momentum of the nickel nucleus must be equal to the magnitude of the momentum of the photon, since the system starts off with zero momentum. $v = \dfrac{p}{m} = \dfrac{6.25\times10^{-22}}{9.95\times10^{-26}} = 6.28\times10^{3}\ \text{m s}^{-1}$
	(c)	A moving nickel nucleus has non-zero linear momentum. By the principle of conservation of momentum, the angle will be the same if the photon was emitted in the same direction or directly opposite direction as the motion of the nickel nucleus. Otherwise, the angle will not be the same.
D4	**(a)**	$s = vt = (3\times10^{8})(1.0\times10^{-8}) = 3000$ m
	(b)	$\Delta x \Delta p \gtrsim h$ $(3000)\Delta p \geq 6.63\times10^{-34}$ $\Delta p \geq 2.21\times10^{-37}$ Ns
		Examiners comments : Many students forgot to give the units of the momentum.
D5	For molecules confined within a piece of material, there is a fixed uncertainty in their position along one axis, Δ*x*. If the molecular motion were to cease at absolute zero, Δ*v* = 0. The product of zero uncertainty in velocity and a nonzero uncertainty in position will be zero, violating the uncertainty principle. Thus, there must be some molecular motion even at absolute zero.	

D6	**(a)(i)**	Electrons can have wavelike properties and have a wavelength. The wavelength of the electron λ and its momentum *p* is related to the through the de Broglie relationship: $p = \frac{h}{\lambda}$ *This part is not in syllabus : The intensity of the wave function / square of the amplitude of the wave function is related to the probability of locating the electron. The higher the value, the higher the probability of finding the electron.
	(a)(ii)	Since, the electrons have wavelike properties like wavelength. They will diffract after passing through the slit, the diffraction pattern formed by the electrons will depend on the slit width *w* and the wavelength of the electrons.
		Examiners Comments: Students must mention diffraction, wavelength and slit width to get credit.
	(b)	The detection / counting of the electrons cannot be explained by wave theory. The electrons are detected/counted discretely.
	(c)(i)	Δy is associated with the uncertainty of the position of the electrons and is related to the slit width.
	(c)(ii)	Δp is associated with the uncertainty of the momentum of the electrons in the *y* direction.
	(d)(i)	The uncertainty in *y*-momentum gives each electron a momentum (velocity) perpendicular to the original direction. The process is random so the beam spreads out with some electrons going to +y and some to –y.
	(d)(ii)	If *w* is smaller then Δy is smaller. Δp_y is therefore larger. so more electrons scatter through larger angles.
	(d)(iii)	Uncertainty in *y*-momentum is still the same. momentum in original direction is larger Using a vector diagram to illustrate:  We can see the deflection angle is smaller.
		Examiners Comments: 2 marks were awarded for clear vector diagrams illustrating the difference. Only explanations related to uncertainty principle were given credit.

Hwa Chong Institution
2024 C2 Physics
Nuclear Physics Tutorial 18A

Discussion Questions

D1 – Question on alpha particle scattering experiment. Concept required from topic of E-field.
(a)

On Fig. 1.3, sketch the path of the α-particle as it passes the gold nucleus.

