9814 H3 Physics Prelim 2021

Suggested Solutions and Markers' Comments

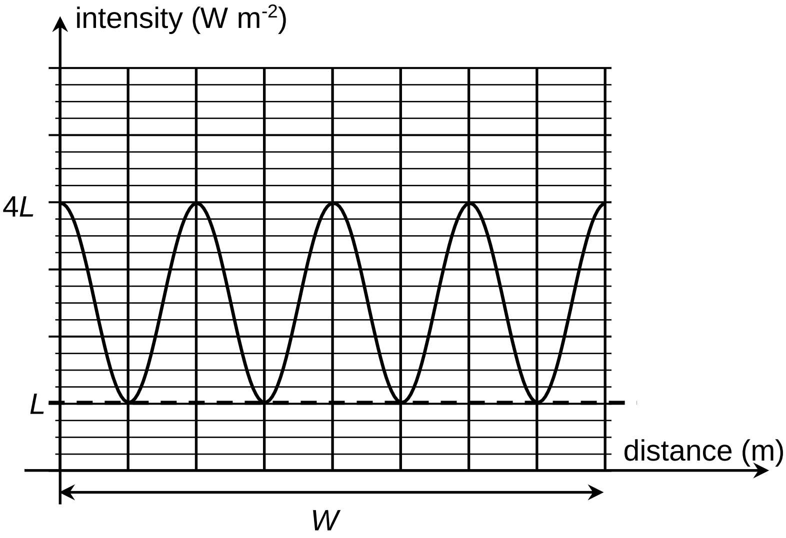
1	**(a)**	$\frac{P_C V_C}{T_C} = \frac{P_A V_A}{T_A}$ $\frac{P_0(3V_0)}{T_C} = \frac{P_0 V_0}{T_0}$ $T_C = 3T_0$	A1
		Comments: Quite well-done. *Majority of candidates managed to get the correct expression for temperature at C. However, the approaches varied. Most used PV=nRT instead of the suggested method. It is expected that candidates compare temperatures at A and C, but there were a few compared temperatures at B and C, and gave the relationship without clear working - the mark was awarded, given the benefit of doubt.*	
	(b)	Work done by gas during process A to B = 0 Work done by gas during process B to C $= \int P dV$ $= \int_{V_o}^{3V_o} \frac{nRT_B}{V} dV$ $= nRT_B \left\lvert \ln V \right\rvert_{V_o}^{3V_o}$ $= 3nRT_o \ln 3$ Work done by gas during process C to A $= -\left\lvert P_o (V_A - V_C) \right\rvert$ $= -P_o(3V_o - V_o)$ $= -2P_oV_o$ Net work done by gas $= 3nRT_o \ln 3 - 2P_oV_o$ $= 3nRT_o \ln 3 - 2nRT_o$ since $P_oV_o = nRT_o$ $= nRT_o(3 \ln 3 - 2)$	B1 B1 B1 A0
		Comments: Quite well done. *Most students could show the correct expression for net work done by gas. We preferred that candidates explicitly state that the net work done is the sum of work done by gas during each process AB, BC, CA before showing the math.* *More candidates had difficulty determining the work done by gas during process BC, as it required integration.* *There were signs that some candidates actually "reverse-engineered" the proof.*	
	(c)	A to B: By First Law of Thermodynamics, as $w = 0$, $\Delta U = q$ Since temperature increases, ΔU and hence q are positive i.e. heat is **added** to the gas. B to C: as temperature is constant, $\Delta U = 0$, $q = -w$	B1

