West Spring Secondary School
PRELIMINARY EXAMINATION 2022

Additional Mathematics **4049 / 02**

Paper 2

Secondary 4 Express / 5 Normal (Academic)

Name ______________________ (   ) **Date** 13 SEP 2022

Class ____________ **Duration** 2 h 15 min

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class in the spaces at the top of this page.

Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

FOR EXAMINER'S USE
/ 90

This document consists of **25** printed pages and **1** blank page.

Setter(s) **Mr Soh Hong Wei**

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1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\cdots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\text{Area of } \Delta = \frac{1}{2}bc \sin A$$

1 The monthly profit, \$$y$, of a bathroom supply company can be approximated by $y=-\frac{1}{10}x^2+120x-5000$, where x is the price of the product sold.

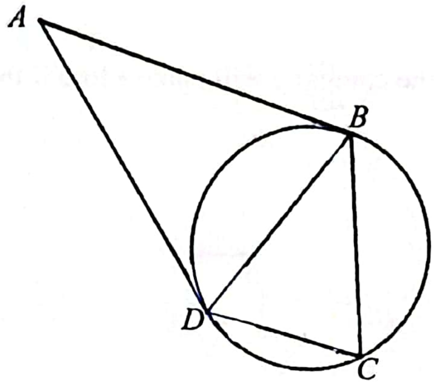
(a) Explain why the company will make a loss if they price their product to sell at \$40. [1]

(b) Express $-\frac{1}{10}x^2+120x-5000$ in the form $a(x-h)^2+k$, where a, h and k are constants to be determined. [3]

(c) How should the company price their product to make a maximum profit?

State the approximate profit at this price. [1]

2 **This diagram is not drawn to scale. Solution by accurate measurement will not be accepted.**



In the diagram, *AB* and *AD* are tangents to the circle and *DB* is the angle bisector of angle *ADC*.

(a) Prove that triangles *ABD* and *BCD* are similar. [3]

(b) Show that $BC = BD$. [2]

3 Factorise $f(x) = x^3 - x^2 - 5x - 3$ completely. [3]

Hence, solve the equation

(a) $f(x)=0$, [2]

(b) $f(x)=(x+1)(x-3)$. [1]

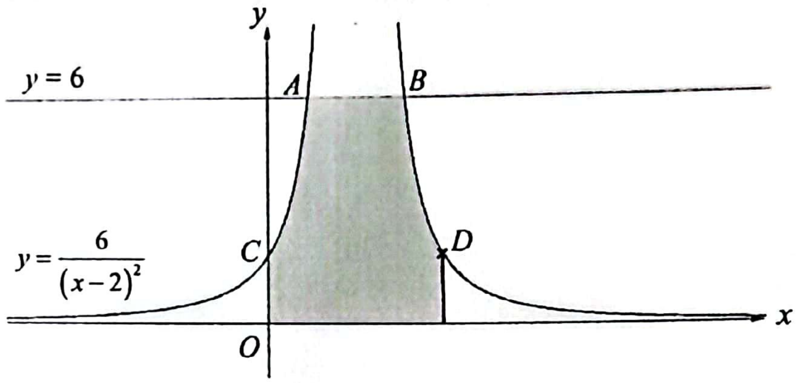
4 (a) Show that $2\cos^2 x - 4\sin^2 x$ can be written as $a\cos 2x + b$, where a and b are integers. [2]

Hence

(b) state the period and amplitude of $2\cos^2 x - 4\sin^2 x$, [2]

(c) sketch the graph of $y = 2\cos^2 x - 4\sin^2 x$ for $0 \leq x \leq 2\pi$ radians. [3]

5



The diagram shows the curve $y = \dfrac{6}{(x-2)^2}$ intersecting the y-axis at point C and intersecting the line $y = 6$ at the points A and B. The point D lies on the curve and has the same y-coordinate as point C.