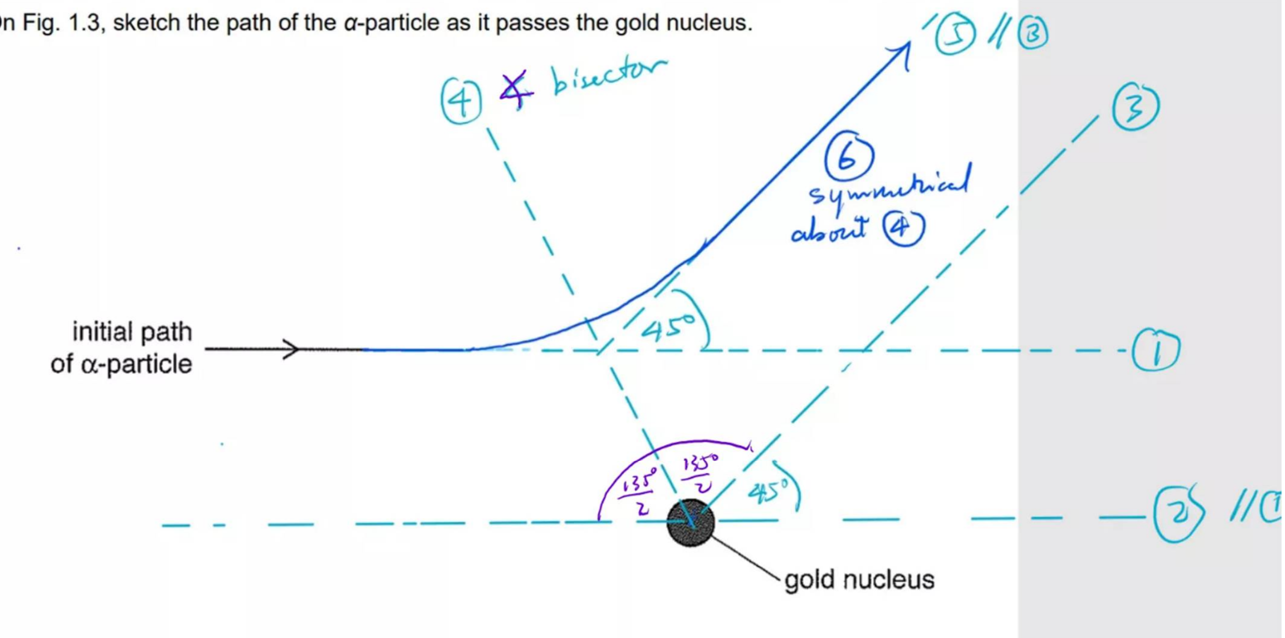


Fig. 1.3

To get a decent drawing, it will be good to either use a protractor or a pair of compasses.

Comments: the great majority of the students indicated a sudden change in direction and many of these did not show the deviation in an appropriate position relative to the nucleus.

(b)
It is observed that most of the alpha particles passes through un-deviated. Some were deflected by very small angle. A very small percentage of alpha deviated by large angle and they must have been scattered by a strong positive charge. The small percentage of large angle scattering suggests that the strong positive charge must have concentrated in a very small space within the atom.

Knowing the mass of the mass of negative charges (electron) was tiny from earlier experiments, we can deduce that most of the mass of the atom must be due to the positive charge carriers and they were concentrated in a tiny space within the atom.

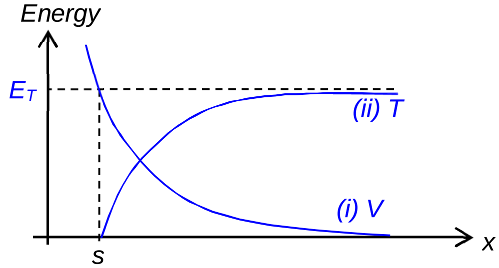
D2 – Question on alpha particle scattering experiment. Concepts required from topics of E-field and Work, Energy & Power.

2a) $F = \dfrac{1}{4\pi\varepsilon_o}\dfrac{(79e)(2e)}{x^2} = \dfrac{3.64\times10^{-26}}{x^2}$ (with x in metres, the force F is in newtons).

2b) $U = \dfrac{1}{4\pi\varepsilon_o}\dfrac{(79e)(2e)}{x} = \dfrac{3.64\times10^{-26}}{x}$ (with x in metres, the potential energy U is in joules).

Comments: students tend to miss out the "e" in the formula and the "square" in the calculation.

2c)



2ciii) $T + V = E_T$ i.e. constant total energy (= electric potential energy at closest approach = $\dfrac{3.64\times10^{-26}}{s}$).

2di) Applying the principle of conservation of energy:
(KE + EPE) at infinity = (KE + EPE) at s

$(1.8 \times 1.6 \times 10^{-13}) + 0 = 0 + \dfrac{3.64\times10^{-26}}{s} \quad \Rightarrow \quad s = 1.26 \times 10^{-13}$ m

Comments: Students must explain the method instead of just writing a bunch of numbers.

2dii) $R_{gold} + R_\alpha < 1.26 \times 10^{-13}$ m
The size of the particles involved (which are the gold nucleus and the alpha particle) must be smaller than 10^{-13} m, or they would touch.
(For knowledge: A gold nucleus has a radius of about 7.5×10^{-15} m, a helium nucleus has radius of 1.7×10^{-15} m)

D3 – Question on analysing a nuclear reaction. With this nuclear reaction James Chadwick discovered the neutron.

3a) $^{13}_{6}\text{C}$ indicates a carbon nucleus that contains 6 protons and 13 – 6 = 7 neutrons.