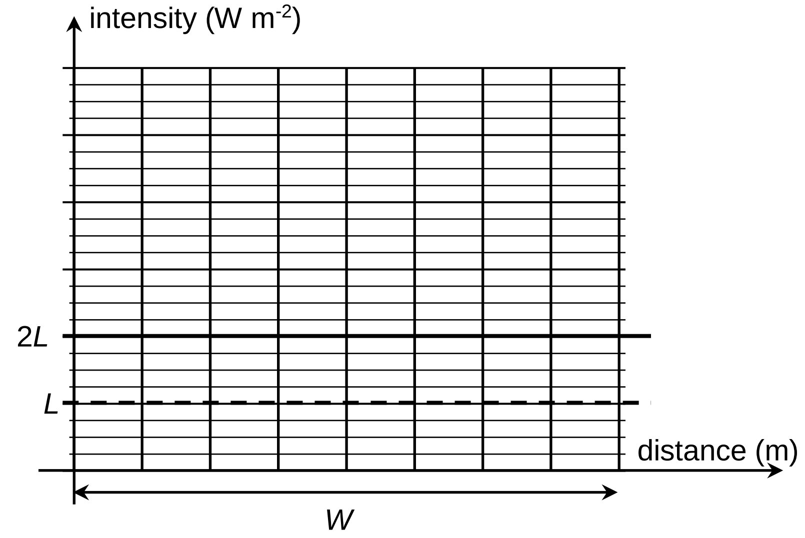
		Since gas is expanding, work done on the gas, w is negative. To keep ΔU = 0, q must be positive i.e. heat must be **added** to the gas. C to A: Temperature decreases, ΔU is negative, gas is compressed, hence w is positive. Hence q must be negative i.e. heat is **lost**.	B1 B1
		Comments: Quite well done. *Most candidates were able to clearly explain whether heat is supplied during each process, analysing work done by gas using the change in volume, the change in internal energy using change in temperature or PV, and applying First Law of Thermodynamics.* *Candidates are reminded to explain/define all notations.* *Marks are deducted for incomplete/wrong explanations and undefined notations.*	
2	**(a)**	By Newton's 3rd law, the tension force (F) acting on both springs is the same. $F = k_1 x_1 = k_2 x_2$ The total extension/compression of the springs is $e = x_1 + x_2$ It can be found that $x_1 = \dfrac{k_2}{k_1 + k_2} e$ Since the system performs simple harmonic motion, $F = ma$, where a is the acceleration of the oscillation. $k_1 x_1 = m\omega^2 e = m\left(\dfrac{2\pi}{T}\right)^2 e$ $k_1\left(\dfrac{k_2}{k_1+k_2}\right)e = m\left(\dfrac{2\pi}{T}\right)^2 e \rightarrow T = 2\pi\sqrt{\dfrac{m(k_1+k_2)}{k_1 k_2}}$	B1 B1 B1 B1
		Comments: There were a handful of remarkable, detailed solutions seen. Thank you very much! *Many only stated that the relationship for the effective spring constant connected in series and did not really show how it was derived using Newton's 2nd Law and applying simple harmonic motion.* *A handful of students did not mention at all about any forces involved in the spring-mass system in their workings. They just did a "reverse-engineering".*	
	(b)	As the objects move down the ramp, by PCOE, the gravitational potential energy will be converted to rotational kinetic energy (because they are rolling without slipping) and translational kinetic energy of the objects. The greater the moment of inertia, the greater the rotational kinetic energy. This results in less energy to be converted to translational kinetic energy. The hollow cylinder has the greatest moment of inertia (MR^2). It has the greatest rotational kinetic energy and the least translational kinetic energy. Hence it moves down the ramp the slowest.	B1 B1 B1

		Likewise, the solid sphere has the least moment of inertia ($\frac{2}{5}MR^2$). It has the least rotational kinetic energy and the greatest translational kinetic energy. Hence, it moves down the ramp the fastest. Therefore, the solid sphere (C) would reach the bottom first. (The moment of inertia of solid cylinder is $\frac{MR^2}{2}$) (Energy loss due to the rolling friction is negligible.) Alternatively, $KE_i + GPE_i = KE_f + GPE_f$ $0 + Mgh = \frac{1}{2}I_{CM}\omega^2 + \frac{1}{2}mv_{CM}^2 + 0$ $\rightarrow v_{CM} = \sqrt{\left(\frac{2gh}{1+\left(I_{CM}/MR^2\right)}\right)}$ The greater the moment of inertia, the less the centre of mass velocity and the translational kinetic energy of the object. Hence, the hollow cylinder moves down the slowest and the solid sphere moves down the fastest.	
		Comments: Many students mentioned that the GPE is converted to both rotational KE and translational KE as the objects roll down and figured out the role played by the moment of inertia in their rotations.	
3	**(a)**	Kepler's Third Law is $$\frac{4\pi^2}{T^2} = \frac{GM}{a^3}$$ 1 solar mass = $M_{solar} = \frac{4\pi^2}{GT^2}a^3 = \frac{4\pi^2}{G}\frac{(1\,\text{AU})^3}{(1\,\text{year})^2}$ Leaving everything in solar-system units, we get $$M = \frac{a^3}{T^2} = \frac{(919)^3}{(14.53)^2} = 3.68\times10^6 \text{ solar masses}$$	M1 A1
		Comments: Most could recall the Kepler's Third law which was also printed in the formulae page. A significant number of students did not cube or square the respective variables in the equations. Some were troubled in converting their answer to solar masses.	
	(b)(i)	Since M is very large, we may assume it to be stationary, while m is in orbit around M. The total energy E_{total} is the sum of the kinetic energy of m and the gravitational potential energy of the system, $$E_{total} = \frac{1}{2}mv^2 - \frac{GMm}{r}$$	A1

