Kent Ridge Secondary School
Secondary 4 Express/5 Normal Academic Preliminary Examination 2024
Add Math Prelim 2024 P2 Mark scheme

Qn	Solutions	Marks	
1a	$\frac{4}{(x^2+1)(x+1)} = \frac{Ax+B}{x^2+1} + \frac{C}{x+1}$ $\frac{4}{(x^2+1)(x+1)} = \frac{(Ax+B)(x+1)}{(x^2+1)(x+1)} + \frac{C(x^2+1)}{(x^2+1)(x+1)}$ $4 = (Ax+B)(x+1) + C(x^2+1)$ Sub x = -1 $4 = C((-1)^2+1)$ $C = 2$ Sub x = 0 $4 = (B)(1) + C(1)$ $B = 2$ Compare coef of x^2: $A + C = 0$ $A = -2$ $\frac{4}{(x^2+1)(x+1)} = \frac{2-2x}{x^2+1} + \frac{2}{x+1}$	A1 M1 M1 M1 A1	
2a	Plot points of **corresponding values of R and $\frac{1}{d^2}$** Draw best fit line through points and the **origin** Find **gradient** of the line gives the **value of *k***	B1 B1 B1 B1	
2bi	Gradient of line $= \frac{1.5}{-10} = -0.15$ Lg y intercept = 3 $\lg y = -0.15t + 3$ $y = 10^{-0.15t+3}$	M1 B1 M1 A1	
2bii	Initial number of particles : $\lg y = 3$ $y = 10^3 = 1000$ Find the point on the straight line when $\lg y = \lg 500 = 2.69$ The time taken is the t value of the point	M1 M1 B1 – their t value (± 0.4)	
3a	$\frac{dy}{dx} = -\sin x + \frac{1}{2}$ $-\sin x + \frac{1}{2} = 0$ $\sin x = \frac{1}{2}$ $\alpha = \frac{\pi}{6}$ $x = \frac{\pi}{6}, \frac{5\pi}{6}$ $x = \frac{5\pi}{6}$	M1 A1 M1 A1	
3bi	Gradient of tangent at x = 0 $\frac{dy}{dx} = -\sin 0 + \frac{1}{2} = \frac{1}{2}$ $y = \cos 0 + 0 = 1$ Equation of tangent $y = \frac{1}{2}x + 1$	M1 M1	

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3bii	$Area = \int_0^{\frac{5\pi}{6}} \frac{1}{2}x + 1 - \cos x - \frac{x}{2}\, dx$ $= [x - \sin x]_0^{\frac{5\pi}{6}}$ $= \frac{5\pi}{6} - \sin\left(\frac{5\pi}{6}\right)$ $= \frac{5\pi}{6} - \frac{1}{2} = 2.12\ (3\ s.f.)$	M1 – mtd to find area trap under tangent M1 – definite integral of curve from 0 to x_b A1 – correct expr of their integrals A1 – correct sub of limits A1	
4a	$\frac{k}{2} + 8x = \frac{1}{2x} + 2kx$ $kx + 16x^2 = 1 + 4kx^2$ $(4k - 16)x^2 - kx + 1 = 0$ $k^2 - 4(4k - 16)(1) = 0$ $k^2 - 16k + 64 = 0$ $k = 8$	M1 M1 M1 A1	
4b	Discriminant: $\sqrt{8}^2 - 4a(a - 1) < 0$ $8 - 4a^2 + 4a < 0$ $a^2 - a - 2 > 0$ $(a - 2)(a + 1) > 0$ Since $a < 0$, $a < -1$	M1 – expr for D M1 – condition for D B1 A1	
5a	$y = e^{2x} \sin 3x$ $\frac{dy}{dx} = 2e^{2x} \sin 3x + 3e^{2x} \cos 3x$	M1 either term seen A1 use of product rule and final ans	
5b	$\frac{d^2y}{dx^2} = 2(2e^{2x} \sin 3x + 3e^{2x} \cos 3x) + 3(2e^{2x} \cos 3x - 3e^{2x} \sin 3x$	M1 use of at one correct product rule of their dy/dx	
5c	$\frac{d^2y}{dx^2} = -5e^{2x} \sin 3x + 12e^{2x} \cos 3x$ $2e^{2x} \sin 3x + 3e^{2x} \cos 3x - 5e^{2x} \sin 3x + 12e^{2x} \cos 3x + ae^{2x} \sin 3x = be^{2x} \cos 3x$ $2 - 5 + a = 0$ $3 + 12 = b$ $a = 3, b = 15$	A1 M1 M1 M1 A1	

