NATIONAL JUNIOR COLLEGE
SENIOR HIGH 2
Preliminary Examination

NAME

SUBJECT CLASS: 2ma2

REGISTRATION NUMBER

H2 MATHEMATICS — 9758 / 01

28 August 2019

3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF26)
Writing Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your name, class and registration number in the boxes above.
Please write clearly and use capital letters.

Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, glue or correction fluid.

Answer **all** the questions.

Write your answers in the spaces provided in the question paper.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an approved graphing calculator is expected, where appropriate.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers. Up to 2 marks may be deducted for improper presentation.

The number of marks is given in the brackets [ ] at the end of each question or part question.

Question Number	Marks Possible	Marks Obtained
1	6	
2	6	
3	7	
4	7	
5	8	
6(a)	3	
6(b)	6	
7	10	
8	10	
9	12	
10	12	
11	13	
Presentation Deduction		–1 / –2
TOTAL	100	

This document consists of **29** printed pages and **3** blank pages.

[Turn over

1 The complex number z has modulus r and argument θ, where $0 < r < 1$ and $\frac{\pi}{2} < \theta < \pi$. The complex number w is such that $\left|\frac{w}{z}\right| = 1$ and $\arg(w) + \arg(z) = \pi$.

Let the points A, B, C, D and E represent the complex numbers z, w, $z - w$, z^* and zz^* respectively, where z^* denotes the conjugate of z.

(i) On a single Argand diagram, illustrate these five points. [5]

(ii) Identify the shape of the quadrilateral $ACDE$. [1]

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2 **Do not use a calculator in answering this question.**

Solve the inequality $\dfrac{2x^2+5x+9}{x-7} \leq x-4$. [4]

Hence, solve the inequality $\dfrac{2+5\mathrm{e}^{x}+9\mathrm{e}^{2x}}{7\mathrm{e}^{x}-1}\geq 4\mathrm{e}^{x}-1$. [2]

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3 Two sightseeing points A and B are located offshore. A cruise ship S travels along a straight path passing through A and B. A lighthouse is located at the shore. If the lighthouse's position is taken as the origin O, the position vectors of A and B are **a** and **b** respectively.

(i) If $AS : SB = \lambda : (1-\lambda)$, find the position vector of S, giving your answer in terms of λ, **a** and **b**. [1]

When the cruise ship is closest to the lighthouse,

(ii) show that $\lambda = \dfrac{\mathbf{a}\cdot(\mathbf{a}-\mathbf{b})}{(\mathbf{b}-\mathbf{a})\cdot(\mathbf{b}-\mathbf{a})}$, [2]

(iii) find its position vector in terms of **a** and **b**, if sightseeing point A is three distance units away from the light house, sightseeing point B is two distance units away from the lighthouse and the angle between **a** and **b** is $60°$. [4]

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4 **(i)** Show that $\dfrac{px^2+(4p-q)x+(4p+q)}{(1-x)(2+x)^2}=\dfrac{p}{1-x}+\dfrac{q}{(2+x)^2}$. [1]

(ii) Find the values of p, q and r such that the series expansion of

$$\frac{px^2+(4p-q)x+(4p+q)}{(1-x)(2+x)^2}+r\ln(1-3x)$$

up to and including the term in x^2 is $11-3x+x^2$. [4]

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(iii) Find the range of the values of x for which the expansion is valid. [2]

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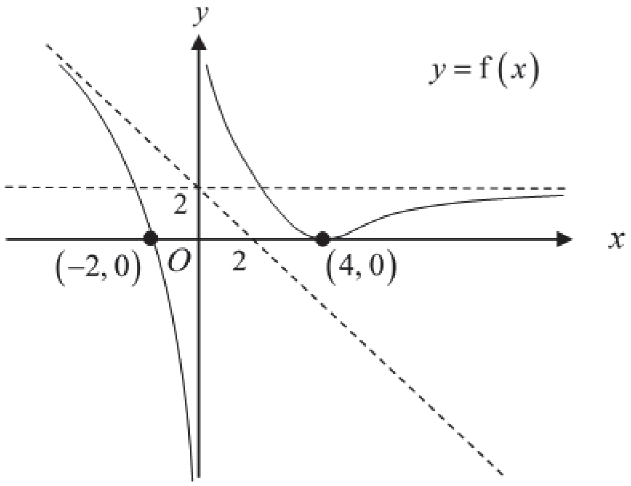
5



The diagram shows a sketch of the curve $y = \mathrm{f}(x)$. The curve cuts the x-axis at (–2, 0) and it has a stationary point at (4, 0). The equations of the asymptotes are $x = 0$, $y = 2$ and $y = 2 - x$.

Sketch, in separate diagrams, the graphs of

(i) $y = \dfrac{1}{\mathrm{f}(x)}$, [3]

(ii) $y = \mathrm{f}'(x)$, [3]

stating the equations of any asymptotes and coordinates of axial intercepts.

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In the curve $y = \mathrm{f}(ax+b)$, there exists an oblique asymptote $y = 4-2x$, find the values of a and b. [2]

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6 **(a)** Relative to the origin *O*, a point *A* has position vector $6\mathbf{i}+t\mathbf{j}+10\mathbf{k}$, where *t* is a real and negative constant. Given that the direction cosine of *A* with respect to the *x*-axis is $\frac{3}{\sqrt{35}}$, find the value of *t*. Deduce the angle between $\overrightarrow{OA}$ and the *yz*-plane. [3]

(b) The planes p_1 and p_2 have equations $\mathbf{r}\cdot\begin{pmatrix}2\\-2\\1\end{pmatrix}=8$ and $\mathbf{r}\cdot\begin{pmatrix}-2\\2\\-1\end{pmatrix}=k$ respectively, where k is a real constant. It is given that the plane p is equidistant from p_1 and p_2.

(i) Find, in terms of k, a vector equation of p in the scalar product form. [3]

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[Turn over

(ii) Given $k = 13$, find the coordinates of the point on line l with equation

$$\frac{x-1}{3} = \frac{2y-1}{18} = \frac{3-z}{1}$$

that is equidistant from p_1 and p_2. [3]

7 Let $\mathrm{f}(x)=\dfrac{x^2+3x+4-a}{x+a}$, where $0<a<\dfrac{3}{2}$.

Sketch, in the separate diagrams, the graphs of

(i) $y=\mathrm{f}(x)$, [3]

(ii) $y=\left|\mathrm{f}(x)\right|$, [3]

stating the equations of the asymptotes and coordinates of the axial intercepts, in terms of a.

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[Turn over

(iii) It is given that $|\mathrm{f}(x)| = m$ has exactly two distinct roots. Find the range of values of m in terms of a. [4]

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[Turn over

8 A curve C has parametric equations

$$x = t^2 - 4t, \quad y = 3t^2 - t^3, \quad t \leq 0 .$$

(i) Sketch C. [1]

A point P on C has parameter p.

(ii) Show that the tangent at P has equation

$$y = -\frac{3}{2}px + \frac{1}{2}p^3 - 3p^2 .$$ [3]

The tangent at P makes an angle of $45°$ with the x-axis.

(iii) Find the exact coordinates of P. [2]

(iv) Find the area of the region bounded by the curve C, its tangent at P and the x-axis. [4]

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9 Functions f and g are defined by

$$\mathrm{f} : x \mapsto 2|x-p|+1 \text{ for } x \in \mathbb{R},\ p>1,$$

$$\mathrm{g} : x \mapsto x(x-q) \text{ for } x \in \mathbb{R},\ q<0.$$

(i) Explain why f does not have an inverse. [2]

The domain of f is now restricted to $x \le k$.

(ii) Write down the largest value of k for which the function f^{-1} exists. Hence find $\mathrm{f}^{-1}(x)$ and state the domain of f^{-1}. [4]

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(iii) Sketch on the same diagram the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$, giving the coordinates of the axial intercepts. Hence solve $\mathrm{f}(x) = \mathrm{f}^{-1}(x)$. [4]

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(iv) Find the range of $\mathrm{g}\,\mathrm{f}^{-1}$. [2]

10 A construction company, which has just won a contract to build foundations at two different sites I and II, wants to buy a new borewell drilling machine. Two brands, MTZ and STK, are considered and tested. To test a machine, it is used to drill a hole for ten hours.

The following data were recorded.

MTZ: The depth drilled is 4.5 m in the first hour, and for each subsequent hour, the depth drilled decreases by a positive constant r%.

STK: The depth drilled is 4.7 m in the first hour, and for each subsequent hour, the depth drilled decreases by 0.1 m.

(i) Show that the total depth drilled by an MTZ borewell drilling machine is

$$\frac{450}{r}\left[1-\left(1-\frac{r}{100}\right)^{10}\right] \text{ m.}$$ [2]

(ii) Find the value of r if the total depth drilled by both machines are the same. [2]

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It is given that the machines perform the same way as recorded at the test site.

(iii) The company chooses a MTZ borewell drilling machine to build a foundation at Site I. Assuming $r = 1$, to maximise the efficiency of the machine, the company will stop drilling if the depth to be drilled in the next hour is less than 4.0 m at this site. How many complete hours should the machine be operated? [2]

Before drilling starts at Site II, the engine of the MTZ borewell drilling machine breaks down.

(iv) To save costs, the company considers to buy a new engine from an alternative manufacturer. The manufacturer claims that with the new engine, the MTZ borewell drilling machine will be able to drill a hole to a theoretical maximum depth of at least 360 m if it is operated continuously. Find the range of values of r for which the alternative manufacturer's claim is false. [3]

(v) The company changes its mind and decides to buy a STK borewell drilling machine to drill the hole at Site II instead. Suppose that the machine performs the same way at Site II as at the test site, explain clearly whether the STK borewell drilling machine will be able to drill a hole of depth 150 m if it is operated continuously. [3]

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11 [It is given that a cone of radius r with height h and slant edge l has lateral surface area πrl and volume $\frac{1}{3}\pi r^2 h$.]

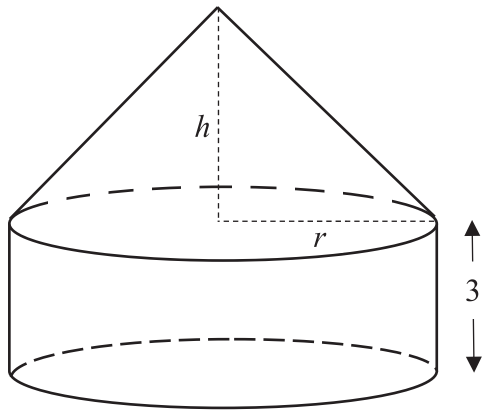

Figure 1

A circus tent consists of a cylindrical shape of radius r metres and height 3 metres and a right conical roof of height h metres as shown in Figure 1.

The conical roof has a fixed volume, V , and a lateral surface area, S, which varies with its height and radius.

(i) Show that $S^2 = \dfrac{9V^2 + \pi^2 r^6}{r^2}$. [3]

(ii) Using differentiation, find the ratio $h : r$ where the least amount of material is used for the conical roof . Justify that this ratio corresponds to the least amount of material used. [5]

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The circus tent is firmly anchored to a horizontal ground using taut ropes as shown in Figure 2. Each rope is inclined at an angle of $\frac{\pi}{6}$ with the ground and makes an angle of $\frac{\pi}{3}$ with the perpendicular height of the cylindrical shape of the tent.

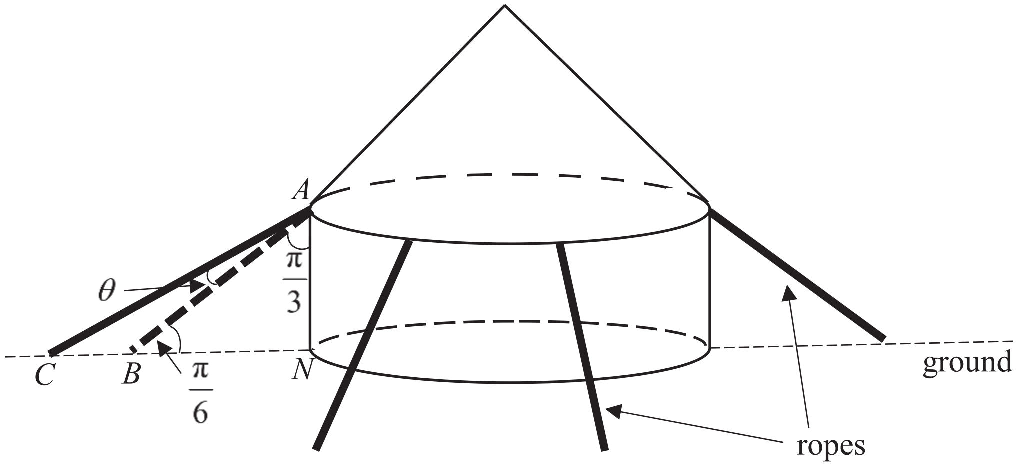

Figure 2

When anchoring a rope to the ground from point A to point B, an error of θ radians was made resulting in the rope being anchored on the ground at point C instead. Point N lies directly below point A such that points C, B and N are collinear.