3b) $\sum m_{LHS}$ = 9.0150 u + 4.0040 u = 13.0190 u $\sum m_{RHS}$ = 13.0075 u
The difference in mass is 0.0115 u.
The equivalent energy released is (0.0115) (1.66 x 10^{-27}) (3.00 x 10^{8})^{2} = 1.7181 x 10^{-12} J.
Taking into account the initial kinetic energy of the α-particles, 8.0 x 10^{-13} J, the total energy available after the reaction is (1.7181 x 10^{-12}) + (8.0 x 10^{-13}) = 2.52 x 10^{-12} J (which is less than 8.8 x 10^{-12} J).
Even assuming that the carbon-13 atom is stationary after the reaction, this is still below the minimum energy of the gamma ray photons, so **(i)** is not possible.

or

The difference in mass is 0.0115 u.
The decrease in rest-mass energy is (0.0115) (1.66 x 10^{-27}) (3.00 x 10^{8})^{2} = 1.7181 x 10^{-12} J.
By the principle of conservation of energy,
initial rest-mass energy + kinetic energy of helium-4 =
final rest-mass energy + kinetic energy of carbon-13 + energy of gamma ray photon
Energy of gamma ray photon =
decrease in rest-masse energy + kinetic energy of helium-4 – kinetic energy of carbon-13
Energy of penetrating radiation = (1.7181 x 10^{-12} J) + (8.0 x 10^{-13} J) – kinetic energy of carbon-13
Energy of penetrating radiation = (2.52 x 10^{-12} J) – kinetic energy of carbon-13
Even assuming that the carbon-13 atom is stationary after the reaction (i.e., its kinetic energy is zero), the maximum energy of the gamma ray photon is only 2.52 x 10^{-12} J, so it is not possible for the photon to have an energy of 8.8 x 10^{-12} J.

D4 – Question on nuclear fission of uranium-235. This nuclear reaction took place in the atomic bomb "Fat Boy" that was dropped in Hiroshima in World War II. The nuclear bomb dropped on Nagasaki used plutonium-239.

4i) The particle c is a neutron ($^{1}_{0}n$); x = 4.

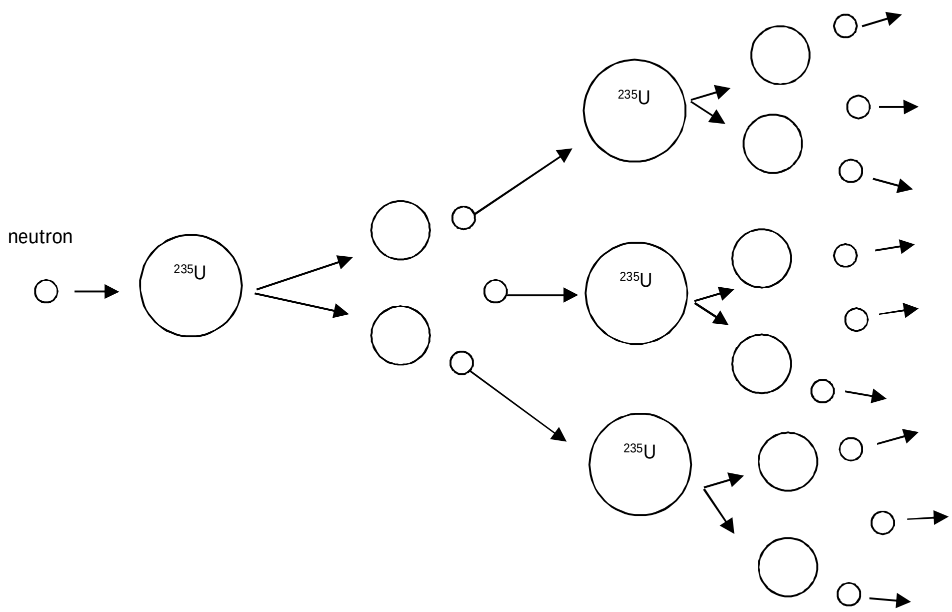
4ii-1) The ***binding energy per nucleon*** of a nucleus is the average energy required to remove a nucleon from the nucleus.

4ii-2) The binding energy of $^{235}_{92}\text{U}$ is 235 x 7.60 = 1786.00 MeV.
The binding energy of $^{139}_{54}\text{Xe}$ = 139 x 8.39 = 1166.21 MeV.
The binding energy of $^{95}_{38}\text{Sr}$ = 95 x 8.74 = 830.30 MeV.
After uranium-235 absorbs a slow-moving neutron, it becomes uranium-236, which spontaneously fissions into xenon-139 and strongtium-95. As this is a spontaneous reaction, the total binding energy increases. The energy released is equal to the increase in binding energy.
Energy released = increase in total binding energy
= (binding energy of products) – (binding energy of reactant)
= (1166.21 + 830.30) – (1786.00)
= 210.51 MeV

4ii-3) Reaction 1. A greater amount of energy released implies that the products of reaction 1 have greater binding energies per nucleon, which have higher stability. Nature favours reactions that result in more stable products.