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6	$$\frac{d}{dx}\left(\frac{x-2}{\sqrt{3x+1}}\right)=\frac{\sqrt{3x+1}-\frac{3(x-2)}{2\sqrt{3x+1}}}{3x+1}$$ $$=\frac{\frac{2(3x+1)}{2\sqrt{3x+1}}-\frac{3(x-2)}{2\sqrt{3x+1}}}{3x+1}$$ $$=\frac{3x+8}{2\sqrt{3x+1}(3x+1)}$$ $$=\frac{3x+8}{2\sqrt{(3x+1)^3}}$$	M1 – quotient rule seen with positive sq root or product seen with negative sq root M1 – simplify with common denominator or taking out common factor M1 – all factors in denominator collected B1	
6b	$$\int_{x_1}^{x_2}\frac{3x+8}{2\sqrt{(3x+1)^3}}dx=\left[\frac{x-2}{\sqrt{3x+1}}\right]_{x_1}^{x_2}$$ $$\int_{x_1}^{x_2}\frac{3x+7}{2\sqrt{(3x+1)^3}}dx+\int_{x_1}^{x_2}\frac{1}{2\sqrt{(3x+1)^3}}dx=\left[\frac{x-2}{\sqrt{3x+1}}\right]_{x_1}^{x_2}$$ $$\int_{x_1}^{x_2}\frac{3x+7}{2\sqrt{(3x+1)^3}}dx$$ $$=\left[\frac{x-2}{\sqrt{3x+1}}\right]_{x_1}^{x_2}-\int_{x_1}^{x_2}\frac{1}{2\sqrt{(3x+1)^3}}dx$$ $$=\left[\frac{x-2}{\sqrt{3x+1}}\right]_{x_1}^{x_2}-\frac{1}{2}\int_{x_1}^{x_2}(3x+1)^{-\frac{3}{2}}dx$$ $$=\left[\frac{x-2}{\sqrt{3x+1}}\right]_0^5-\left[\frac{1}{2}\frac{(3x+1)^{-\frac{1}{2}}}{3\left(-\frac{1}{2}\right)}\right]_0^5$$ $$=\frac{3}{4}-\frac{-2}{1}-\left(-\frac{1}{3(4)}+\frac{1}{3(1)}\right)$$ $$=2\frac{1}{2}$$	M1 – seen or implied M1 – any equivalent form To show 7 = 8-1 or 8 = 7+ 1 M1 – standard integral M1 – show the correct limits substituted into a valid integral A1	
7a	$\angle DAE=\angle ABD$ (angles in alternate segment) $\angle ADE=\angle BAD$ (alternate angles of parallel lines) triangle ABD is similar to triangle DAE (AA similarity)	B1 B1 B1	
7b	$\angle BAD=\angle DCB$ (corresponding angles of similar triangles) $\angle BAD+\angle DCB=180°$ (angles in opposite segment)	B1 B1	

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	$\angle BAD = \angle DCB = 90°$ BD is diameter (angle in semicircle = 90°)	B1	
8a	$3(3^{x+1}) = 10 - 3^{-x}$ Let $u = 3^x$ $3(3u) = 10 - \frac{1}{u}$ $9u^2 - 10u + 1 = 0$ $3^x = \frac{1}{9}$ or $3^x = 1$ $x = -2\ or\ 0$	M1 – breakdown 3^{x+1} M1 – general QE M1 – eqn in *x* A1	
8b	$\log_{100} x + \lg y = 3$ $\frac{\lg x}{\lg 100} + \lg y = 3$ $\frac{\lg x}{2} + \lg y = 3$ $\lg \sqrt{x} + \lg y = 3$ $\lg \sqrt{x}\, y = 3$ $\sqrt{x} y = 10^3$ $y = \frac{1000}{\sqrt{x}}$	M1 – change base M1 – step before simplifying to one log term M1 – one log term A1	
9a	$(x - 2.5)^2 + (-\frac{1}{2}x + 5)^2 = \frac{365}{4}$ $x^2 - 5x + 6.25 + \frac{1}{4}x^2 - 5x + 25 = \frac{365}{4}$ $\frac{5}{4}x^2 - 10x + 31.25 = \frac{365}{4}$ $5x^2 - 40x + 125 = 365$ $5x^2 - 40x - 240 = 0$ $x = 12, x = -4$ $A(-4, 12)$	M1v - substitution M1 – general QE A1 A1	
9b	centre of circle (2.5,5) $y = 2x + c$ Sub centre of circle (2.5,5) $5 = 2(2.5) + c$ $c = 0$	B1 M1 – grad $\perp$ seen B1 A1	
9c	Sub y = 0 into AB $0 = -\frac{1}{2}x + 10$ $x = 20$ D(20,0) M, Mid point AD = (8,6) Distance ME^2 = $(8 - 2.5)^2 + (6 - 5)^2 = \frac{125}{4} < \frac{365}{4}$	M1 M1 M1 M1	
10a	Sub t = 0, v = 1	B1, B1	
10b	$a = -8e^{-2t} + 1 = 0$ $e^{-2t} = \frac{1}{8}$	M1 M1	

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	$$-2t = \ln\frac{1}{8}$$ $$-2t = \ln 1 - \ln 8 = -\ln 8$$ $$t = \frac{1}{2}\ln 8$$ $$v = 4e^{-\ln 8} + \frac{1}{2}\ln 8 - 3 = -1.46$$	B1 A1	
10c	Since **velocity changes from positive to negative**, the particle **did change** its direction of motion	B1 B1	
10d	$$s = -2e^{-2t} + \frac{t^2}{2} - 3t + c$$ Sub t= 0, s = 6 $$6 = -2 + c$$ $$c = 8$$ Sub t= 2 $s = -2e^{-4} + 2 - 6 + 8 = 3.96$m	M1 integrate exp term M1 integrate power term M1 A1	