(iii) Show that the length of the rope AC is $\dfrac{3}{\sin\left(\dfrac{\pi}{6}-\theta\right)}$ metres. [2]

(iv) To rectify the error, part of the rope AC is to be cut off so that the remaining part can be anchored tautly on the ground at point B.

Given that θ is a sufficient small angle, deduce an approximation for the length of the rope to be cut off, in ascending powers of θ up to and including the term in θ^2. [3]

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NATIONAL JUNIOR COLLEGE
SENIOR HIGH 2
Preliminary Examination

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SUBJECT CLASS	2ma2	REGISTRATION NUMBER	

H2 MATHEMATICS

9758 / 02

17 September 2019

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3	10	
4	13	
5	5	
6	6	
7	7	
8	9	
9	9	
10	12	
11	12	
Presentation Deduction		–1 / –2
TOTAL	100	

This document consists of **26** printed pages and **2** blank pages.

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Section A: Pure Mathematics [40 marks]

1 It is given that

$$f(x) = \begin{cases} ax, & \text{for } 0 \le x < 2, \\ \dfrac{12a}{x} - 4a, & \text{for } 2 \le x < 3, \end{cases}$$

where a is a positive constant and that $f(x) = f(x+3)$ for all real values of x.

(i) Find the exact value of $f\left(\dfrac{44}{3}\right)$ in terms of a. [2]

(ii) Sketch the graph of $y = f(x)$ for $-3 \le x \le 3$, indicating clearly the coordinates of the maximum points and axial intercepts. [3]

(iii) Find $\int_{-2}^{2} \mathrm{f}\left(x\right) \mathrm{d}x$ in terms of a, leaving your answer in the exact form. [3]

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2 Find the general solution of the differential equation

$$\frac{d^2 y}{dx^2} = \frac{2}{\sqrt{1-4x^2}}, \text{ where } -\frac{1}{2} < x < \frac{1}{2}.$$ [5]

The particular solution curve $y = \mathrm{f}(x)$ has a minimum point at the origin.

(i) Find $\mathrm{f}(x)$. [2]

(ii) Sketch the graph of this particular solution. [2]

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[Turn over

3 **Do not use a calculator in answering this question.**

(a) **(i)** Given that $x+y\mathrm{i}=\sqrt{8-6\mathrm{i}}$, determine the possible values of x and y, where x and y are real numbers. [2]

(ii) Solve $w^2+2\mathrm{i}w-9+6\mathrm{i}=0$, giving your answer in the form $a+b\mathrm{i}$, where a and b are real numbers. [2]

(b) **(i)** Show that if z_1 is a root of the equation $z^4 - pz^2 + q = 0$, where p and q are constants, then $-z_1$ is a also a root of this equation. [1]

(ii) Verify that $3\mathrm{e}^{\mathrm{i}\left(\frac{\pi}{6}\right)}$ is a root of the equation $z^4 - 9z^2 + 81 = 0$. Hence express $z^4 - 9z^2 + 81$ as the product of two quadratic expressions in z with exact real coefficients. [5]

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4 If k is a nonzero integer, find $\int_{-\pi}^{\pi} \sin kx \, \mathrm{d}x$ and $\int_{-\pi}^{\pi} \cos kx \, \mathrm{d}x$. [2]

Let m and n be positive integers.

(i) Show that

$$\int_{-\pi}^{\pi} \sin(mx)\sin(nx)\,\mathrm{d}x = \begin{cases} \pi & \text{if } m = n, \\ 0 & \text{if } m \neq n. \end{cases}$$ [4]

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(ii) By considering the formula for $\cos(mx+nx)$, evaluate

$$\int_{-\pi}^{\pi} \cos(mx)\cos(nx)\,\mathrm{d}x\,.$$ [2]

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[Turn over

Suppose a is a positive integer.

(iii) Show that $\int_{-\pi}^{\pi} \left|\sin(ax)\right| \, dx = 4$. [2]

Hence, find the exact volume of the solid formed when the region bounded by the curve $y=\left|\sin\left(ax\right)\right|+1$, the x-axis and the lines $x=-\pi$ and $x=\pi$, is rotated completely about the x-axis. [3]

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[Turn over

Section B: Probability and Statistics [60 marks]

5 The random variable X is the number of successes in n independent trials of an experiment in which the probability of a success at any one trial is p.

(i) Show that $\dfrac{\mathrm{P}(X=k+1)}{\mathrm{P}(X=k)}=\dfrac{(n-k)p}{(k+1)(1-p)}$, where $k=0,1,2,\ldots,(n-1)$. [2]

(ii) Using the result in part **(i)**, find the most probable number of successes when $n = 99$ and $p = \dfrac{7}{9}$. [3]

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[Turn over

6 A group of 8 people consists of 4 married couples.

(a) The group stands in a line. Find the number of different possible orders in which no two men stand next to each other. [2]

(b) The group stands in a circle. Find the number of different possible orders in which each man stands next to his wife. [2]

(c) The group forms a committee consisting of two teams of four people each. Find the number of ways that the committee can be formed such that neither team consists of only men or women. [2]

7 A survey was conducted with a large number of employees who were asked to select one SkillsFuture programme that they would like to enroll in to further develop their skills. The survey showed that 29% would like to enroll in human resource programme, $100p$% would like to enroll in financial management programme and the rest would like to enroll in infocomm technology programme.

Fifteen employees who took part in the survey were randomly selected. Find the probability that

(i) exactly twelve employees said that they would like to enroll in either infocomm technology programme or financial programme, [2]

(ii) no less than five employees said that they would like to enroll in human resource programme. [2]

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[Turn over

Suppose now that four employees who took part in the survey are randomly selected, write down an equation in terms of p such that the probability of one or two employees saying that they would like to enroll in financial management programme is 0.3. Hence find the value of p. [3]

8 Joe has won a game. For his prize, he is given k identical sealed envelopes out of which he is allowed to open exactly three and keep their contents. Three of the envelopes each contain \$1 and the rest each contain \$10. He chooses three envelopes at random. Let \$$X$ be the amount of prize money that he receives when he wins the game once.

The probability distribution of X is given in the table below

x	3	12	21	a
$\mathrm{P}(X=x)$	0.05	0.45	0.45	0.05

(i) Find the values of a and k. [2]

(ii) Explain the concept of randomness in the context of this question. [1]

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[Turn over

Joe won the game n times where $n \geq 20$.

(iii) Find the least value of n such that the probability that the mean amount of prize money that Joe receives is within \$2 of the expected amount of money that he will receive after winning one game, is greater than 0.95. [6]

9 *A temperature anomaly* is the change in global surface temperature relative to the average temperature from Year 1951 to 1980.

The table gives the temperature anomaly in degree celsius from Year 2011 to 2018.

t, number of years since 1980	31	32	33	34	35	36	37	38
Temperature Anomaly, θ°C	0.57	0.61	0.64	0.73	0.86	0.98	0.90	0.82

source: https://climate.nasa.gov

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(i) Draw a scatter diagram for these values, labelling the axes. [2]

(ii) Calculate the value of the product moment correlation coefficient for this set of data. [1]

(iii) State, giving a reason, which of the least squares regression lines, θ on t or t on θ, should be used to express a possible linear relation between θ and t. [1]

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[Turn over

(iv) Calculate the equation of the regression line chosen in part **(iii)**. Hence, interpret, in context, the value of the gradient of the regression line. [2]

(v) Use the regression line to predict the temperature anomaly for Year 2025. Comment on the reliability of this estimate. [2]

(vi) Explain why in this context a linear model would probably not be suitable for long term predictions. [1]

10 To determine if a patient has heart disease, a test called *cardiac fluoroscopy* is used. This test checks for narrowing of heart arteries due to build-up of calcium. Suppose that a *cardiac fluoroscopy* test gives either a positive or negative result. Let

T_0 be the event of obtaining a negative test result indicating that no artery is narrowing,

T_1 be the event of obtaining a positive test result indicating that at least one artery is narrowing,

S be the event that heart disease is present in the patient, and

S' be the event that heart disease is not present in the patient.

Based on studies, the conditional probabilities are given in the following table.

i	$\mathrm{P}(T_i \mid S)$	$\mathrm{P}(T_i \mid S')$
0	0.42	q_1
1	q_2	0.04

(i) Find q_1 and q_2. [2]

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[Turn over

Suppose $\mathrm{P}(S) = p$.

(ii) Show that $\mathrm{P}(S | T_0) = \dfrac{7p}{16 - 9p}$. [3]

(iii) Prove by differentiation that $\mathrm{P}(S | T_0)$ is an increasing function for $0 < p < 1$, and explain what this statement means in the context of the question. [3]

(iv) It is further given that $p = 0.3$ for a group of patients. One randomly chosen patient from this group takes the *cardiac fluoroscopy* test twice and both test results are positive. Find the probability that the patient has heart disease. You may assume the test results are independent of each other. [4]

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[Turn over

11 In view of the rise in the number of accidents involving Personal Mobility Devices (PMDs) in the past year, regulations on PMDs are being tightened to step up safety for pedestrians on footpaths. Since February 2019, the speed limit for riding a PMD on footpaths has been lowered to 10 km/h. In June 2019, a concerned citizen believes that the mean speed of PMD riders still exceeds the new speed limit. To test his belief, a team of enforcement officers capture the speeds, x km/h, of a random sample of 250 PMD riders at five hotspots. The findings are summarised by

$$\sum(x-10)=38.9, \qquad \sum(x-10)^2=390.31.$$

Assume that the speed of a PMD rider follows a normal distribution.

(i) Calculate unbiased estimates of the population mean and variance of the speed of PMD riders on footpaths. [2]

(ii) Test at the 4% level of significance whether the belief should be accepted. [4]

(iii) Explain, in the context of the question, the meaning of 'at the 4% level of significance'. [1]

(iv) Without any further test, explain if the conclusion in part **(ii)** will change if instead we are testing that the mean speed of PMD riders on footpaths is not 10 km/h. [1]

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Three months later, to investigate if the mean speed of PMD riders on footpaths differs from the regulated speed limit, another large random sample is collected at the same hotspots. It is found that the mean speed captured is 9.5 km/h, with standard deviation 3.2 km/h.