		Comments: Surprisingly, a significant number of students just expressed the total energy for a circular orbit in H2 physics. The question did not specify that the orbit was circular, hence a general expression for the total energy was expected.	
	(b)(ii)	It is given that, $U_{eff} = E_{total} - (KE)_r$. Decomposing the velocity v in its radial and tangential components v_r and v_t, we get $$E_{total} = \frac{1}{2}mv_r^2 + \frac{1}{2}mv_t^2 - \frac{GMm}{r}$$ Using that the angular momentum $L = r \times p = rmv \sin\theta$, where $v \sin\theta = v_t$ is the perpendicular component of the motion, we get $$E_{total} = \frac{1}{2}mv_r^2 + \frac{L^2}{2mr^2} - \frac{GMm}{r}$$ Taking $(KE)_r = \frac{1}{2}mv_r^2$, we find that $$U_{eff} = \frac{L^2}{2mr^2} - \frac{GMm}{r}$$	A1 A1 A1
		Comments: "Reversed-engineering" students had a hard time to find the effective potential. Some just threw in L into their equations without properly using the concept of angular momentum.	
	(c)(i)	Determining the escape velocity v_{esc} by applying the principle of conservation of energy, total energy at altitude of 160 000 m = total energy at infinity $$\frac{1}{2}mv^2 - \frac{GMm}{r} = 0$$ $$v_{esc} = \sqrt{\frac{2GM}{r}} = \sqrt{\frac{2(6.67\times10^{-11})(7.36\times10^{22})}{(1.6\times10^5 + 1.74\times10^6)}} = 2273 \text{ m s}^{-1}$$ Since the speed of the Apollo 11 was larger than the escape speed from an altitude of 160 000 m, the spacecraft would not stay within the Moon's gravitational well.	B1 A1
		Comments: Most of the students recognized that the Apollo 11's total energy must be zero or greater than zero and showed properly that its speed is much larger than the escape speed from the Moon's gravitational well.	
	(c)(ii)	By the principle of conservation of angular momentum, $mv_A r_A = mv_P r_P$ $(1670)(1.1\times10^5 + 1.74\times10^6) = v_A r_A$ $r_A = \frac{3.09\times10^9}{v_A}$ (in base SI units) By PCOE,	B1 B1

		$\frac{1}{2}mv_P^2 - \frac{GMm}{r_P} = \frac{1}{2}mv_A^2 - \frac{GMm}{r_A}$ $\frac{1}{2}1670^2 - \frac{(6.67\times10^{-11})(7.36\times10^{22})}{(1.1\times10^5 + 1.74\times10^6)} = \frac{1}{2}v_A^2 - \frac{(6.67\times10^{-11})(7.36\times10^{22})}{3.09\times10^9}v_A$ Solving the quadratic equation yields $v_A = 1510 \text{ m s}^{-1}$	B1 A1
		Comments: There were plenty of well-written solutions with clear presentation of the principles of conservation of angular momentum and energy. Those who did not use the concept of angular momentum in their workings had a very tough time with this question.	

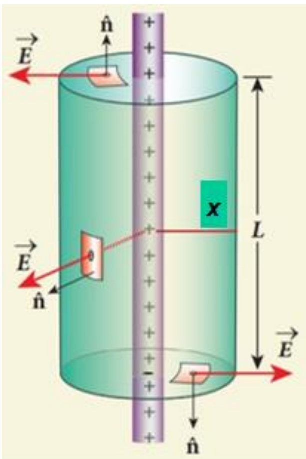
4	**(a)**	$\lambda = c \div f = (3.00\times10^8)\div(1500\times10^3) = 200 \text{ m}$ $\Delta y = \frac{L\lambda}{d} = \frac{(22000)(200)}{2500} = 1760 \text{ m}$ $W = 4\Delta y = 4(1760) = 7040 \text{ m}$	B1 B1 A1
		Comments: Since Fig. 4.2 shows the maxima to be regularly spaced, we know we can use the fringe spacing formula. *Some students used pythagoras to work out the path difference. The same answer is obtained, but it was a lot more work.*	
	(b)	It is the intensity detected by the receiver when one transmitter is operating.	B1
		Comments: None.	
	(c)	 Voltage of 60 V → Amplitude of 1A → Intensity of 1L Voltage of 90 V → Amplitude of 1.5A Voltage of 30 V → Amplitude of 0.5A C.I. → $1.5A + 0.5A = 2.0A$ → $4L$ D.I. → $1.5A - 0.5A = 1.0A$ → L	B1 B1 A1