D5 – Data Analysis Question on chain reaction for nuclear fission of uranium-235. Question looks at the nuclear reaction in fission reactors used for power generation. The problem of nuclear waste storage is also looked at.

5ai) A chain reaction is a sequence of reactions where a reactive product causes additional reactions to take place. In a chain reaction, positive feedback leads to a self-amplifying chain of events. In the diagram below, a neutron causes a uranium nucleus to fission, which leads to the emission of three new neutrons to be produced, which can in turn cause uranium nuclei to fission, etc.



5aii) The neutrons are the particles which trigger the reactions. Since all neutrons from each reaction are captured, the **number of reactions in each step is larger than the number in the previous step, and the number of reactions increases exponentially**. **The greater the number of reactions, the greater the amount of energy released**. At the final stage of the reaction, **the number of reactions is the largest** and the amount of energy released will be the largest at this stage.

5bi) In general, logarithmic scales **allow a large range (spanning several orders)** to be displayed **without small values being compressed down into bottom of the graph**. In this case, if a linear scale had been used, the features and **variation** of the percentage yield when the nucleon number is between about 110 and 125 **would not be observable**. It would appear as a horizontal line.

5bii) From Fig. 8.2, for nucleon number 82, percentage yield is about 0.08. Nucleon numbers 108, 127 and 152 also give a yield of 0.08%.

5biii) For the nuclear equation given, take a = 82.
To balance the mass numbers, c = 235 + 1 – 82 – (2 or 3) = 152 or 151.
With reference to the answer in (ii), P has a yield of 0.08%, and Q is expected to have the same yield.
Hence, c = 152.

5biv) 6%. Graphs should be read to half the smallest square, which is challenging for a logarithmic scale. In principle, 7% should also be acceptable – but it leads to a problem in the next part.

5bv) For products having equal masses, the mass number of the products is [235 + 1 – 2] / 2 = 117. (If the products have equal masses, it is not possible for 3 neutrons to be emitted.)
From Fig. 8.2, the yield of nucleon number 117 is 0.01 %.
[Percentage yield of products in (iv)] / [percentage yield of equal-mass products] = 6 / 0.01 = 600.

5ci) Emission of high-energy photons.

5cii1) Estimating the mass of a uranium atom as 235 u, the number of nuclei ≈ $\frac{1kg}{235(1.66\times10^{-27})}$ = 2.56 x 10^{24}.

5cii2) Total energy from reactions = (number of uranium nuclei) x (energy generated per reaction)
= (2.56 x 10^{24}) (167 x 1.6 x 10^{-13}) = 6.85 x 10^{13} J.

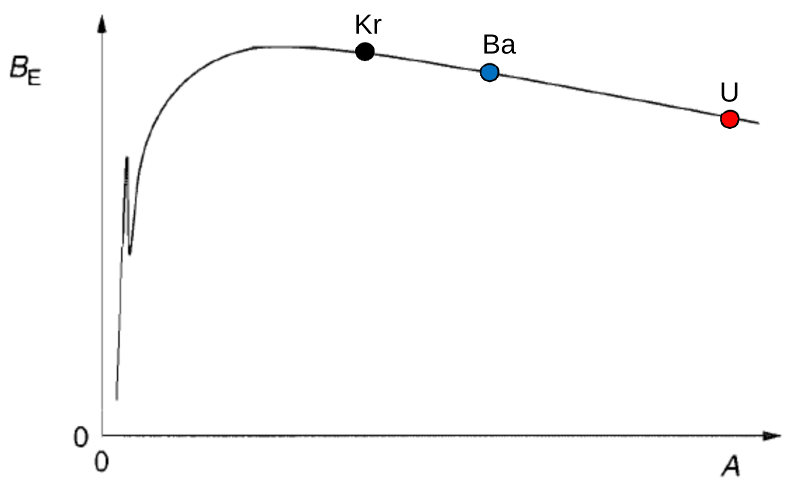
Since the efficiency in the generation of electrical energy is 25%, the electrical energy generated from 1 kg of uranium is 0.25 x (6.85 x 10^{13}) = 1.71 x 10^{13} J.

5cii3) (Average power output) = (total energy generated) / (time taken)
= (1.71 x 10^{13}) / (24 x 3600) = 1.98 x 10^8 W = 198 MW.

D6 – Question on the binding energy per nucleon.

6a) 8.8 MeV

6b)



It is important to remember certain characteristics of this curve, such as the maximum value and where it occurs (at A ~ 56) and the fact that uranium is near the end of the curve. Any plotting relative to these values will do.