(v) Find the smallest sample size that will provide significant evidence at the 5% level of significance that the mean speed of PMD riders on footpaths differs from the regulated speed limit. [4]

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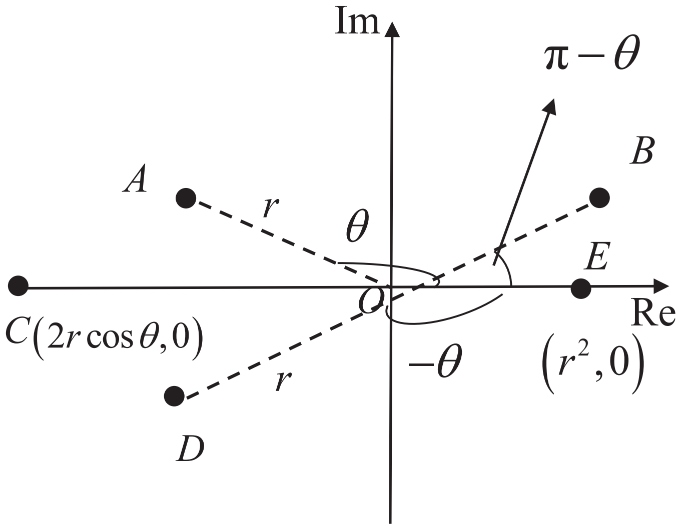
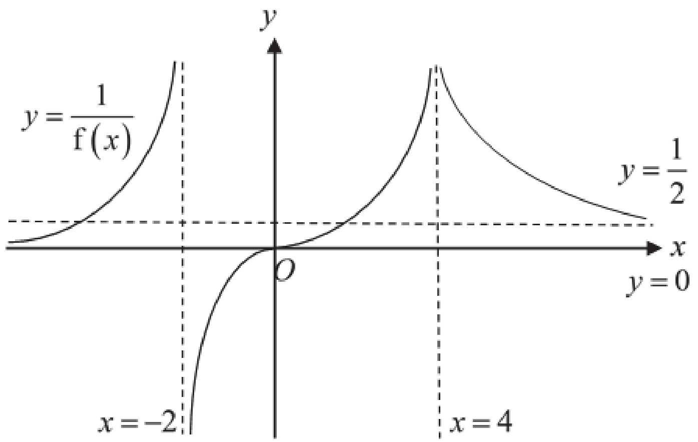
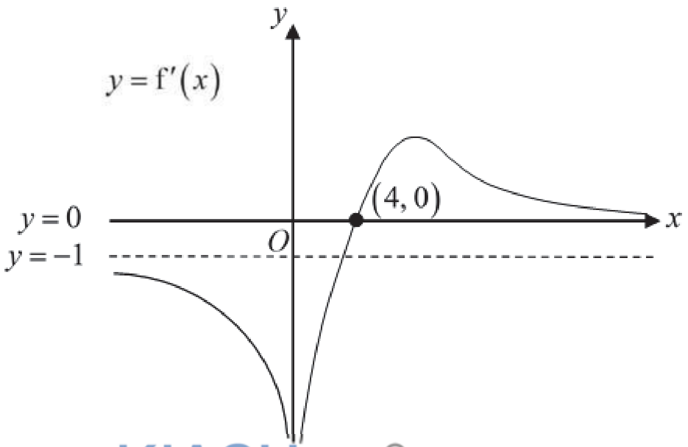
1	**Suggested Solution**	**Comments**
(i)	 	Most students are not able to figure out how to find $z-w$. Method 1 Let $z=a+bi$ then $w=-a+bi$ (due to symmetry from diagram) We know that the $z=re^{i\theta}=a+bi=r\cos\theta+i(r\sin\theta)$ So real component of z is $r\cos\theta$. $z-w=a+bi-(-a+bi)=2a=2r\cos\theta$ Method 2 $z-w=re^{i\theta}-re^{i(\pi-\theta)}=r\left(e^{i\theta}-e^{i(\pi-\theta)}\right)$ $=r\left(e^{i\theta}-e^{i(-\theta)}e^{i\pi}\right)=r\left(e^{i\theta}-(-1)e^{i(-\theta)}\right)$ $=r\left(e^{i\theta}+e^{i(-\theta)}\right)=r\left(\cos\theta+i\sin\theta+\cos(-\theta)+i\sin(-\theta)\right)$ $=r(\cos\theta+i\sin\theta+\cos\theta-i\sin\theta)$ $=2r\cos\theta$ Note that the lines joining O to the different points e.g. A, are dotted. Points A and B need to be symmetrical about imaginary axis and point points A and D need to be symmetrical about the real axis and BOD should be collinear.
(ii)	$ACDE$ is a kite.	Please note the geometrical properties of a kite e.g. two pairs of equal-length sides that are adjacent to each other.

2	Suggested Solution	Comments
(i)	$$\frac{2x^2+5x+9}{x-7}\le x-4$$ $$\frac{2x^2+5x+9}{x-7}-(x-4)\le 0$$ $$\frac{2x^2+5x+9-(x-4)(x-7)}{x-7}\le 0$$ $$\frac{2x^2+5x+9-x^2+11x-28}{x-7}\le 0$$ $$\frac{x^2+16x-19}{x-7}\le 0$$ $$\frac{(x+8)^2-83}{x-7}\le 0$$ $$\frac{\left(x+8+\sqrt{83}\right)\left(x+8-\sqrt{83}\right)}{x-7}\le 0$$ $$x\le -8-\sqrt{83} \text{ or } \sqrt{83}-8\le x<7$$	Firstly, GC is no allowed for this question as per instruction. Do take note! Do not cross multiply, i.e. $2x^2+5x+9\le(x-4)(x-7)$. One cannot be sure of the sign for $x-7$. $x\le -8-\sqrt{83}$ or $\sqrt{83}-8\le x<7$ cannot be the final answers. $x=7$ must be excluded because $\frac{2x^2+5x+9}{x-7}$ cannot admit 7!
(ii)	Replace x with e^{-x}, $\frac{2e^{-2x}+5e^{-x}+9}{e^{-x}-7}\le e^{-x}-4$ $$\frac{2e^{-x}+5+9e^{x}}{1-7e^{x}}\le e^{-x}-4$$ $$\frac{2+5e^{x}+9e^{2x}}{7e^{x}-1}\ge 4e^{x}-1$$ $$e^{-x}\le -8-\sqrt{83}\ (\text{reject}) \text{ or } \sqrt{83}-8\le e^{-x}<7$$ $$\ln\left(\sqrt{83}-8\right)\le -x<\ln 7$$ $$-\ln 7<x\le -\ln\left(\sqrt{83}-8\right)$$	Let $x=e^{-x}$. This is very poor presentation. Use a different variable and go for $u=e^{-x}$ say. $e^{-x}\le -8-\sqrt{83}\ (\text{reject})$ or $\sqrt{83}-8\le e^{-x}<7$ We are solving an equality so answer must be in terms of x.

3	Suggested Solution	Comments
(i)	By Ratio Theorem, $\overrightarrow{OS}=\dfrac{\lambda\mathbf{b}+(1-\lambda)\mathbf{a}}{\lambda+(1-\lambda)}=\mathbf{a}+\lambda(\mathbf{b}-\mathbf{a})$	Most students were able to apply the Ratio Theorem here.
(ii)	If the cargo ship is closest, $\overrightarrow{OS}\perp\overrightarrow{AB}$ and thus, $\overrightarrow{OS}\cdot\overrightarrow{AB}=0$. Consequently, $\overrightarrow{OS}\cdot\overrightarrow{AB}=(\mathbf{a}+\lambda(\mathbf{b}-\mathbf{a}))\cdot(\mathbf{b}-\mathbf{a})$ $0=\mathbf{a}\cdot(\mathbf{b}-\mathbf{a})+\lambda(\mathbf{b}-\mathbf{a})\cdot(\mathbf{b}-\mathbf{a})$ $\lambda=\dfrac{\mathbf{a}\cdot(\mathbf{a}-\mathbf{b})}{(\mathbf{b}-\mathbf{a})\cdot(\mathbf{b}-\mathbf{a})}$	Although students are generally able to identify that the vectors must be perpendicular and thus, $\overrightarrow{OS}\cdot\overrightarrow{AB}=0$. Manipulation of the vector algebra is generally convoluted as students expands the terms unnecessarily. Some students used incorrect vector notation for $\mathbf{b}\cdot\mathbf{b}$ and write $\mathbf{b}^2$, which is meaningless.
(iii)	$\overrightarrow{OS}=\mathbf{a}+\lambda(\mathbf{b}-\mathbf{a})$ $=\mathbf{a}+\dfrac{\mathbf{a}\cdot(\mathbf{a}-\mathbf{b})}{(\mathbf{b}-\mathbf{a})\cdot(\mathbf{b}-\mathbf{a})}(\mathbf{b}-\mathbf{a})$ $=\mathbf{a}+\dfrac{\mathbf{a}\cdot\mathbf{a}-\mathbf{a}\cdot\mathbf{b}}{\mathbf{b}\cdot\mathbf{b}+\mathbf{a}\cdot\mathbf{a}-2\mathbf{a}\cdot\mathbf{b}}(\mathbf{b}-\mathbf{a})$ $=\mathbf{a}+\dfrac{\lvert\mathbf{a}\rvert^2-\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\cos(60^\circ)}{\lvert\mathbf{b}\rvert^2+\lvert\mathbf{a}\rvert^2-2\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\cos(60^\circ)}(\mathbf{b}-\mathbf{a})$ $=\mathbf{a}+\dfrac{9-(3)(2)\left(\frac{1}{2}\right)}{9+4-2(3)(2)\left(\frac{1}{2}\right)}(\mathbf{b}-\mathbf{a})$ $=\mathbf{a}+\dfrac{6}{7}(\mathbf{b}-\mathbf{a})$ $=\dfrac{1}{7}\mathbf{a}+\dfrac{6}{7}\mathbf{b}$	

4	Suggested Solution	Comments
(i)	$\dfrac{p}{1-x}+\dfrac{q}{(2+x)^2}=\dfrac{p(2+x)^2+q(1-x)}{(1-x)(2+x)^2}$ $=\dfrac{4p+4px+px^2+q-qx}{(1-x)(2+x)^2}$ $=\dfrac{px^2+(4p-q)x+(4p+q)}{(1-x)(2+x)^2}$	Many students tried to do partial fraction method which is a waste of time. Note that you can show from left to right or vice versa.
(ii)	$\dfrac{px^2+(4p-q)x+(4p+q)}{(1-x)(2+x)^2}+r\ln(1-3x)$ $=\dfrac{p}{1-x}+\dfrac{q}{(2+x)^2}+r\ln(1-3x)$ $=p(1-x)^{-1}+q\left[2^{-2}\left(1+\dfrac{x}{2}\right)^{-2}\right]+r\ln(1-3x)$ $=p\left(1+x+x^2+...\right)$ $+\dfrac{1}{4}q\left[1+(-2)\left(\dfrac{x}{2}\right)+\dfrac{(-2)(-3)}{1\cdot 2}\left(\dfrac{x}{2}\right)^2+...\right]$ $+r\left[-3x-\dfrac{(-3x)^2}{2}+...\right]$ $\approx p\left(1+x+x^2\right)+\dfrac{1}{4}q\left(1-x+\dfrac{3}{4}x^2\right)+r\left(-3x-\dfrac{9}{2}x^2\right)$ $=\left(p+\dfrac{1}{4}q\right)+\left(p-\dfrac{1}{4}q-3r\right)x+\left(p+\dfrac{3}{16}q-\dfrac{9}{2}r\right)x^2$ $\equiv 4-4x+x^2$ Comparing coefficients,	Note the factoring of 2 out gives 2^{-2} and there is no need for maclaurin's series expansion – juse use the Binomial series expansion. Note the $\dfrac{x}{2}$ in $\left(1+\dfrac{x}{2}\right)^{-2}$.

	$\begin{cases} p+\frac{1}{4}q+0r=11 \\ p-\frac{1}{4}q-3r=-3 \\ p+\frac{3}{16}q-\frac{9}{2}c=1 \end{cases}$ By GC, $p=7, q=16, r=2$	
(iii)	$\lvert -x \rvert < 1$ and $\left\lvert \frac{x}{2} \right\rvert < 1$ and $-1 < -3x \le 1$ Thus $-\frac{1}{3} \le x < \frac{1}{3}$	Please present in such a way to show the marker that you are considering the intersection of values in three inequalities as well as the use of 'and'.

5	Suggested Solution	Comments
(i)		Generally well done. **Common mistakes:** 1. Graph did not approach the x-axis as $x \to -\infty$. 2. Did not label $y=0$. 3. Did not label the origin / x/y intercept.
(ii)		Generally well done. **Common mistakes:** 1. Graph did not approach $y=-1$ as $x \to -\infty$. 2. Did not label $x=0$ and $y=0$. 3. Curve not smooth (weird curvature) at (4, 0).
	$f(x)=2-x$ Replace x by $x-2$ $f(x-2)=4-x$ Replace x by $2x$ $f(2x-2)=4-2x$ Hence $a=2,\ b=-2$.	Badly done. Most students are unable to identify the correct replacement technique to transform the equation of the asymptote.