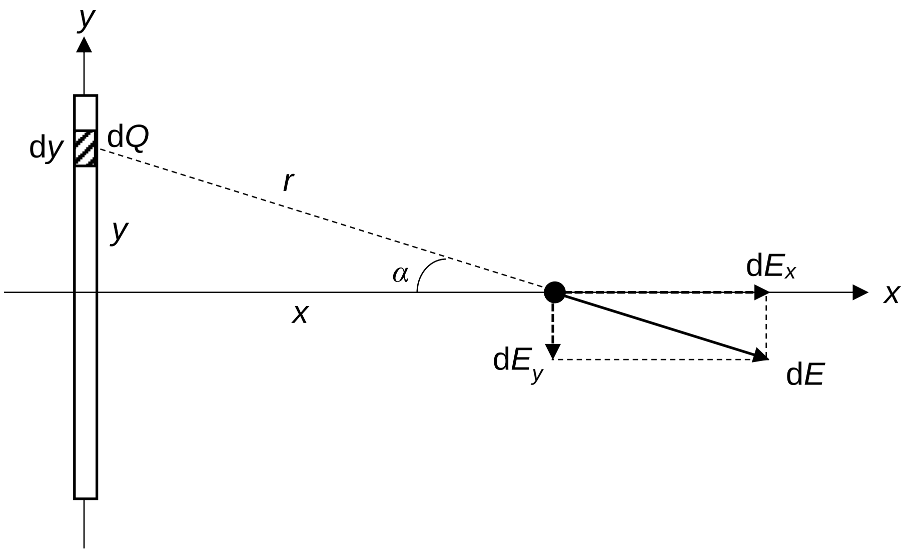
		Comments: This part requires students to realize that the voltage is proportional to the amplitude, not intensity, of the emitted wave. This deduction can be made by noting that it should be V^2/R that corresponds to power and thus intensity.	
	(d)		A1
		Comments: Without interference between the two waves, the resultant intensity is simply the simple summation of the individual instensities.	
	(e)	When not interfering, the average intensity is 2L. When interfering, the average intensity is $\frac{0+4L}{2}=2L$. Superposition does not violate PCOE. OR The area under the graph are the same for Fig. 4.2 and Fig. 4.4. This shows that the total power is constant.	B1
		Comments: None.	
5	**(a)**	1. $p\sin\phi=\frac{h}{\lambda}\sin\theta$ 2. $\frac{h}{\lambda_0}=\frac{h}{\lambda}\cos\theta+p\cos\phi$	A2
		Comments: $0=p\sin\phi+\frac{h}{\lambda}\sin\theta$ *is not accepted because p represents the magnitude of the electron's momentum. It cannot be the vector notation of momentum. If not* $\sin\phi$ *and* $\sin\theta$ *would not make sense.*	
	(b)	Since some of the energy is passed to the electron, the photon should have less energy and thus longer wavelength.	B1
		Comments: I decided to reject answers that argued that the photon lost momentum (instead of energy). While it clear that the photon must have lost energy (since the electron gained KE), it is not so clear that the photon lost momentum (it is clear that the momentum changed direction, but how do we know that its magnitude become smaller?).	

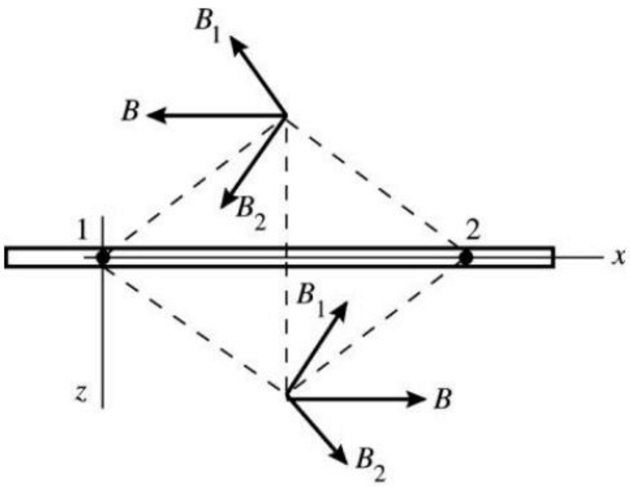
	(c)(i)	$\Delta\lambda = \frac{h}{mc}(1-\cos\theta)$ $= \frac{6.63\times10^{-34}}{(9.11\times10^{-31})(3.00\times10^{8})}(1-\cos180°)$ $= 4.852\times10^{-12}$ m $\lambda_{max} = 6.91\times10^{-11} + 4.852\times10^{-12} = 7.40\times10^{-11}$ m	B1 A1
		Comments: Half the students thought maximum increase in wavelength occurs at $\theta = 90°$. *Exam stress perhaps?*	
	(c)(ii)	$\Delta E = \frac{hc}{\lambda_0} - \frac{hc}{\lambda_{max}} = (6.63\times10^{-34})(3.00\times10^{8})(\frac{1}{6.91\times10^{-11}} - \frac{1}{7.40\times10^{-11}})$ $\Delta E = 1.91\times10^{-16}$ J	B1 A1
		Comments: With ECF, most students scored full marks for this part.	
	(c)(iii)	$\text{impulse} = \Delta p = \left(p_f - \left(-p_i\right)\right) = p_i + p_f$ $= \frac{h}{\lambda_0} + \frac{h}{\lambda} = (6.63\times10^{-34})(\frac{1}{6.91\times10^{-11}} + \frac{1}{7.40\times10^{-11}}) = 8.96\times10^{-24}$ N s	B1 A1
		Comments: None.	
6	(a)	The objects in a closed system are **only interacting** with each other and there is no net external force acts on the system.	B1
		Comments: Many could explain the meaning of closed system very well.	
	(b)(i)	For PCOLM to hold, the total momentum of the ball should remain constant and hence, no change in momentum should take place, i.e., $\Delta p_{ball} = 0$. Take $(\rightarrow +)$ $\Delta p = m(-v) - m(+v) = -2mv \neq 0$. Therefore, the momentum of the ball is not conserved.	B1 B1
		Comments: A number of students had difficulties in showing that the total momentum of the ball had changed. Some just used velocities, without mentioning the momentum in their workings.	
	(b)(ii)	The wall is at rest before the collision. RSOA=RSOS $v - 0 = v_{wall} - v^{f}_{ball} \rightarrow v_{wall} = v + v^{f}_{ball}$ by PCOLM, $mv + 0 = Mv_{wall} + mv^{f}_{ball} \rightarrow v_{wall} = \frac{mv - mv^{f}_{ball}}{M}$	B1 B1