6c)

- The binding energies per nucleon of barium-141 and krypton-92 are significantly higher than the binding energy per nucleon of uranium-235.
- Since, BE × A = Binding energy of the nuclei, and as the number of nucleons is almost the same, the total binding energy of the fission products' nuclei is larger than that of the parent nucleus.
- An increase in binding energy implies a decrease in mass, which implies a release of energy.

It is the total binding energy (rather than binding energy per nucleon) that will decide if there is going to be a net release of energy.

Hwa Chong Institution
2024 C2 Physics Nuclear Physics Tutorial 18B
Discussion Questions

D1 N91/I/30; J95/I/30: Medical application of radioactivity (cancer treatment)

For the same treatment effect, the number of decayed nuclei ΔN must be the same.

Since $A = -\frac{\Delta N}{\Delta t}$, $\Delta N = -A\Delta t$ (assuming activity is relatively stable in a short time interval).

Let A_i be the activity when the source is first used, t_i = 10 minutes the treatment time when the source is first used, A_f the activity after 2 years, and t_f the treatment time after 2 years. Then

$$A_i\Delta t_i = A_f\,\Delta t_f$$
$$\Delta t_f = \frac{A_i}{A_f}(10 \text{ minutes})$$

Method 1,

$$A_f = A_i e^{-\lambda t}$$
$$\frac{A_f}{A_i} = e^{-\lambda t}$$
$$\frac{A_i}{A_f} = e^{\lambda t} = e^{\left(\frac{\ln 2}{\tau_{1/2}}\right)t}$$
$$\Delta t_f = e^{\left(\frac{\ln 2}{4}\right)(2)}(10 \text{ minutes}) = (1.414)(10 \text{ minutes}) = 14.1 \text{ minutes}$$

Answer: C

Method 2:

$$\frac{A_f}{A_i} = \left(\frac{1}{2}\right)^{t/\tau_{1/2}}$$
$$\frac{A_i}{A_f} = \frac{1}{\left(\frac{1}{2}\right)^{t/\tau_{1/2}}}$$
$$\Delta t = \frac{1}{\left(\frac{1}{2}\right)^{2/4}}(10 \text{ minutes}) = (1.414)(10 \text{ minutes}) = 14.1 \text{ minutes}$$

Answer: C

Method 3:

If we need 10 minutes of radiation when the source is first used, we will need 20 minutes 4 years later, since the half-life is 4 years (the activity drops by a factor of 2 in 4 years). Hence, after 2 years, we need more than 10 minutes, but less than 20 minutes. **Answer: C**

D2 From the graph,

The activity of X after 4 hours is 0.9 x 1000 = 900 Bq.
Since the decay is exponential, the activity will drop by the same fraction every four hours.
Hence, the activity of X after 8 hours is 0.9 x 0.9 x 1000 = 810 Bq.

The activity of Y after 2 hours is 0.9 x 1000 = 900 Bq.
Since the decay is exponential, the activity will drop by the same fraction every two hours.
Hence, the activity of Y after 8 hours is 0.9^4 x 1000 = 656.1 Bq.

$$\frac{\text{activity of X after 8 hours}}{\text{activity of Y after 8 hours}} = \frac{\left(\frac{9}{10}\right)^2}{\left(\frac{9}{10}\right)^4} = \frac{100}{81}$$

Answer: D

D3 $A = \lambda N$

$$\lambda = \frac{A}{N} = \frac{\left(\frac{\Delta N}{\Delta t}\right)}{N} = \frac{\Delta N}{N\Delta t}$$

Answer: C

D4 N90/I/30

$$A = A_o e^{-\lambda t} \Rightarrow \ln A = \ln A_o - \lambda t$$

Since the graph shows ln *A* vs *t*, the gradient is $-\lambda$. Since the same radioactive sample is injected into the vessel, the decay constant λ is the same, so the gradient should be the same after the injection.

Answer: B

D5 N99/II/7. Potassium-Argon Dating

D5 a) The half-life of a radioactive nuclide is the **average** time taken for half of the original number of nuclei in a sample of the radioactive nuclide to decay.

D5 bi) ${}^{40}_{19}\text{K} \rightarrow {}^{40}_{18}\text{Ar} + {}^{0}_{1}\text{e}$

D5 bii) Positron (anti-electron)

D5 ci) All the argon comes from the potassium.
Current number of potassium : original number of potassium = 1 : 8
$1 = 8e^{-\frac{\ln 2}{1.4\times10^9}t}$ or observe that $\frac{1}{2^3} = \frac{1}{8}$, so $t = 3\ (1.4 \times 10^9)$ years
$\Rightarrow$ $t = 4.2 \times 10^9$ years

D5 cii) Current number of potassium : original number of potassium = 1 : *x* where *x* > 8
Hence, the time that has passed is longer than that found in (ci): our value is an underestimate.