6	Suggested Solution	Comments
(a)	$\left\lvert\overrightarrow{OA}\right\rvert^2 = 6^2+t^2+10^2 = 136+t^2$ $\cos\theta = \dfrac{6}{\sqrt{136+t^2}} = \dfrac{3}{\sqrt{35}}$ $\dfrac{36}{136+t^2} = \dfrac{9}{35}$ $1260 = 1224+9t^2$ $9t^2 = 36$ $t = \pm 2$ $t = -2 \quad (\because t<0)$ Angle with the yz-plane is $90^\circ - \cos^{-1}\dfrac{3}{\sqrt{35}} \approx 30.5^\circ$	You must be familiar with the definition and formula of Direction Cosine. If point A has a position vector $\overrightarrow{OA}$, then the direction cosine along the x, y and z-axis is given by $\dfrac{\overrightarrow{OA}}{\left\lvert\overrightarrow{OA}\right\rvert}\cdot\mathbf{i}$, $\dfrac{\overrightarrow{OA}}{\left\lvert\overrightarrow{OA}\right\rvert}\cdot\mathbf{j}$, $\dfrac{\overrightarrow{OA}}{\left\lvert\overrightarrow{OA}\right\rvert}\cdot\mathbf{k}$. $\cos\theta$ is positive so θ acute with θ being the angle between the vector and the x-axis. With some visualisation, one can see that the angle of the yz plane is $90^\circ - \cos^{-1}\theta$. Although the question says deduce, some of you computed the angle of a line and a plane using existing methods.
(b) (i)	$(4,0,0)$ is a point on p_1, $(0,0,-k)$ is a point on p_2. The midpoint $(2,0,-0.5k)$ is on p. A normal vector of p is $\begin{pmatrix}2\\-2\\1\end{pmatrix}$, so a vector equation of p is $\mathbf{r}\cdot\begin{pmatrix}2\\-2\\1\end{pmatrix} = \begin{pmatrix}2\\0\\-0.5k\end{pmatrix}\cdot\begin{pmatrix}2\\-2\\1\end{pmatrix} = 4-0.5k$ i.e. $\mathbf{r}\cdot\begin{pmatrix}2\\-2\\1\end{pmatrix} = 4-0.5k$	There are many approaches to this • Using ratio theorem • Using the distance between plane and origin • Using the distance between 2 planes

(b) **(ii)**	Let N be the point of intersection of l and p. $\frac{x-1}{3}=\frac{2y-1}{18}=\frac{3-z}{1} \Rightarrow \frac{x-1}{3}=\frac{y-0.5}{9}=\frac{z-3}{-1}$ Vector equation of l: $\mathbf{r}=\begin{pmatrix}1\\0.5\\3\end{pmatrix}+\lambda\begin{pmatrix}3\\9\\-1\end{pmatrix},\ \lambda\in\mathbb{R}$. Vector equation of p: $\mathbf{r}\cdot\begin{pmatrix}2\\-2\\1\end{pmatrix}=4-0.5(13)=-2.5$ Let $\overrightarrow{ON}=\begin{pmatrix}1+3\lambda\\0.5+9\lambda\\3-\lambda\end{pmatrix}$ for some $\lambda\in\mathbb{R}$. $\begin{pmatrix}1+3\lambda\\0.5+9\lambda\\3-\lambda\end{pmatrix}\cdot\begin{pmatrix}2\\-2\\1\end{pmatrix}=-2.5$ $2+6\lambda-1-18\lambda+3-\lambda=-2.5$ $-13\lambda=-6.5$ $\lambda=0.5$ $\overrightarrow{ON}=\begin{pmatrix}1+3(0.5)\\0.5+9(0.5)\\3-0.5\end{pmatrix}$ The coordinates are $(2.5, 5, 2.5)$	Part (ii) is dependent on (i). If the equation of p is found in (ii), a point that is common on l and p will be equidistant between p_1 and p_2. However, the question can still be solved if the equation of p is not found i.e. by considering the distance between l with p_1 and l with p_2. Since l is not parallel to either p_1 or p_2, we can find a point on l such that it is equidistance to p_1 and p_2.

<table>
<tr><th>7</th><th>Suggested Solution</th><th>Comments</th></tr>
<tr><td>(i)</td><td>

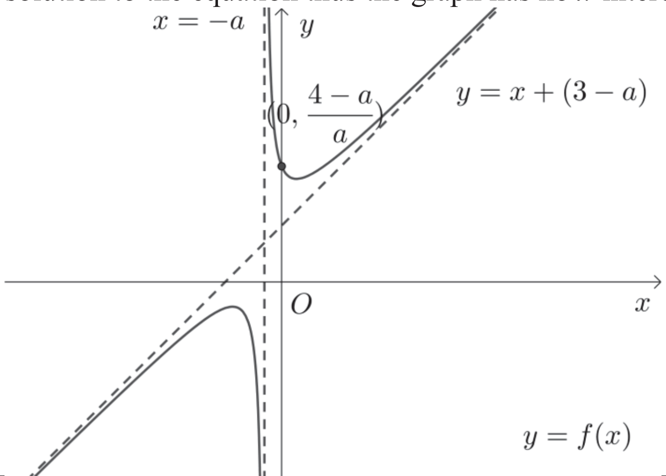
$\mathrm{f}(x)=\dfrac{x^2+3x+4-a}{x+a}=x+(3-a)+\dfrac{a^2-4a+4}{x+a}$

The asymptotes are $x=-a$ and $y=x+(3-a)$

For y-intercept, let $x=0$. Then $y=\dfrac{4-a}{a}$.

For x-intercept, let $y=0$. Then $x^2+3x+4-a=0$.

But $3^2-4(4-a)=9-16+4a=4a-7<0$ as $0<a<1.5$, there is no solution to the equation thus the graph has no x-intercept.



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Students need to master long division with unknowns at the denominator.

Students are to recognise that f is a rational function consisting of a vertical and an oblique asymptote.

Common mistake:

1. Coordinates of intercept not labelled.

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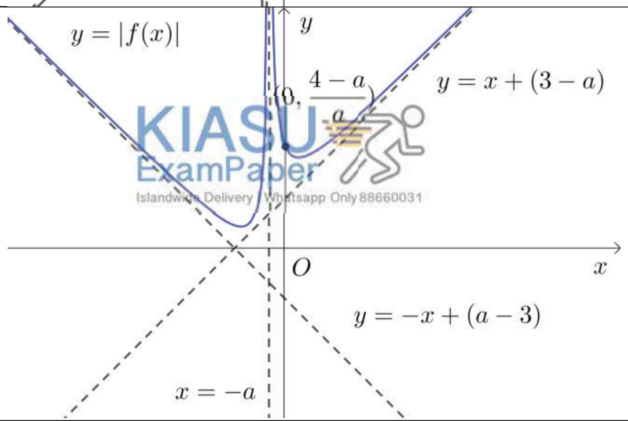

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Almost all students sketched the graph for $y=|\mathrm{f}(x)|$ correctly.

Common mistakes:

1. Equation of the reflected asymptote found wrongly.
2. The reflected asymptote should be a reflection of the original oblique asymptote about the x-axis.

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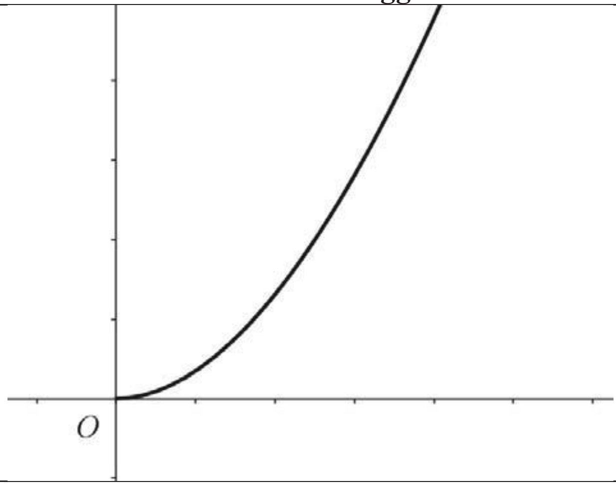
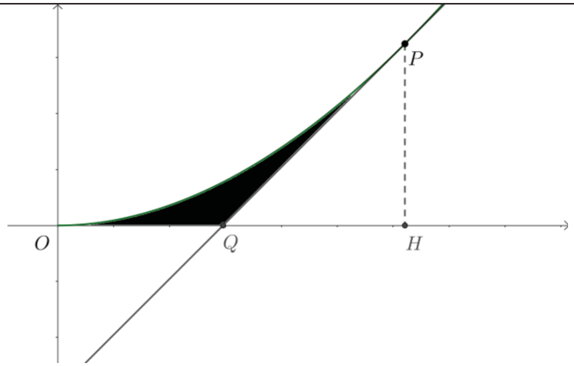
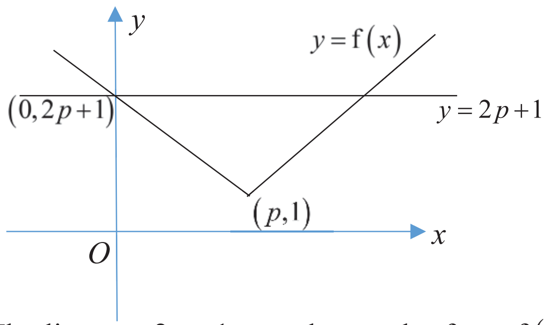
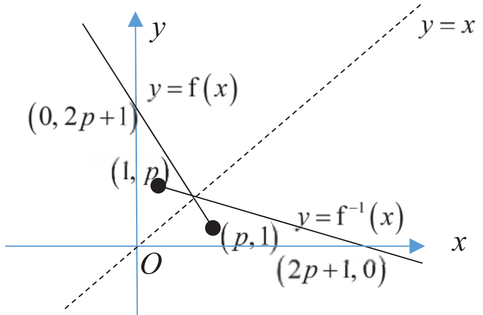
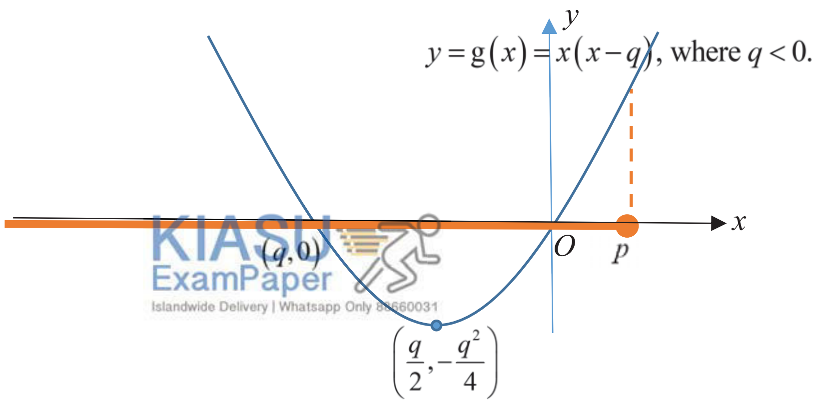
(iii)	Method 1 $\mathrm{f}(x)=x+(3-a)+\dfrac{a^2-4a+4}{x+a}$ $=x+(3-a)+\dfrac{(a-2)^2}{x+a}$ At stationary points, $\mathrm{f}'(x)=1-\dfrac{(a-2)^2}{(x+a)^2}=0$ $(x+a)^2=(a-2)^2$ $x+a=a-2$ or $x+a=-(a-2)$ $x=-2$ or $x=2-2a$ Correspondingly, the y-coordinates of the points: $\mathrm{f}(-2)=-2+(3-a)+\dfrac{(a-2)^2}{a-2}=1-a+a-2=-1$ $\mathrm{f}(2-2a)=(2-2a)+(3-a)+\dfrac{(a-2)^2}{(2-2a)+a}$ $=5-3a+\dfrac{(a-2)^2}{2-a}$ $=5-3a+2-a$ $=7-4a$ On $y=\lvert\mathrm{f}(x)\rvert$, the stationary points are $(-2,1)$ and $(2-2a,7-4a)$. Since $0<a<\dfrac{3}{2}$ $\therefore 7-4a>1$. Hence the line $y=m$ cuts the graph of $y=\lvert\mathrm{f}(x)\rvert$ at two distinct points	From the graph in **(ii)**, students should differentiate and find the stationary points to determine the range of values of m. However, many students differentiated wrongly, likely due to error in long division in the first part of the question.

	when $1 < m < 7-4a$. Method 2 Consider the equation $\mathrm{f}(x) = m$ to have no real root $\dfrac{x^2+3x+4-a}{x+a} = m$ $x^2+3x+4-a = mx+am$ $x^2+(3-m)x+(4-a-am) = 0$ $(3-m)^2-4(4-a-am)$ $= m^2-6m+9-16+4a+4am$ $= m^2+(4a-6)m+(4a-7)$ $= (m-(-1))\left[m-(7-4a)\right] < 0$ $-1 < m < 7-4a$ Referring to the graph of $y = \mathrm{f}(x)$ and $y = \lvert \mathrm{f}(x) \rvert$, since $0 < a < \dfrac{3}{2}$ $\therefore 7-4a > 1$. We can deduce that $1 < m < 7-4a$ for $\lvert \mathrm{f}(x) \rvert = m$ to have exactly two distinct roots.	Most students used Method 2. However, students are not sure what to do with the modulus sign. They should realise that the modulus can be removed to simplify their calculation.