		$v_{wall} = \frac{mv - mv_{ball}^f}{M} = v + v_{ball}^f$ $mv - mv_{ball}^f = Mv + Mv_{ball}^f$ $v_{ball}^f = \frac{m-M}{m+M}v = -\frac{M-m}{M+m}v$ $v_{wall} = v + \frac{m-M}{m+M}v = \frac{mv+Mv+mv-Mv}{m+M} = \frac{2m}{M+m}v$	A1 A1
		Comments: Except a handful of weaker students, this part is very well done. A number of students left their final answer not in the simplest form.	
	(b)(iii)	As the wall is infinitely larger than the ball (*m*<<*M*), $v_{ball}^f = -v\frac{M-m}{M+m} \to -v$ $v_{wall} = \frac{2mv}{M+m} \to 0$	B1
		Comments: This part is well done.	
	(c)(i)	$\frac{Mv_c^2}{2} + \frac{mv^2}{2} = \Delta E_{\text{int}}$ $\frac{(m+M)u^2}{2} + \Delta E_{\text{int}} = \frac{M(u-v_c)^2}{2} + \frac{m(v+u)^2}{2}$	A1 A1
		Comments: This part is well done.	
	(c)(ii)	In S^0 frame, $\Delta KE_{cannon,\ \text{S}^0\text{frame}} = \frac{1}{2}Mv_c^2 - 0 = \frac{1}{2}Mv_c^2$ In *S* frame, $\Delta KE_{cannon,\ \text{S frame}} = \frac{1}{2}M(u-v_c)^2 - \frac{1}{2}Mu^2 = \frac{1}{2}M(-2uv_c + v_c^2)$ The results are not the same. Hence, the change in kinetic energy of the cannon in the two inertial frames of reference is not the same. We could conclude that the change in kinetic energy depends on the inertial reference frame, i.e., it is *not invariant*.	A1 A1 B1
		Comments: Although it was sufficient to show that the change in kinetic energy of the cannon in both frames is not the same, a number of students either could not conceptualize or left this part blank. *Some students wanted to prove that the change in kinetic energy of the cannon in S frame depended on u, which is the speed of S frame. If there was no calculation error or u was briefly defined, this method was also accepted.*	

	(c)(iii)	In S^0 frame, $\frac{Mv_c^2}{2}+\frac{mv^2}{2}=\Delta E_{\text{int}}$ In S frame, $\frac{(m+M)u^2}{2}+\Delta E_{\text{int}}=\frac{M(u-v_c)^2}{2}+\frac{m(v+u)^2}{2}$ $\Delta E_{\text{int}}=\frac{M(u-v_c)^2}{2}+\frac{m(v+u)^2}{2}-\frac{(m+M)u^2}{2}$ $\Delta E_{\text{int}}=\frac{M(u^2-2uv_c+v_c^2)+m(v^2+2vu+u^2)-mu^2-Mu^2}{2}$ Simplify mu^2 , Mu^2 and using PCOLM, $0=mv-Mv_c \rightarrow mv=Mv_c$, we get $\Delta E_{\text{int}}=\frac{-2Mv_cu+2mvu+Mv_c^2+mv^2}{2}=\frac{Mv_c^2+mv^2}{2}$ The result, ΔE_{int}, in S frame is equal to the one in S^0 frame. Hence, ΔE_{int} is independent of an inertial reference frame, i.e., is invariant.	B1 M1 B1
		Comments: There were plenty of good presentation of answers. Thank you! *Some did not use the principle of conservation of linear momentum properly.* *The students were expected to show that the change in internal energy in both frames are the same. Yes, it was that easy. However, a number of students left this part blank.*	

7	**(a)(i)**	$\lambda=\frac{Q}{2a}$	B1
		Comments: Yay! Everyone scored this mark.	
	(a)(ii)	Consider a Gaussian cylinder of radius x and length $2a$, Applying Gauss's law: $\oint \vec{E}\cdot d\vec{A}=\frac{Q_{in}}{\varepsilon_0}$ $E_x(2\pi x)(2a)=\frac{\lambda(2a)}{\varepsilon_0}$ Hence, the electric field at the position (x, 0, 0), where $x<<a$, is given by $E_x=\frac{\lambda}{2\pi\varepsilon_0 x}$ 	B1 B1 A1
		Comments: Although generally well done, students are reminded to include the chosen Gaussian surface and present the Gauss's law clearly to answer a "show" question.	