D6 J97/II/8 (part); J92/III/6 (Modified): Application of beta radiation in checking thickness of metal.

D6 i) The number of β-particles detected by the detector increases with *decreasing thickness of foil*. The foil gets too thin. Hence, the rollers are too close to each other, and a feedback signal should be sent to increase their separation.

D6 ii) γ radiation can easily penetrate thick aluminium foil, so the count rate registered at the detector will not vary with the thickness of the foil.

D6 iii) 1) $A = A_o e^{-\lambda t} \Rightarrow \frac{A}{A_o} = e^{-\frac{\ln 2}{14}\left(\frac{8}{24}\right)} = 0.984$

(Or: number of half-lives in 8 hours = $\frac{8}{24} \div 14 = 1/24$, then $\frac{A}{A_o} = \frac{1}{2^{1/24}} = 0.984$)

D6 iii) 2) Since the activity is weaker, the detector will register a lower count rate. The system will think the sheet is too thick. Hence, to correct for it, a feedback signal will be sent to decrease the separation between the rollers, which will reduce the thickness the aluminium sheets.

As a first-order approximation, since the relative change of activity is small (about 2%), the expected relative change in thickness is expected to be similar, also 2%.

If the activity is 0.984 times the initial value, the system might want to decrease the thickness by a factor 0.984. Hence, the thickness at the end of the day will be 1 – 0.984 = 0.016 ≈ 2% less than the thickness at the beginning of the day.

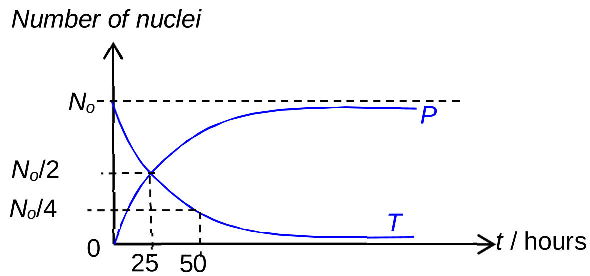
D6 iv) 1) The fractional error in the count rate will be reduced, since the count rate is high.
Background subtraction is not necessary, as the background is negligible compared to the source's activity.
Changes in count rate due to changes in thickness of the material are proportional to the activity.
Any detected change in count rate will be more significant, i.e., monitoring will be more sensitive.
The high activity may pose a health hazard.

2) If the half-life is short, the activity will change considerably over a short period of time. If no adjustments are made, this will lead to aluminium foil that is too thin, as in (iii) part 2.
Furthermore, the radioactive source will have to be replaced more often.

D7 N2000/II/8(part) Data Analysis

D7 ai) In the nucleus of thorium-231, a neutron transforms into a proton and an electron (and an antineutrino, ${}^{1}_{0}\text{n} \rightarrow {}^{1}_{1}\text{p} + {}^{0}_{-1}\text{e} + \overline{\nu}$), transmuting the thorium-231 into protactinium-231. As electrons (and neutrinos) cannot exist in the nucleus of an atom, it (and the antineutrino) is ejected (beta emission).

D7 aii) Since the half-life of Pa-231 is much longer than that of Th-231, we can assume that Pa is stable within a time interval of 125 hours.



D7 aiii) Assume all the Pa-231 comes from Th-231.
Current number of Th-231 : original number of Th-231 = 1 : 4

$1 = 4e^{-\frac{\ln 2}{25}t} \Rightarrow$ t = 50 hour   Alternatively, $t = 2T_{½} = 2(25)$ = 50 hours.
Or, from the graph, by observation, t = 50 hours.

D7 bi) Monotonically increasing graph. Since S is stable, its number should keep increasing as more D will decay to form S.

D7 bii1) Since $A = \lambda N$, maximum activity is when the number of D is the largest, i.e., at t = 9.0 years.

D7 bii2) $A = [(\ln 2)/T]\, N = [(\ln 2) / (15 \times 365 \times 24 \times 3600)]\,(0.68 \times 1.2 \times 10^{15}) = 1.196 \times 10^{6}$ Bq = 32.3 μCi

D7 biii) The change in number of D at any time depends on
(1) the rate of production of D from P;
(2) the rate of decay of D to form S.
Initially, the number of parent nuclei is large, so (1) is larger than (2), and D increases.
Eventually, the number of parent nuclei is so small that (1) is smaller than (2), and D decreases.
At the time in-between when (1) equals (2), D is at its maximum number.