8	Suggested Solution	Comments
(i)		This part is not done well by some of the students. **Common mistakes:** 1. Forget to set the values of t in GC. 2. Fail to show the feature that the only x-intercept of the curve is at the origin by making the curve lie on the x-axis for a segment. Students need to remember to adjust the window setting in GC to 1. sketch any parametric curve, 2. display key features clearly on the screen.
(ii)	$x = t^2 - 4t, \quad y = 3t^2 - t^3, \quad t \le 0$ $\frac{dx}{dt} = 2t - 4, \frac{dy}{dt} = 6t - 3t^2, \frac{dy}{dx} = \frac{6t - 3t^2}{2t - 4} = \frac{-3t(t-2)}{2(t-2)} = -\frac{3t}{2}$ At P, the equation of tangent is $y - (3p^2 - p^3) = -\frac{3}{2}p\left[x - (p^2 - 4p)\right]$ $y = -\frac{3}{2}px + \frac{3}{2}p^3 - 6p^2 + 3p^2 - p^3$ $y = -\frac{3}{2}px + \frac{1}{2}p^3 - 3p^2$	This part is well done by most of the students.
(iii)	From the sketch of C, the gradient of the tangent must be 1. $-\frac{3}{2}p = 1 \Rightarrow p = -\frac{2}{3}$. The coordinates of P are $\left(\frac{28}{9}, \frac{44}{27}\right)$	This part is not done well by many students. **Common mistakes:** 1. Wrongly assume the ratio of y to x is 1 for the gradient of the tangent. 2. Fail to make use the result of part **(ii)**. 3. Forget to find the coordinates after finding the parameter value.

	 Area $= \int_0^{\frac{28}{9}} y \, \mathrm{d}x -$ area of triangle PQH $= \int_0^{-\frac{2}{3}} \left(3t^2 - t^3\right)\left(2t-4\right) \mathrm{d}t - \frac{1}{2}\left(\frac{44}{27}\right)\left(\frac{44}{27}\right)$ $\approx 1.731687 - 1.327846$ $= 0.403841$ ≈ 0.404	This part is poorly done. **Common mistakes:** 1. See the region as simply an area under the parametric curve. 2. See the region as an area between the curve and the line over the same interval 3. Cannot recall the formula to find the area under a parametric curve. 4. Make mistakes in calculating the height or base of the triangle. In addition, whenever a question does not specify 'exact' or 'without using a calculator', it is encouraged to use a GC to evaluate the definite integral numerically, to save time.

9	Suggested Solution	Comments
(i)	 The line $y=2p+1$ cuts the graph of $y=\mathrm{f}(x)$ twice. Hence f is not one-one function and its inverse does not exist. OR The line $y=l$, where $l>1$, cuts the graph of $y=\mathrm{f}(x)$ twice. Hence f is not one-one function and its inverse does not exist.	Students need to be mindful that when using graphical method to show whether f is one-one function, they need to sketch the graph of f and the horizontal line on the same diagram. **Common Mistakes** 1. Did not label the important feature of the graph of f. In this case, the vertex and the axial intercept of f. 2. Did not sketch the mentioned horizontal line on the same diagram as graph of f. 3. Did not state the range of l where $y=l$ cuts the graph of f at least once.
(ii)	Largest value of k is p. Based on restricted domain of f i.e. $x\le p$, such that inverse of f exists, $y=2\lvert x-p\rvert+1$ $\Rightarrow y=-2(x-p)+1$ $\Rightarrow y-1=-2(x-p)$ $\Rightarrow x-p=\dfrac{1-y}{2}$ $\therefore x=p+\dfrac{1-y}{2}$ $\therefore \mathrm{f}^{-1}(x)=p+\dfrac{1-x}{2}$ $\therefore \mathrm{D}_{\mathrm{f}^{-1}}=\mathrm{R}_{\mathrm{f}}=[1,\infty)$	Need to learn to choose the right expression for f based on the restricted domain of f. $\lvert x-p\rvert=\begin{cases}(x-p), & \text{where } (x-p)\ge 0 \Leftrightarrow x\ge p\\ -(x-p), & \text{where } (x-p)<0 \Leftrightarrow x<p\end{cases}$

(iii)	 	Graph of f and inverse of f must be symmetrical about the line $y = x$. **Common Mistakes:** 1. Did not sketch the line of reflection $y = x$ 2. Not aware that image of of (x, y) reflected about the line $y = x$ gives (y, x).
	$\mathrm{f}(x)=\mathrm{f}^{-1}(x)$ $\Rightarrow \mathrm{f}^{-1}(x)=x$ $\Rightarrow p+\frac{1-x}{2}=x$ $\Rightarrow 2p+1-x=2x$ $\Rightarrow 3x=2p+1$ $\therefore x=\frac{2p+1}{3}$ OR $\mathrm{f}(x)=\mathrm{f}^{-1}(x)$ $\Rightarrow \mathrm{f}(x)=x$ $\Rightarrow -2(x-p)+1=x$ $\Rightarrow -2x+2p+1=x$ $\Rightarrow 3x=2p+1$ $\therefore x=\frac{2p+1}{3}$	From the diagram in (iii), students are to observe that the point(s) of intersection of $\mathrm{f}(x)=\mathrm{f}^{-1}(x)$ lie on the line $y = x$. **Common Mistakes**: 1. Students did not choose the correct expression for f when solving the equation $\mathrm{f}(x)=x$.
(iv)	 $\mathrm{D}_{\mathrm{gf}^{-1}}=\mathrm{D}_{\mathrm{f}^{-1}}=[1,\infty)\xrightarrow[\mathrm{f}^{-1}]{}(-\infty, p]\xrightarrow[\text{into graph of g}]{}\left[-\frac{q^2}{4},\infty\right)=\mathrm{R}_{\mathrm{gf}^{-1}}$	Students are encouraged to use the mapping method to find the range of the composite function gf^{-1} and thus it is important to obtain the correct graph of g. **Common Mistakes**: 1. Students are not able to determine the coordinates of turning point of quadratic curve correctly. 2. Many assume that $\mathrm{R}_{\mathrm{gf}^{-1}}=\mathrm{R}_{\mathrm{g}}$ without verifying whether $\mathrm{R}_{\mathrm{f}^{-1}}=\mathrm{D}_{\mathrm{g}}$. That is to say students must verify that $\mathrm{R}_{\mathrm{f}^{-1}}=\mathrm{D}_{\mathrm{g}}$ is indeed true before applying $\mathrm{R}_{\mathrm{gf}^{-1}}=\mathrm{R}_{\mathrm{g}}$.

	Thus $R_{gf^{-1}} = \left[-\frac{q^2}{4}, \infty\right)$. Note: $D_{f^{-1}} = R_f = [1, \infty)$ and $R_{f^{-1}} = D_f = (-\infty, p]$	

10	Suggested Solution	Comments
(i)	Total depth $= 4.5 + 4.5\left(1-\frac{r}{100}\right) + 4.5\left(1-\frac{r}{100}\right)^2 + ... + 4.5\left(1-\frac{r}{100}\right)^9$ $= \frac{4.5\left[1-\left(1-\frac{r}{100}\right)^{10}\right]}{1-\left(1-\frac{r}{100}\right)}$ $= \frac{450}{r}\left[1-\left(1-\frac{r}{100}\right)^{10}\right]$	A lot of students did not understand the phrase "reduced by r %" and used $\frac{r}{100}$ as the common ratio.
(ii)	$\frac{450}{r}\left[1-\left(1-\frac{r}{100}\right)^{10}\right] = \frac{10}{2}[2(4.7)+9(-0.1)] = 42.5$ By GC, $r = 1.28$.	Some students failed to recognise that the value of the common difference is –0.1, not 0.1 when computing the total depth that the STK machine can drill. Sad to say, some students tried to solve the equation manually or manipulated the equation wrongly and then used GC to solve. This shows a lack of awareness of how to use the GC.
(iii)	Let the number of hours the machine has drilled be n. To satisfy requirement, $4.5(0.99)^{n-1} \geq 4.0$ From GC, $n \leq 12$ (or $n \leq 12.7$ algebraically) It should operate 12 complete hours.	Many students tried to solve $4.5(0.99)^n \geq 4.0$ while thinking that $4.5(0.99)^n$ gives the depth of the hole drilled in the n-th hour, not realising that this contradicts what they have written in **(i)**. Some students used a complicated method, which is to use the formula in **(i)**.

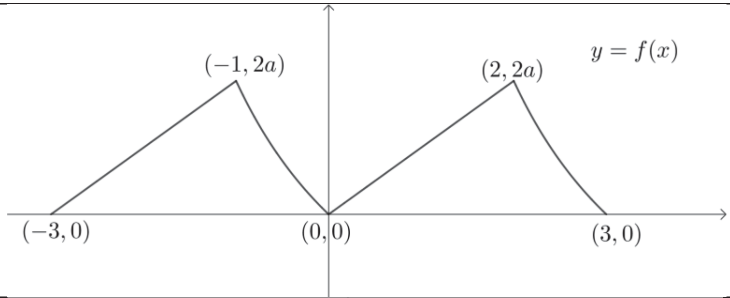
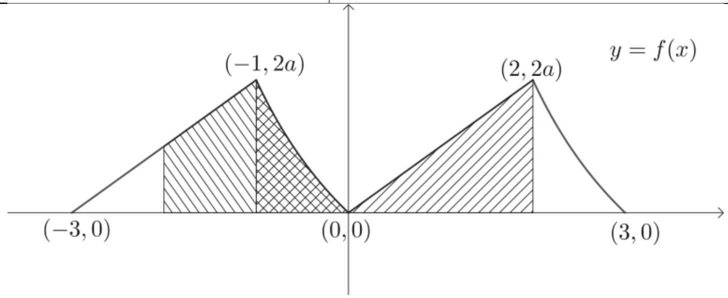
(iv)	For the alternative manufacturer's claim to be false, $\dfrac{4.5}{1-\left(1-\dfrac{r}{100}\right)}<360$ $r>1.25$	While most students were able to recognise that sum of infinity must be used, they gave the common ratio as r or $1-r$, instead of $1-\dfrac{r}{100}$. Some of them used the correct common ratio in **(i)** correct but used the wrong one here.
(v)	<u>Method 1</u> For the STK machine, it can only drill at most 47 hours, as it will drill $4.7+(48-1)(-0.1)=0$ m in the 48th hour. Total depth drilled is $\dfrac{47}{2}\left[2\times4.7+(47-1)(-0.1)\right]=112.8$. Therefore, it is not possible to drill a hole to 150 m deep. <u>Method 2</u> The total distance drilled by the STK machine in m hours is $\dfrac{m}{2}\left[2\times4.7+(m-1)(-0.1)\right]=4.75m-0.05m^2$. Let $4.75m-0.05m^2=150$ $4.75m-0.05m^2=150$ $0.05m^2-4.75m+150=0$ discriminant $=4.75^2-4\times150\times0.05=-7.4375<0$ There is no real solution for m, so it is not possible to drill a hole to 150 m deep.	Equal number of students attempted both methods. The errors usually occur when computing the maximum total depth or manipulating the equation. Some students tried to use "sum to infinity", not realising that this does not make sense because the sum of infinity does not exist for an AP. Moreover the common difference is negative, so in this context it means that the machine can only drill up to a certain extent, so it is pointless to talk about drilling "forever".

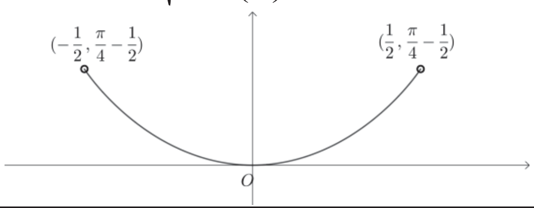
11	Suggested Solution	Comments
(i)	$V=\frac{1}{3}\pi r^2 h \Rightarrow h=\frac{3V}{\pi r^2}$ $S=\pi r\sqrt{h^2+r^2}$ $=\pi r\sqrt{\left(\frac{3V}{\pi r^2}\right)^2+r^2}$ $=\pi r\sqrt{\frac{9V^2+\pi^2 r^6}{\pi^2 r^4}}$ $=\frac{\pi r}{\pi r^2}\sqrt{9V^2+\pi^2 r^6}$ $S^2=\frac{9V^2+\pi^2 r^6}{r^2}$ (Shown)	Well-done except for a handful who mistook V and S to include the cylindrical body.
(ii)	$S^2=\frac{9V^2+\pi^2 r^6}{r^2}=9V^2 r^{-2}+\pi^2 r^4$ or let $y=9V^2 r^{-2}+\pi^2 r^4$ Differentiating w.r.t. r, $2S\frac{\mathrm{d}S}{\mathrm{d}r}=9V^2\left(-2r^{-3}\right)+4\pi^2 r^3$ ---------- (1) $\frac{\mathrm{d}y}{\mathrm{d}r}=-18V^2 r^{-3}+4\pi^2 r^3$ For $\frac{\mathrm{d}S}{\mathrm{d}r}=0$, $9V^2\left(-2r^{-3}\right)+4\pi^2 r^3=0$ $\Rightarrow -18V^2+4\pi^2 r^6=0$ $\Rightarrow r=\sqrt[6]{\frac{9V^2}{2\pi^2}}$	Many students were still not proficient in doing implicit differentiation when they tried to make S the subject by taking square root of the RHS, before finding $\frac{\mathrm{d}S}{\mathrm{d}r}$. Many students were also unaware that V is a constant and hence $\frac{\mathrm{d}}{\mathrm{d}r}(V)=0$ which simplifies subsequent workings.