	(a)(iii)	From diagram: $y = x\tan\alpha \rightarrow y^2 = x^2\tan^2\alpha \rightarrow y^2 + x^2 = x^2\left(1+\tan^2\alpha\right) = x^2\sec^2\alpha$ $\dfrac{dy}{d\alpha} = x\sec^2\alpha$, $r^2 = \sqrt{x^2+y^2}$  From symmetry, the <u>y-component of the electric field is cancelled</u> and only the *x*-component, dE_x remains. $dE_x = \dfrac{\lambda dy\cos\alpha}{4\pi\varepsilon_o\left(y^2+x^2\right)} = \dfrac{\lambda x\sec^2\alpha d\alpha\cos\alpha}{4\pi\varepsilon_o x^2\sec^2\alpha} = \dfrac{\lambda\cos\alpha d\alpha}{4\pi\varepsilon_o x}$ $E_x = \dfrac{\lambda}{4\pi\varepsilon_o x}\int_{-\alpha_a}^{\alpha_a}\cos\alpha d\alpha = \dfrac{\lambda}{4\pi\varepsilon_o x}\left[\sin\alpha\right]_{-\alpha_a}^{\alpha_a} = \dfrac{\lambda}{2\pi\varepsilon_o x}\dfrac{a}{\sqrt{a^2+x^2}}$	B1 B1 B1 B1 A1
		Comments: It is crucial that students deduce from the given diagram or even the expression E_x given in the question that only the electric field in the x direction needs to be considered. Some students could not complete the integration to arrive at the final expression required. With more practice and exposure, I believe students will be able to observe a pattern in such integration for different configurations and overcome this difficulty.	
	(a)(iv)	For an infinitely long line / length of charge ($x \ll a$), we can ignore x^2/a^2 in the denominator, then $E_x = \dfrac{\lambda}{2\pi\varepsilon_o x}\dfrac{a}{\sqrt{a^2+x^2}} = \dfrac{\lambda}{2\pi\varepsilon_o x}\dfrac{a}{\sqrt{a^2\left(1+\dfrac{x^2}{a^2}\right)}} = \dfrac{\lambda}{2\pi\varepsilon_o x}\dfrac{1}{\sqrt{1}} = \dfrac{\lambda}{2\pi\varepsilon_o x}$ This is the magnitude of the field of an infinitely long charged wire at any point at a perpendicular distance *x* from the charged wire in any direction.	B1 B1
		Comments: Generally well done except for some students who probably forgot that they have to present how the two E_x expressions are actually equivalent in this context.	

	(b)(i)	The field is the same strength at both ends of the dipole, so we can just use the torque equation. The field is axially inward to the line of charge, which means that it is perpendicular to the dipole moment, so the magnitude of the torque: $$\tau = p\,E \sin 90^{o} = kp\lambda/r$$	B1
		Comments: A few students did not realise that only the magnitude is needed. Generally well managed part.	
	(b)(ii)	The field from the long line of charge is not uniform – it is stronger closer to the line. Therefore the "front" of the dipole, which is closer to the line of charge, feels a strong force than the "rear." The line of charge is negative, which means the front of the dipole (which is the positive charge) is attracted more than the rear is repelled, and the dipole feels a net force toward the line of charge.	B1
		Comments: This part was not well done. Perhaps these students had not interpreted the question correctly to arrive at the correct answer due to time constraints.	
	(b)(iii)	The force on each charge has a magnitude of qE, and the force acts in the direction of the displacement for both charges, hence the change in potential energy, $$\Delta U_{dipole} = -2W_{(A\rightarrow B)} = -2Fx = -2(qE)(d/2)(\cos\theta) = -(qd)E\cos\theta$$	B1
		Comments: While most students understood the need to find the difference between the final and initial potential energies, some did not have the negative sign in their final expression.	
	(c)(i)	Consider the individual currents in pairs, where the currents in each pair are equidistant on either side of the point where ***B*** is being calculated. Figure below shows that for each pair the z-components cancel, and that above the sheet the field is in the -x-direction and that below the sheet it is in the +x-direction. 	B1
		Comments: Generally well done.	
	(c)(ii)	By symmetry, the magnitude of ***B*** a distance *a* above the sheet must be equal to the magnitude of ***B*** a distance *a* below the sheet. Apply Ampere's law, taking a rectangular path, *I* is directed out of the page, so for *I* to be positive the integral around the path is taken in the anti-clockwise direction.	

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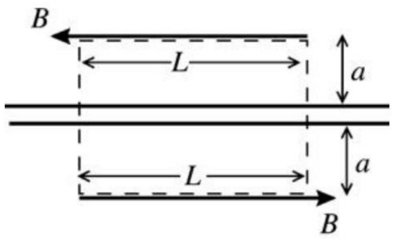

$$\oint \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{l}} = \mu_0 I_{\text{encl}}$$

Since B is parallel to the sheet, on the sides of the rectangle that have length 2*a*, $\oint \vec{\boldsymbol{B}} . \vec{d\boldsymbol{l}} = 0$. On the long sides of length *L*, ***B*** is parallel to the side, in the direction we are integrating around the path, and has the same magnitude, ***B***, on each side.