D7iv) From the text, initially, there are 1.2×10^{15} nuclei of the parent isotope in the sample. From the graph, after ten years, there is 10%, or 1.2×10^{14} nuclei of the parent isotope left. After 30 years, there is 0.1%, or 1.2×10^{12} nuclei of the parent isotope left, producing an activity of $A = \lambda N \approx$ 9,000 Bq.
After 100 years, 1.2×10^{15} nuclei of the parent isotope have been reduced to only about 10^{5}. The activity from the parent isotope is then much lower than 1 Bq, which means it is unmeasurable. Hence, it is no longer possible to reliably measure any relative activities of parent and daughter.

D8 N02/III/5 (parts) Carbon Dating

D8 a) The most straightforward way to determine the half-life of a radioactive sample would be to measure the activity twice, with some time in-between, and compare the activities.

However, as radioactive decay is random, the activity over a short period of time is never really constant, and may show significant fluctuations.
This problem can be countered by measuring the activity over a longer period. However, if that period becomes too long, the activity itself may have decreased significantly within it.

Note: Examiners' Report

Answers to this part often did no more than state what is meant by 'random'. What the mark scheme required was the realisation, stated one way round or its opposite, that if a long period of time is used for counting decays, so ironing out the problems of randomness, then the activity itself is decreasing.

D8 b) $N = N_o e^{-\lambda t} \Rightarrow \frac{1}{8.6\times10^{10}} = \frac{1}{3.3\times10^{10}} e^{-\frac{ln\,2}{1.8\times10^{11}}t} \Rightarrow$ t = 2.49 x 10^{11} s = 7900 years.
Alternatively, the remaining number of carbon-14 is 3.3 / 8.6 = 0.384 times the original value, so 0.5^n = 0.384, where n is the number of half-lives that have passed. n ln 0.5 = ln 0.384, n = 1.38. Hence, t = 1.38 x (1.8 x 10^{11}) = 2.49 x 10^{11} s = 7900 years (to 2 s.f.).

D8 ci) As the ions pass through the magnetic field of flux density B, they feel a magnetic force, $F_B = B\,q\,v$. As the charge q and the speed v are the same, they feel the same magnetic force F_B. By Newton's Second Law, $F_{net} = m\,a$, for the same net force F_{net}, the more massive carbon-14 ions have a smaller acceleration. Hence, the carbon-14 ions get deflected less than the carbon-12 ions.

D8 cii) Before and after the ions pass through the region of magnetic field, there is no net force acting on them (assuming negligible gravitational force). By Newton's First Law, the ions will travel in a straight line at a constant speed before entering and after leaving the magnetic field.
When the ions are in the magnetic field, the magnetic force provides the centripetal force for the ions to travel in a circular path. As the magnetic force is always perpendicular to the ions' velocity, it does not do any work, so the speed of the ions does not change.

D8 ciii) This deflection method of measuring the ratio of carbon-14 to carbon-12 is more reliable as all the carbon-12 and carbon-14 atoms in a sample can counted, giving a small fractional uncertainty. In the former method, only a fraction of the decaying carbon-14 atoms will be detected, and the total number of carbon-14 atoms inferred. Furthermore, as the decay process is random, a detected count rate may be higher or lower than the average value. All these increase the fractional uncertainty in the ratio of carbon-14 to carbon-12 atoms.

D9 N03/II/8 (continued from question D4 of Tutorial 18A) Nuclear Waste

D9 d) The activity of a radioactive nuclide depends on the half-life and the number of radioactive nuclide present ($A = \lambda N$). The total number of emissions is thus determined by the number and type of daughter products in the radioactive decay.

Considering _equal numbers_ of the two fission products, as the decay series of Mo-99 has relatively long half-lives of days to years, it will require a longer period for the activity of a sample of Mo-99 to reduce to an acceptable level. Hence, this nuclear waste needs to be stored for lower. On the other hand, since the (initial) activity is lower, to as much shielding is required.