	Differentiating (1) w.r.t. r, $2S\frac{\mathrm{d}^2S}{\mathrm{d}r^2}+2\left(\frac{\mathrm{d}S}{\mathrm{d}r}\right)^2=9V^2\left(6r^{-4}\right)+12\pi^2r^2$ Or $\frac{\mathrm{d}^2y}{\mathrm{d}r^2}=54V^2r^{-4}+12\pi^2r^2>0$ When $r=\sqrt[6]{\frac{9V^2}{2\pi^2}}$ and $\frac{\mathrm{d}S}{\mathrm{d}r}=0$, $\frac{\mathrm{d}^2S}{\mathrm{d}r^2}=\frac{1}{2S}\left(\frac{3V^2}{2r^4}+12\pi^2r^2\right)>0.$ Thus S is minimum when $r=\sqrt[6]{\frac{9V^2}{2\pi^2}}$. $\frac{h}{r}=\left(\frac{3V}{\pi r^2}\right)\left(\frac{1}{r}\right)=\frac{3V}{\pi r^3}$ When $r=\sqrt[6]{\frac{9V^2}{2\pi^2}}$, $\frac{h}{r}=\frac{3V}{\pi}\left(\sqrt[6]{\frac{2\pi^2}{9V^2}}\right)^3$ $=\frac{3V}{\pi}\left(\sqrt{\frac{2\pi^2}{9V^2}}\right)$ $=\frac{3V}{\pi}\left(\frac{\sqrt{2}\pi}{3V}\right)$ $=\sqrt{2}$ $\therefore h:r=\sqrt{2}:1$	Most students attempted to check for minimum by using the first derivative test but failed to realise that the test failed in this context because of the unknown numerical value of r and hence we were not able to find the numerical values of the first derivative around the neighbourhood of the minimum point that explained the gradient of the tangents. Many students did not leave their answer in ratio, as required by the question.

(iii)	$(AB)\cos\frac{\pi}{3}=3 \Rightarrow AB=3(2)=6$ $\angle ACB+\theta=\frac{\pi}{6} \Rightarrow \angle ACB=\frac{\pi}{6}-\theta$ and $\angle ABC=\pi-\frac{\pi}{6}=\frac{5\pi}{6}$ By sine rule, $\frac{AC}{\sin\frac{5\pi}{6}}=\frac{AB}{\sin\left(\frac{\pi}{6}-\theta\right)}$ $\Rightarrow AC=\frac{6\times\frac{1}{2}}{\sin\left(\frac{\pi}{6}-\theta\right)}=\frac{3}{\sin\left(\frac{\pi}{6}-\theta\right)}$ (Shown)	The method to answer this question is not unique and the provided solution uses the sine rule. Some students easily used the sine ratio on the right angle triangle *ANC*.

(iv)	$AC=\dfrac{3}{\sin\left(\dfrac{\pi}{6}-\theta\right)}$ $=\dfrac{3}{\sin\dfrac{\pi}{6}\cos\theta-\cos\dfrac{\pi}{6}\sin\theta}$ $\approx\dfrac{3}{\dfrac{1}{2}\left(1-\dfrac{1}{2}\theta^2\right)-\dfrac{\sqrt{3}}{2}\theta}$ $=6\left[1-\left(\sqrt{3}\theta+\dfrac{1}{2}\theta^2\right)\right]^{-1}$ $\approx 6\left[1+\left(\sqrt{3}\theta+\dfrac{1}{2}\theta^2\right)+\left(\sqrt{3}\theta+\dfrac{1}{2}\theta^2\right)^2\right]$ $\approx 6\left[1+\left(\sqrt{3}\theta+\dfrac{1}{2}\theta^2\right)+\left(\sqrt{3}\theta\right)^2\right]$ $=6+6\sqrt{3}\theta+3\theta^2+18\theta^2$ $=6+6\sqrt{3}\theta+21\theta^2$ The length (in metres) to be cut off is $AC-AB\approx 6+6\sqrt{3}\theta+21\theta^2-6=6\sqrt{3}\theta+21\theta^2$	Most students failed to go beyond the approximations using small angles on $\sin\theta$ and $\cos\theta$. Common mistake includes writing '$\sin\left(\dfrac{\pi}{6}-\theta\right)\approx\dfrac{\pi}{6}-\theta$'. Those who attempted to do Maclaurin series increased the complexity of the solution that led to numerical mistakes and hence the final answer.

1	Suggested Solution	Marker's Comments
(i)	$\mathrm{f}\left(\frac{44}{3}\right)=\mathrm{f}\left(\frac{8}{3}+3\times 4\right)=\mathrm{f}\left(\frac{8}{3}\right)=\frac{12a}{\frac{8}{3}}-4a=\frac{a}{2}$	Note the clear and efficient way of showing that the graph repeats itself after 3 units i.e. $\mathrm{f}\left(\frac{44}{3}\right)=\mathrm{f}\left(\frac{8}{3}+3\times 4\right)$.
(ii)		Common mistakes: - hollow point for (3,0) - since graph has a period of 3 units, the maximum point should have x-coordinate -1 and 2. Please draw all graphs with a dark pencil.
(ii)	 $\int_{-2}^{2}\mathrm{f}(x)\,\mathrm{d}x=\frac{1}{2}(2)(2a)+\frac{1}{2}(1)(a+2a)+\int_{-1}^{0}\mathrm{f}(x)\,\mathrm{d}x$ $=2a+\frac{3}{2}a+\int_{2}^{3}\mathrm{f}(x)\,\mathrm{d}x$ $=\frac{7}{2}a+\int_{2}^{3}\frac{12a}{x}-4a\,\mathrm{d}x$ $=\frac{7}{2}a+\left[12a\ln\lvert x\rvert-4ax\right]_{2}^{3}$ $=\frac{7}{2}a+12a\ln 3-12a-12a\ln 2+8a$ $=12a\ln\left(\frac{3}{2}\right)-\frac{1}{2}a$	Many students did not manage to come out with this expression $\int_{2}^{3}\frac{12a}{x}-4a\,\mathrm{d}x$. Due to the graph having a period of 3 units, the curve between $x=-1$ and $x=0$ will appear to be the same as in the interval $x=2$ to $x=3$. But this does not mean that $\int_{2}^{3}\frac{12a}{x}-4a\,\mathrm{d}x=\int_{-1}^{0}\frac{12a}{x}-4a\,\mathrm{d}x$. Be careful and notice that there are three partitions also.

2	Suggested Solution	Marker's Comments
(i)	$\frac{d^2y}{dx^2}=\frac{2}{\sqrt{1-4x^2}}$ $\frac{dy}{dx}=\int\frac{2}{\sqrt{1-4x^2}}\,dx=\int\frac{2}{\sqrt{1-(2x)^2}}\,dx=\sin^{-1}(2x)+C$ $y=\int\sin^{-1}(2x)+C\,dx$ $=\int 1\cdot\sin^{-1}(2x)\,dx+Cx+D$ $=x\sin^{-1}(2x)-\int x\left(\frac{2}{\sqrt{1-4x^2}}\right)dx+Cx+D$ $=x\sin^{-1}(2x)+\frac{1}{4}\int(-8x)(1-4x^2)^{-\frac{1}{2}}\,dx+Cx+D$ $=x\sin^{-1}(2x)+\frac{1}{4}\frac{(1-4x^2)^{\frac{1}{2}}}{\frac{1}{2}}+Cx+D$ $=x\sin^{-1}(2x)+\frac{1}{2}\sqrt{1-4x^2}+Cx+D$	Most students knew that they had to refer to MF26 to integrate $\frac{2}{\sqrt{1-(2x)^2}}$. However, many mistakes were made. Common mistakes: • When using the formula for integrating $\frac{1}{\sqrt{1-x^2}}$ to integrate $\frac{1}{\sqrt{1-(2x)^2}}$, students forgot to divide by 2. • When taking out factor of 4 from $\sqrt{1-4x^2}$, students forgot to square root the 4. • When differentiating $\sin^{-1}(2x)$, many students gave $\frac{1}{\sqrt{1-(2x)^2}}$ instead of $\frac{2}{\sqrt{1-(2x)^2}}$. There were many other minor mistakes in algebraic manipulation.
(b) (i)	When $x=0$, $y=0$ and $\frac{dy}{dx}=0$. $0=\sin^{-1}(0)+C\Rightarrow C=0$. $0=0\sin^{-1}(0)+\frac{1}{2}\sqrt{1-0}+C(0)+D\Rightarrow D=-\frac{1}{2}$ $f(x)=x\sin^{-1}(2x)+\frac{1}{2}\sqrt{1-4x^2}-\frac{1}{2}$	There were quite a lot of students who did not seem to understand the term "particular solution curve". They tried to "guess" an equation of a curve that has a minimum point at the origin, often giving the equation $y=kx^2$.
(b) (ii)	$\frac{1}{2}\sin^{-1}1+\frac{1}{2}\sqrt{1-4\left(\frac{1}{2}\right)^2}-\frac{1}{2}=\frac{\pi}{4}-\frac{1}{2}$ 	Most students did not label the end points, or did not bother to indicate the domain.

3	Suggested Solution	Marker's Comments
(a)(i)	Let $(x+yi)^2 = 8-6i$, $x, y \in \mathbb{R}$. $x^2 + 2xyi - y^2 = 8-6i$ $\begin{cases} x^2 - y^2 = 8 \\ 2xy = -6 \end{cases} \Rightarrow \begin{cases} x^2 - y^2 = 8 & \ldots(1) \\ y = -\dfrac{3}{x} & \ldots(2) \end{cases}$ Substitute (2) into (1), $x^2 - \dfrac{9}{x^2} = 8$ $x^4 - 8x^2 - 9 = 0$ $(x^2-9)(x^2+1) = 0$ $x^2 = 9$ or $x^2 = -1$ (rejected) $x = 3$ or $x = -3$ Correspondingly, $y = -1$ or $y = 1$. $x = 3, y = -1$ or $x = -3, y = 1$	
(a)(ii)	$w^2 + 2iw - 9 + 6i = 0$ $w^2 + 2iw = 9 - 6i$ $w^2 + 2iw + i^2 = 9 - 6i + i^2$ $(w+i)^2 = 8 - 6i$ From (i), $w + i = 3 - i$ or $w + i = -3 + i$ $w = 3 - 2i$ or $w = -3$	
(b)(i)	Substitute $z = -z_1$ into $z^4 - pz^2 + q$. $(-z_1)^4 - p(-z_1)^2 + q$ $= z_1^4 - pz_1^2 + q = 0$ (since z_1 is a root) Or $z^2 = (-z_1)^2, z^4 = (-z_1)^4$ Therefore $-z_1$ is also a root.	

(b)(ii)

$$z = 3e^{i\left(\frac{\pi}{6}\right)}$$

$$z^2 = \left[3e^{i\left(\frac{\pi}{6}\right)}\right]^2 = 9e^{i\left(\frac{\pi}{3}\right)} = 9\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) = 9\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$$

$$z^4 = \left[3e^{i\left(\frac{\pi}{6}\right)}\right]^4 = 81e^{i\left(\frac{2\pi}{3}\right)} = 81\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right) = 81\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$$

Substitute these values into $z^4 - 9z^2 + 81$

$$81\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) - 9\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) + 81$$

$$= -\frac{81}{2} + \frac{81\sqrt{3}}{2}i - \frac{81}{2} - \frac{81\sqrt{3}}{2}i + 81$$

$$= 0$$

Thus $3e^{i\left(\frac{\pi}{6}\right)}$ is a root.