Thus $\oint \vec{\boldsymbol{B}} . \vec{d\boldsymbol{l}} = 2BL$, *n* conductors per unit length and current *I* out of the page in each conductor gives $I_{\text{encl}} = InL$. Hence

$$\oint \vec{\boldsymbol{B}} . \vec{d\boldsymbol{l}} = 2BL = \mu_o InL$$

$$\therefore \quad B = \frac{\mu_o In}{2}$$
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<td>*Comments: There were students who wrong magnetic field directions. A few seemed to confuse their Ampere's Law expression with Gauss's Law's. For students attempting B1 topic questions, you are required to be well-versed in these 2 laws as well as their usage in order to score well.*</td>
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<td>**(c)(iii)**</td>
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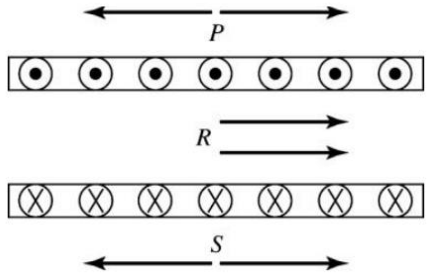

B is independent of the distance from the sheets, as deduced from (ii).

Above the 2 sheets at *P*, the fields cancel. Hence no net *B* field.

In between the 2 sheets at R, the 2 fields add up to yield $B = \mu_o nI$ to the right.

Below the 2 sheets at *S*, the fields again cancel. Hence no net *B* field.

The 2 sheets with currents in opposite directions produce a uniform B field between the sheets and zero field outside the 2 sheets.
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<td>*Comments: There were a few students who left this page blank. Perhaps the diagram had looked deceivingly challenging and intimidated them. A few had their answers reversed for P & S with that for R!*</td>
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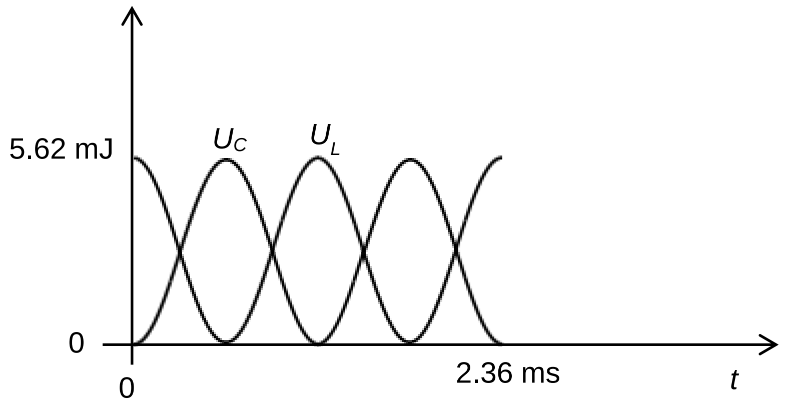
8	**(a)**	Moment of inertia of a rod through one end = $\frac{ML^2}{3}$ (given in data and formula pages) Using parallel-axis theorem Moment of inertia of a rod through its centre of mass + $M\left(\frac{L}{2}\right)^2 = \frac{ML^2}{3}$ Moment of inertia of a rod through its centre of mass = $\frac{ML^2}{12}$ OR $\delta I = r^2\delta m = r^2\frac{M}{L}\delta r$ $I = \frac{M}{L}\int_{-L/2}^{L/2} r^2 dr = \frac{M}{L}\frac{r^3}{3}\Big\|_{-L/2}^{L/2} = \frac{ML^2}{12}$	B1 A1
		Comments: Most students managed to answer this question correctly.	
	(b)(i)	Using the parallel-axis theorem: Moment of inertia of rod about pivot = $\frac{ML^2}{12} + M\left(\frac{L}{4}\right)^2$ $= \frac{7ML^2}{48} = \frac{7(0.300)(2.00)^2}{48} = 0.175$ kg m^{-2}	B1 B1
		Comments: Most students managed to answer this question correctly.	
	(b)(ii)	By conservation of angular momentum, $m_{gum}v_{gum,before}\frac{l}{4} = m_{gum}v_{gum,after}\frac{l}{4} + I_{rod}\omega$ $(0.100)(40.0)(0.50) = (0.100)(\omega\frac{l}{4}) + (0.175)\omega$ $\omega = 10.0$ rad s^{-1} OR Impulse on gum X distance to pivot + angular impulse on rod = 0 (0.100)((0.50)ω - 40.0)(0.50) + (0.175)(ω- 0) = 0 ω = 10.0 rad s^{-1}	M1 B1
		Comments: Most students managed to answer this question correctly.	