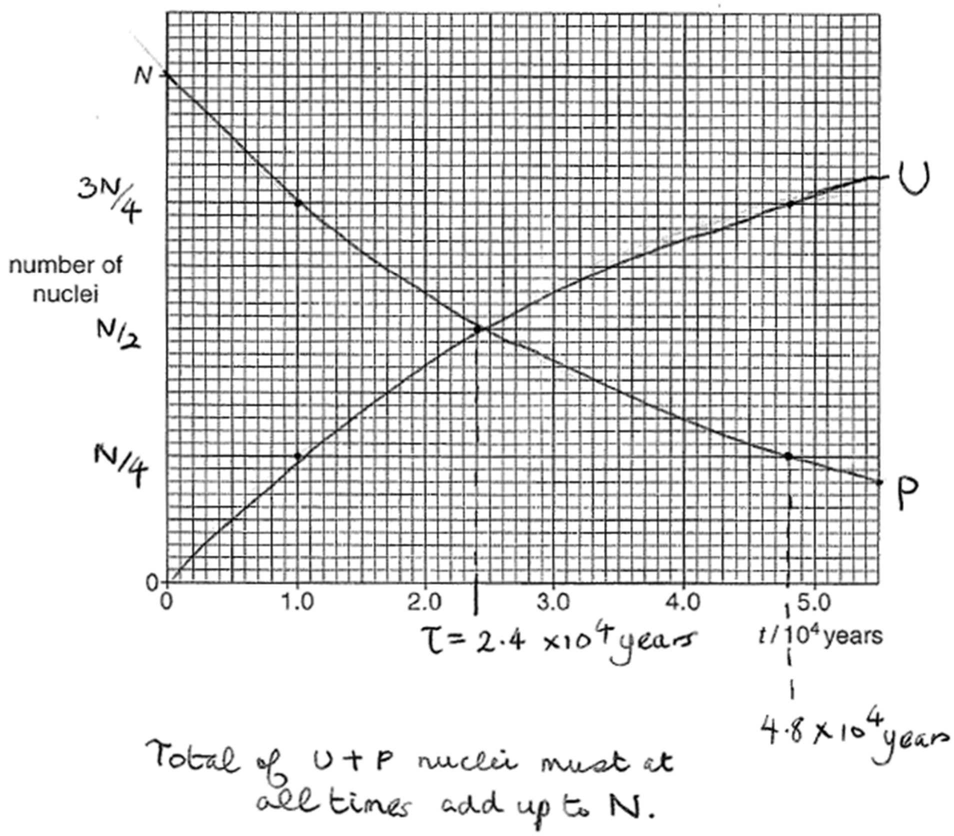
The nuclides in the decay series of Xe-140 have relatively half-lives of seconds to days. Thus, for the same initial number of atoms, this decay series will have a higher activity. Hence, this nuclear waste does not have to be stored for as long (the activity will be reduced to an acceptable level much faster), but it will require more shielding because of the higher initial activity of the decay series.

D10(a) **(i)** Isotopes refer to nuclides which have the same number of protons but different numbers of neutrons.

(ii) The half-life of a radioactive nuclide is the average time taken for half of the original number of nuclei in a sample of the radioactive nuclide to decay.

(iii) $^{239}_{94}Pu \rightarrow ^{235}_{92}U + ^{4}_{2}\alpha$

(b)



(c) A sample of plutonium-239 is presumably initially at rest, so the total momentum is zero. By the principle of conservation of linear momentum, the total momentum of the particles must still be zero after the reaction. Since the alpha particle has (kinetic) energy, it is moving and has momentum. The uranium particle will have momentum of the same magnitude (in the opposite direction), and will therefore also carry (kinetic) energy.

There could also be a release of gamma radiation.

[Note: since the kinetic energy $K = p^2/(2m)$, the ratio of the kinetic energies is inversely proportional to the ratio of the masses. As the alpha particle carries away 5.15 MeV, the uranium atom has 4 / 235 x 5.15 = 0.09 MeV. Since 5.26 – 5.15 – 0.09 = 0.02 MeV, it seems that there is indeed still some high-energy radiation, on top of the two particles.]

Challenging Question

C1 N88/II/12 (part) Medical use of Radioactive Nuclide

C1 i) $C \propto A \propto N$

After 138 days, 3/4 of the iron has been excreted, so 1/4 is left.

That 1/4 of the original iron is only 1/8 as active as initially.

Hence, C = 1/8 x 1/4 C_0 = 1/32 x 690 = 21.6 counts per minute.

ii) The number of radioactive half-lives that have passed is always 1.5 times the number of biological half-lives that have passed.

Taking n to be the number of biological half-lives that have passed,

$$C = \frac{1}{2^{1.5\,n}}\frac{1}{2^{n}}C_0 \Rightarrow n = 0.20942$$

The time passed is (0.20942) (69) = 14.5 days.

iii) Let m be the effective number of half-lives, n the number of biological half-lives, and p the number of radioactive half-lives that have passed at a certain time. Then, from part ii), we see that $m = n + p$. Hence,

$$\frac{1}{T} = \frac{1}{T_r} + \frac{1}{T_b} = \frac{T_b + T_r}{T_b T_r} \Rightarrow T = \frac{T_r T_b}{T_r + T_b}$$