By **(b)(i)** and conjugate root theorem,

$3e^{i\left(-\frac{\pi}{6}\right)}, -3e^{i\left(\frac{\pi}{6}\right)}, -3e^{i\left(-\frac{\pi}{6}\right)}$ are also the roots of this equation.

One quadratic factor is

$$\left[z - 3e^{i\left(\frac{\pi}{6}\right)}\right]\left[z - 3e^{i\left(-\frac{\pi}{6}\right)}\right] = z^2 - \left(3e^{i\left(\frac{\pi}{6}\right)} + 3e^{i\left(-\frac{\pi}{6}\right)}\right)z + 3e^{i\left(\frac{\pi}{6}\right)}3e^{i\left(-\frac{\pi}{6}\right)}$$

$$= z^2 - 2\left[3\cos\left(\frac{\pi}{6}\right)\right]z + 9$$

$$= z^2 - 3\sqrt{3}z + 9$$

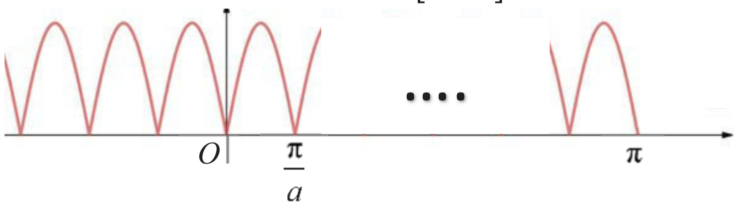
And the other quadratic factor is

$$\left[z + 3e^{i\left(\frac{\pi}{6}\right)}\right]\left[z + 3e^{i\left(-\frac{\pi}{6}\right)}\right] = z^2 + \left[3e^{i\left(\frac{\pi}{6}\right)} + 3e^{i\left(-\frac{\pi}{6}\right)}\right]z + 3e^{i\left(\frac{\pi}{6}\right)}3e^{i\left(-\frac{\pi}{6}\right)}$$

$$= z^2 + 2\left[3\cos\left(\frac{\pi}{3}\right)\right]z + 9$$

$$= z^2 + 3\sqrt{3}z + 9$$

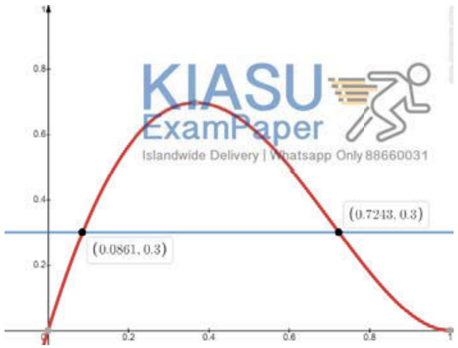
4	Suggested Solution	Marker's Comments
	$\int_{-\pi}^{\pi} \sin kx \, dx = \left(-\frac{\cos kx}{k}\right)_{-\pi}^{\pi}$ $= \left(-\frac{\cos k\pi}{k}\right) - \left[-\frac{\cos(-k\pi)}{k}\right]$ $= \frac{\cos(-k\pi)}{k} - \frac{\cos k\pi}{k}$ $= 0 \quad (\because \ \cos A = \cos(-A))$ $\int_{-\pi}^{\pi} \cos kx \, dx = \left(\frac{\sin kx}{k}\right)_{-\pi}^{\pi}$ $= \left(\frac{\sin k\pi}{k}\right) - \left[\frac{\sin(-k\pi)}{k}\right]$ $= 0 - 0$ $= 0$	
(i)	$\sin(mx)\sin(nx) = \frac{1}{2}\left[\cos(m-n)x - \cos(m+n)x\right]$ $\int_{-\pi}^{\pi} \sin(mx)\sin(nx) \, dx$ $= \int_{-\pi}^{\pi} \frac{1}{2}\left[\cos(m-n)x - \cos(m+n)x\right] dx$ $= \frac{1}{2}\int_{-\pi}^{\pi} \cos(m-n)x \ dx - \frac{1}{2}\int_{-\pi}^{\pi} \cos(m+n)x \, dx$ $= \frac{1}{2}\int_{-\pi}^{\pi} \cos(m-n)x \ dx - \frac{1}{2}(0)$ If $m \neq n$, $\int_{-\pi}^{\pi} \cos(m-n)x \, dx = 0$ using the first part result. If $m = n$, $\int_{-\pi}^{\pi} \cos(m-n)x \, dx = \int_{-\pi}^{\pi} 1 \, dx = \pi - (-\pi) = 2\pi$ Therefore, $\int_{-\pi}^{\pi} \sin(mx)\sin(nx) \, dx = \begin{cases} \pi & \text{if } m = n, \\ 0 & \text{if } m \neq n. \end{cases}$	

(ii)	$\cos(mx+nx)=\cos(mx)\cos(nx)-\sin(mx)\sin(nx)$ $\int_{-\pi}^{\pi}\cos(mx+nx)\,\mathrm{d}x$ $=\int_{-\pi}^{\pi}\cos(mx)\cos(nx)\,\mathrm{d}x-\int_{-\pi}^{\pi}\sin(mx)\sin(nx)\,\mathrm{d}x$ LHS $=\int_{-\pi}^{\pi}\cos(m+n)x\,\mathrm{d}x=0$, therefore $\int_{-\pi}^{\pi}\cos(mx)\cos(nx)\,\mathrm{d}x=\begin{cases}\pi & \text{if } m=n,\\ 0 & \text{if } m\neq n.\end{cases}$	
(iii)	There are $2a$ repeated graphs on $[-\pi,\pi]$.  $\int_{-\pi}^{\pi}\lvert\sin(ax)\rvert\,\mathrm{d}x=2a\int_{0}^{\frac{\pi}{a}}\lvert\sin(ax)\rvert\,\mathrm{d}x$ $=2a\int_{0}^{\frac{\pi}{a}}\sin(ax)\,\mathrm{d}x$ $=2a\left[-\frac{\cos(ax)}{a}\right]_{0}^{\frac{\pi}{a}}$ $=2a\left(-\frac{\cos\pi}{a}+\frac{\cos 0}{a}\right)$ $=2a\left(\frac{1}{a}+\frac{1}{a}\right)$ $=2a\left(\frac{2}{a}\right)$ $=4$	

(iv)	$\pi\int_{-\pi}^{\pi}\left[\lvert\sin(ax)\rvert+1\right]^2 \ \mathrm{d}x$ $=\pi\int_{-\pi}^{\pi}\left[\sin(ax)\right]^2+2\lvert\sin(ax)\rvert+1 \ \mathrm{d}x$ $=\pi\int_{-\pi}^{\pi}\left[\sin(ax)\right]^2 \ \mathrm{d}x+2\pi\int_{-\pi}^{\pi}\lvert\sin(ax)\rvert \ \mathrm{d}x+\pi\int_{-\pi}^{\pi}1 \ \mathrm{d}x$ $=\pi(\pi)+2\pi(4)+\pi\left[x\right]_{-\pi}^{\pi}$ $\quad$ (by letting $m=n=a$) $=3\pi^2+8\pi$	

5	Suggested Solution	Marker's Comments
(i)	$\frac{P(X=k+1)}{P(X=k)}=\frac{\binom{n}{k+1}p^{k+1}(1-p)^{n-k-1}}{\binom{n}{k}p^{k}(1-p)^{n-k}}$ $=\frac{\frac{n!}{(k+1)!(n-k-1)!}p^{k+1}(1-p)^{n-k-1}}{\frac{n!}{(k)!(n-k)!}p^{k}(1-p)^{n-k}}$ $=\frac{\frac{1}{(k+1)}p}{\frac{1}{(n-k)}(1-p)}$ $=\frac{(n-k)p}{(k+1)(1-p)}$ or $=\frac{n!p^{k+1}(1-p)^{n-k-1}}{(k+1)!(n-k-1)!}\times\frac{(k)!(n-k)!}{n!p^{k}(1-p)^{n-k}}$ $=\frac{p}{(k+1)}\times\frac{(n-k)}{(1-p)}$ $=\frac{(n-k)p}{(k+1)(1-p)}$	This is a show question. To get the full marks, you must show every step clearly, especially the simplification. Please see the left column on examples on how to show the simplifications.
(ii)	Assuming k is the mode, then $P(X=k+1)<P(X=k)$. $\frac{P(X=k+1)}{P(X=k)}<1$ $\frac{\frac{7}{9}(99-k)}{(k+1)\left(1-\frac{7}{9}\right)}<1$	Many students missed out the statement 'assuming k is the mode'.

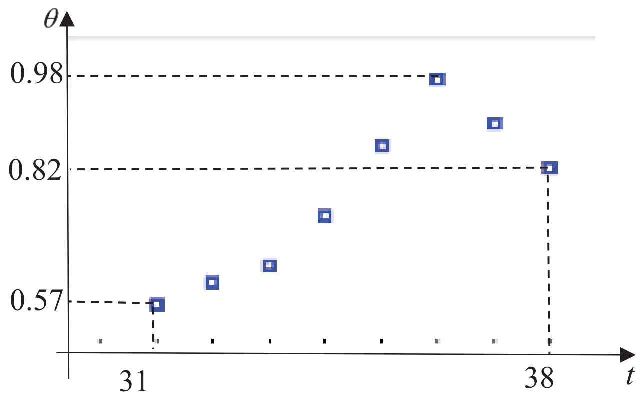
	$77-\frac{7}{9}k<\frac{2}{9}k+\frac{2}{9}$ $k>76\frac{7}{9}$ i.e. $\mathrm{P}(X=77)>\mathrm{P}(X=78)>\mathrm{P}(X=79)>\ldots$ This result also implies k cannot be less than $76\frac{7}{9}$ or otherwise $\mathrm{P}(X=k+1)>\mathrm{P}(X=k)$. Thus, $k=77$	Note the instruction on 'using the result in part (i)' so you should not be using the GC to find the answer.
6	**Suggested Solution**	**Marker's Comments**
(a)	$4!\times\binom{5}{4}\times 4!=2880$ $4!\times 2^4=384$	
(b)	$(4-1)!\times 2^4=96$	
(c)	$\dfrac{\binom{8}{4}\binom{4}{4}}{2!}-1=35-1=34$	

7	Suggested Solution	Marker's Comments
(i)	Let A be the number of young employees, out of 15, who would like to enroll in either infocomm technology programme or financial programme. 1 – 0.29 = 0.71 $A \sim \mathrm{B}(15, 0.71)$ $\mathrm{P}(A=12)=0.182098 \approx 0.182$	• Students are to identify that event A follows a binomial distribution. • For students who did not identify the distribution, most obtained the correct answer using 'Required Prob = $\binom{15}{12}(0.71)^{12}(0.29)^3 = 0.182$.'
(ii)	Let H be the number of young employees, out of 15, who would like to enroll in human resource programme. $H \sim \mathrm{B}(15, 0.29)$ $\mathrm{P}(H \geq 5)=1-\mathrm{P}(H \leq 4)$ $=1-0.450030.54997$ $=0.45003$ ≈ 0.450	• 'not less than 5' equates to '≥ 5'. **Common Mistakes:** 1. Students used $\mathrm{P}(H>5)$ instead of $\mathrm{P}(H \geq 5)$. 2. Students wrote $\mathrm{P}(H \geq 5)=1-\mathrm{P}(H \leq 5)$.
(iii)	Let M be the number of young employees, out of 4, who would like to enroll in financial management programme. $M \sim \mathrm{B}(4, p)$ $\mathrm{P}(M=1 \text{ or } 2)=0.3$ $\mathrm{P}(M=1)+\mathrm{P}(M=2)=0.3$ $\binom{4}{1}p(1-p)^3+\binom{4}{2}p^2(1-p)^2=0.3$ $4p(1-p)^3+6p^2(1-p)^2=0.3$  By GC, $p \approx 0.0861$ or $p \approx 0.7243$ (rejected as $0.7243 > 0.71$). So $p = 0.0861$ (to 3 s.f.)	• Most students are able to rewrite the expression found in MF26 to form the required equation $\binom{4}{1}p(1-p)^3+\binom{4}{2}p^2(1-p)^2=0.3$. • However many are unable to use the GC to solve the quartic equation. Most obtained only 1 answer which is the incorrect one. • For students who obtained more than 1 answer, quite a few students did not realise that $p \approx 0.7243$ is rejected as $p \leq 0.71$ (from context).