	(b)(iii)1	Find the moment of inertia of combined bodies ($I_{comb.}$) by using the parallel-axis theorem: = 0.175 + 0.1x0.5^2 = 0.200 kg m^2 By PCOE, $\frac{1}{2}I_{comb.}\omega^2+0=\frac{1}{2}I_{comb.}\omega_{new}^2+m_{comb.}g\left(\frac{L}{4}-\frac{L}{4}\cos 60°\right)$ $\frac{1}{2}(0.200)(10.0)^2=\frac{1}{2}(0.200)\omega_{new}^2+(0.400)(9.81)(0.5)(1-\cos 60°)$ $\omega_{new}=9.50\text{ rad s}^{-1}$	B1 B1 B1
		Comments: Some students mistakenly wrote "sin60° " instead of "cos60° ".	
	(b)(iii)2	$\tau=mg\sin\theta\frac{L}{4}=I\alpha$ $\alpha=\dfrac{(0.400)(9.81)\sin 60°\left(\frac{2.00}{4}\right)}{0.200}=8.50\text{ rad s}^{-2}$	B1 A1
		Comments: Some students mistakenly wrote "sin60° " instead of "cos60°".	
	(b)(iii)3	centripetal acceleration = $a_c=\omega^2\frac{L}{4}=(9.50)^2(0.5)=45.1\text{ m s}^{-2}$ tangential acceleration = $a_t=\alpha\frac{L}{4}=(8.50)(0.5)=4.25\text{ m s}^{-2}$ resultant acceleration = $a=\sqrt{(a_c)^2+(a_t)^2}=\sqrt{45.1^2+4.25^2}=45.3\text{ m s}^{-2}$	A1 A1 A1
		Comments: Some students forgot about the centripetal acceleration.	
	(b)(iii)4	velocity of centre of mass v_{cm} = 0.5 x ω_2 = 4.75 m s^{-1} vertical velocity of centre of mass = 4.75 x sin 60° = 4.11 m s^{-1} time taken for same level = 4.11 x 2 /9.81 = 0.838 s number of revolutions = 0.838 x 9.5 / 2π = 1.27 rev = 456° = 360 ° + 96° It rotates 96° in ACW direction from its position in Fig 8.4. The rod will be at approx. (180 ° – (96° + 60°) = 24 °) with respect to the vertical and the top end will be B.	B1 B1 B1 A1 A1 A1
		Comments: This is a difficult part. Some students did not resolve the velocity of the centre of mass in a vertical direction before applying the kinematic eqn in a vertical direction.	

9	**(a)(i)**	0 V.	A1
		Comments: The inductor is a short-circuit, hence no voltage across it.	
	(a)(ii)	$I = \dfrac{50.0}{250} = 0.200$ A	A1
		Comments: Good.	
	(b)(i)	Energy alternates between magnetic field energy stored in the inductor and electric field energy stored in the capacitor. $\frac{1}{2}CV_{\text{max}}{}^2 = \frac{1}{2}LI_{\text{max}}{}^2$ $\frac{1}{2}(0.500\times10^{-6})(150)^2 = \frac{1}{2}L(0.200)^2$ L = 0.281 H	B1 B1 A1
		Comments: When an explanation is sought, usually some English sentence is required.	
	(b)(ii)	When current is half the maximum value, energy stored in the inductor is ¼ the total energy. So ¾ of the total energy is stored in the capacitor. $U_C = \frac{3}{4}U_{total}$ $\frac{q^2}{2C} = \frac{3}{4}\left(\frac{1}{2}\times0.281\times0.200^2\right)$ $q = 6.49\times10^{-5}$ C	B1 B1 A1
		Comments: Well-attempted. Some used the i-t and q-t equations to arrive at the same answer.	
	(b)(iii)	$2\pi f_0 = \frac{1}{\sqrt{LC}}$ $f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.281\times0.500\times10^{-6}}}$ $f_0 = 425$ Hz	B1 A1
		Comments: This is a recall question. Well-done.	

	(b)(iv)	$U_{total} = \frac{1}{2}(0.281)(0.200)^2 = 5.62$ mJ $T = \frac{1}{424.6} = 2.36$ ms 	B1 B1 A2
		Comments: Two cycles per period for the energy graphs. Some confused the two graphs – at t = 0 there is no voltage across the capacitor, hence no energy stored in it.	
	(c)(i)	$\frac{dU}{dt} = -i^2R$ $U = \frac{1}{2}Li^2 + \frac{1}{2C}q^2$, hence $\frac{dU}{dt} = Li\frac{di}{dt} + \frac{q}{C}\frac{dq}{dt}$ $Li\frac{di}{dt} + \frac{q}{C}\frac{dq}{dt} + i^2R = 0$ $L\frac{d^2i}{dt^2} + \frac{i}{C} + \frac{di}{dt}R = 0$ $\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0$ (shown)	B1 A1
		Comments: Some proceeded on the basis of Kirchoff voltage law, which is fine since it is fundamentally a conservation of energy law.	
	(c)(ii)	$\frac{di}{dt} = -\gamma I_{max} e^{-\gamma t}\cos(\omega_D t) - \omega_D I_{max} e^{-\gamma t}\sin(\omega_D t)$ $\frac{d^2i}{dt^2} = \gamma^2 I_{max} e^{-\gamma t}\cos(\omega_D t) + 2\omega_D I_{max} e^{-\gamma t}\sin(\omega_D t) - \omega_D{}^2 I_{max} e^{-\gamma t}\cos(\omega_D t)$ Group terms in sin and simplifying: $2\gamma - \frac{R}{L} = 0$ Group terms in cos and simplifying: $\gamma^2 - \omega_D{}^2 - \frac{R}{L}\gamma + \frac{1}{LC} = 0$	B1 B1 B1

		$\omega_D{}^2 = \frac{1}{LC} - \gamma^2$ $\gamma = \frac{R}{2L}$ $\omega_D = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$	A1
		Comments: Just brute force will do.	