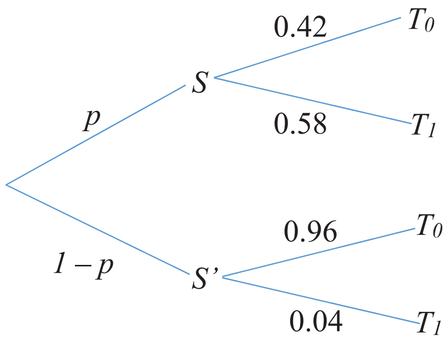
<table>
<tr><th>8</th><th>Suggested Solution</th><th>Marker's Comments</th></tr>
<tr><td>(i)</td><td>$a = 30$.
$\frac{3}{k} \times \frac{2}{k-1} \times \frac{1}{k-2} = 0.05$ (the table). By GC, $k = 6$</td><td>You need to know that the probability is formulated without replacement.</td></tr>
<tr><td>(ii)</td><td>Every envelope has an equal chance of being selected by Joe.</td><td></td></tr>
<tr><td>(iii)</td><td>Let X be the amount of money that Joe receives after winning a game.
By GC, $\mathrm{E}(X) = 16.5$ and $\mathrm{Var}(X) = 36.45$.
Since $n \geq 20$ is large, by Central Limit Theorem,
$\overline{X} \sim N\left(16.5, \frac{36.45}{n}\right)$ approximately.
$\mathrm{P}\left(16.5 - 2 \leq \overline{X} \leq 16.5 + 2\right) > 0.95$
$\mathrm{P}\left(14.5 \leq \overline{X} \leq 18.5\right) > 0.95$
<u>Method 1 (GC Table, good for small n)</u>
<table>
<tr><th>n</th><th>$\mathrm{P}\left(14.5 \leq \overline{X} \leq 18.5\right)$</th></tr>
<tr><td>34</td><td>0.94659316</td></tr>
<tr><td>35</td><td>0.94998281</td></tr>
<tr><td>36</td><td>0.95314596</td></tr>
</table>
Least $n = 36$
Method 2 (Standardisation, tedious but always working)</td><td>Expected amount here signifies the need to use Central Limit Theorem since the distribution for the amount is not given.

Due to Central Limit Theorem, $\overline{X}$ follows a Normal Distribution APPROXIMATELY. The latter is an important keyword.</td></tr>
</table>

	$\mathrm{P}\left(\frac{14.5-16.5}{\sqrt{\frac{36.45}{n}}} \le \frac{\bar{X}-16.5}{\sqrt{\frac{36.45}{n}}} \le \frac{18.5-16.5}{\sqrt{\frac{36.45}{n}}}\right) > 0.95$ $\mathrm{P}\left(\frac{-2}{\sqrt{\frac{36.45}{n}}} \le Z \le \frac{2}{\sqrt{\frac{36.45}{n}}}\right) > 0.95$ $\mathrm{P}\left(Z \le \frac{-2}{\sqrt{\frac{36.45}{n}}}\right) < 0.025$ $\frac{-2}{\sqrt{\frac{36.45}{n}}} < -1.96$ $\sqrt{\frac{36.45}{n}} > \frac{2}{1.96}$ $n > 35.00658$ Least $n = 36$	

9	Suggested Solution	Marker's Comments
(i)		Well attempted except for some students did not indicate the range clearly on the scatter diagram. There are some who labelled the axes wrongly and did not use the variables provided in the context.
(ii)	$r = 0.851386 \approx 0.851$	
(iii)	The regression line θ on t should be used as t is the independent (or controlled) variable (Temperature varies with time)	Many students are able to choose the correct regression line with the correct reason.
(iv)	θ on t: $\theta = 0.0517857143t - 1.022857$ $\therefore \theta = 0.0518t - 1.02$ The temperature anomaly increases by **approximately** 0.0518°C every year.	Majority of students missed out the keyword "**approximately**".
(v)	For Year 2025, $t = 2025 - 1980 = 45$, predicted value of $\theta = 0.0517857143(45) - 1.022857$ $= 1.307500$ ≈ 1.31 As $t = 45$ is not within the data range [31, 38], the predicted value is not reliable.	Students did not realise that they need to state the data range of the given value.

(vi)	In many years to come, factors like various countries could have done more to reduce environment pollution, hence a linear model may not be suitable for long term predictions. OR In many years to come, if people are not putting in diligent effort to minimise environment pollution, the global temperature may change drastically hence a linear model may not be suitable for long term predictions. OR It is not possible for temperature anomaly to increase to an infinite value in long run.	Quite a number of students answered this part (taking into account of the context) well.

10	Suggested Solution	Marker's Comments
(i)	Since conditional probabilities given the same event must sum up to 1, we have $q_1 = 0.96$ and $q_2 = 0.58$	
(ii)	 $\mathrm{P}(S \mid T_0) = \dfrac{\mathrm{P}(S \cap T_0)}{\mathrm{P}(T_0)}$ $= \dfrac{0.42p}{0.42p + 0.96(1-p)}$ $= \dfrac{0.42p}{0.42p + 0.96 - 0.96p}$ $= \dfrac{0.42p}{0.96 - 0.54p}$ $= \dfrac{0.06(7p)}{0.06(16-9p)}$ $= \dfrac{7p}{16-9p}$	
(iii)	$\dfrac{\mathrm{d}}{\mathrm{d}p}\left(\mathrm{P}(S \mid T_0)\right) = \dfrac{\mathrm{d}}{\mathrm{d}p}\left(\dfrac{7p}{16-9p}\right)$ $= \dfrac{7(16-9p) - (-9)(7p)}{(16-9p)^2}$ $= \dfrac{112}{(16-9p)^2} > 0$ for all $0 < p < 1$	

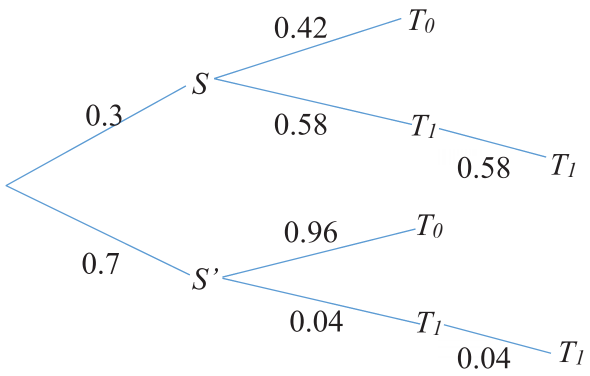
Therefore, $P(S \mid T_0)$ is an increasing function of p.

The higher the proportion of patients with heart disease, the higher is the probability that a patient has heart disease given a negative test result.

OR

If the majority of the population have heart disease, the negative test results are no longer indicative of the absence of heart disease in a patient.

(iv)



$$\begin{aligned}
&\mathrm{P}(\text{heart disease} \mid \text{both test results are positive}) \\
&= \frac{\mathrm{P}(\text{heart disease} \cap \text{both test result are positive})}{\mathrm{P}(\text{both test results are positive})} \\
&= \frac{\mathrm{P}(\text{heart disease} \cap \text{both test results are positive})}{\mathrm{P}(\text{heart disease} \cap \text{both test results are positive}) + \mathrm{P}(\text{no heart disease} \cap \text{both test results are positive})} \\
&= \frac{0.3 \times 0.58 \times 0.58}{0.3 \times 0.58 \times 0.58 + 0.7 \times 0.04 \times 0.04} \\
&= \frac{0.10092}{0.10092 + 0.00112} \\
&= \frac{0.10092}{0.10204} \\
&\approx 0.98902 \\
&\approx 0.989
\end{aligned}$$

11	Suggested Solution	Marker's Comments
(i)	Unbiased estimate of the population mean, $\bar{x} = \frac{38.9}{250} + 10 = 10.1556 \approx 10.2$ Unbiased estimate of the population variance, $s^2 = \frac{1}{249}\left(390.31 - \frac{38.9^2}{250}\right) \approx 1.543201 \approx 1.54$	Well-done for most students. Mistakes involved writing $\mu = \bar{x}$ and $\sigma^2 = s^2$.
(ii)	Test $H_0 : \mu = 10$ against $H_1 : \mu > 10$ Level of significance: 4% (upper-tailed) Under H_0, $\bar{X} \sim N\left(10, \frac{1.543201}{250}\right)$ approximately. $Z = \frac{\bar{X} - 10}{\sqrt{\frac{1.543201}{250}}} \sim N(0,1)$ approximately From GC, p-value $\approx 0.0238 < 0.04$ **OR** critical value, $z_{0.96} \approx 1.75$ and observed test statistic $\approx 1.98 > 1.75$ Thus, we reject H_0 and conclude that there is sufficient evidence at the 4 % level of significance that the mean speed of PMD riders exceeds the new speed limit.	Although students were generally able to obtain the correct p-value or z test statistic value from the GC, some had clearly shown that they understood little of the concepts underlying hypothesis testing by making the following mistakes: - Test $H_0 : x = 10$ against $H_1 : x > 10$ - Applying CLT to say that $\bar{X}$ follows a normal distribution because of the large sample size (this is redundant as X is already assumed to be normally distributed by the question). - $\bar{X} \sim N\left(10.2, \frac{1.543201}{250}\right)$ and thus failed to understand that $E\left(\bar{X}\right) = \mu = 10$ (under H_0) instead of the sample mean 10.2. - Using the 3 s.f. values in part (i) to calculate the p-value or z test statistic that reduces the accuracy; students should bear in mind that at least 5 s.f. is needed for the use of intermediate values. - Writing p-value as p.
(iii)	There is a probability of 0.04 (or 4% chance) of wrongly rejecting the claim that the mean speed of PMD riders does not exceed (or is) 10 km/h.	Generally well-done.

(iv)	Test $\text{H}_0 : \mu = 10$ against $\text{H}_1 : \mu \neq 10$ Level of significance: 4% (2-tailed) For a 2-tailed test, p-value $\approx 2(0.0238) = 0.0476 > 0.04$ The conclusion will change.	Badly done. Most students wrongly concluded that p-value $= 0.0238 > \frac{1}{2}(0.04) = 0.02$. The intention might have been to compare only the right tail but students should show that p-value is doubled in a 2-tailed test rather than 'remaining as the same value' as in part (ii).
(v)	Let n be the sample size. $s^2 = \frac{n}{n-1}\left(3.2^2\right) = \frac{10.24n}{n-1}$ Test $\text{H}_0 : \mu = 10$ against $\text{H}_1 : \mu \neq 10$ Under H_0, $\overline{X} \sim \text{N}\left(10, \frac{10.24}{n-1}\right)$ approximately. Test statistic: $Z = \frac{\overline{X} - 10}{\sqrt{\frac{10.24}{n-1}}} \sim \text{N}(0,1)$ approximately Level of significance: 5% (2-tailed) To reject H_0, $\frac{9.5-10}{\sqrt{\frac{10.24}{n-1}}} \leq -1.96$ or $\frac{9.5-10}{\sqrt{\frac{10.24}{n-1}}} \geq 1.96$ $\Rightarrow \sqrt{\frac{n-1}{10.24}} \geq 3.92$ $\Rightarrow \sqrt{\frac{n-1}{10.24}} \leq -3.92$ (N.A.) $\Rightarrow n \geq 158.35$ $\Rightarrow n \geq 159$ Thus, smallest $n = 159$.	Badly done although a similar question had been done in the assignment. The main mistake was treating the given value 3.2 as population standard deviation instead of sample standard deviation, and hence wrongly writing $s^2 = \frac{3.2^2}{n}$. Surprisingly, majority of the students also wrongly wrote $\overline{X} \sim \text{N}\left(9.5, \frac{3.2^2}{n}\right)$ where they thought $E\left(\overline{X}\right) = 9.5$, although such an occurrence was less common in part (ii). Many students only worked with the left tail although it's a 2-tailed test. This might result from observing that sample mean 9.5 is less than population mean 10, but an appropriate explanation for only considering the left tail should be stated in the solution in order not to compromise the evidence of understanding the rejection criteria of a 2-tailed test. Some fumbled with the inequality sign because $\overline{x} - \mu = 9.5 - 10 < 0$ and working with the right tail will result in no real solution. However, this group of students 'forced' their way through by writing statements like '$\frac{-0.5}{\sqrt{\frac{10.24}{n-1}}} \geq 1.96 \Rightarrow \left(\frac{-0.5}{\sqrt{\frac{10.24}{n-1}}}\right)^2 \leq (1.96)^2$' to remove the negative sign, without realising that LHS is a negative value that cannot be greater than RHS which is a positive value and hence no solution. Thus, the idea of taking squares on both sides of an inequality is not well-understood.