(a) Find the coordinates of A, B, C and D. [4]

(b) Find the area of the shaded region. [4]

6 (a) Given that for all values of x, $2x^3 - x^2 - 13x + 2 = (Ax+1)(x+B)(x+2) + C$, find the values of A, B and C. [4]

(b) Find the value of k given that the expression $3x^3 - x + k$, where k is a constant, leaves a remainder of 3 when divided by $x+1$. [2]

(c) If $x-2$ is a factor of $x^3 + (p+1)x^2 - p^2$ but not a factor of $x^3 + px^2 - 8x - 16$, find the value of p. [3]

7 The equation of a curve is $y = 2x^2 + kx - 5$.

(a) Show that the curve will intersect the x-axis at two real and distinct points for all real values of k. [3]

For **parts (b)** and **(c)**, let $k = 3$.

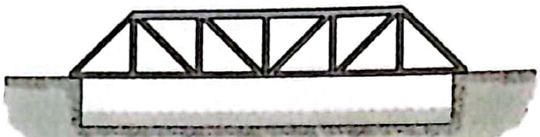
(b) The line $y = mx - m$ is a tangent to the curve at the point P.

(i) Find the value of the constant m. [3]

(ii) Find the coordinates of P. [2]

(c) Find the set of values of x for which the curve $y > 0$. Illustrate your solution on a number line. [3]

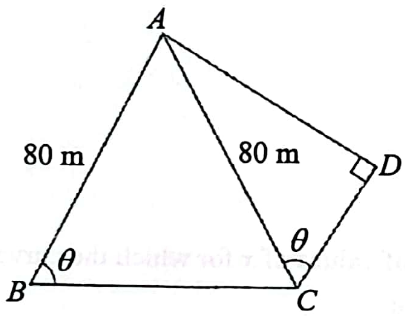
8 Truss bridges are engineered and built since the 18th century to allow more accessibility across places divided by rivers or valleys.



Picture taken from https://www.britannica.com/technology/truss-

The diagram below shows a component of a particular truss bridge design.

Angle $ABC =$ angle $ACD = \theta$, angle $ADC = 90°$ and $AB = AC = 80\,\text{m}$.



(a) Show that the total length, L metres, of material required to build the above bridge component is given by $L = 160 + 240\cos\theta + 80\sin\theta$. [2]

(b) Express L in the form $160 + R\cos(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$ [3]

(c) State the maximum value of L. [1]

(d) Find the value of θ when $L = 360$ m. [2]

9 A particle moves in a straight line so that its distance, s m, from a fixed point A on the line is given by $s = 2t^2 - 4t + 9$, for $t \leq 3$, where t is the time in seconds after passing through a point B on the line. Find

(a) the distance AB, [1]

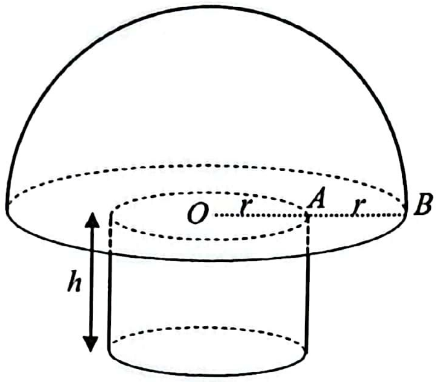
(b) the distance from A of the particle when it is instantaneously at rest, [3]

(c) the total distance travelled by the particle in the period $t = 0$ to $t = 3$. [2]

At $t = 3$, the acceleration of the particle is changed to $(t - 8)$ m/s^2 and the instantaneous velocity remains unchanged.

(d) Find the next value of t at which the particle comes to instantaneous rest. [3]

10 [Surface area of a sphere = $4\pi r^2$; Volume of a sphere = $\frac{4}{3}\pi r^3$]



The diagram shows a solid mushroom figurine made from a hemisphere and a right circular cylinder. The radii of the cylinder and the hemisphere are OA and OB respectively, such that $OA = AB = r$ cm. The cylinder has a height of h cm.

The figurine is to be made by using 80π cm^3 of clay.

(a) Express h in terms of r. [2]

(b) Show that the total surface area, A cm^2, of the figurine is given by $A = \dfrac{160\pi}{r} + \dfrac{4\pi r^2}{3}$. [2]

(c) Given that r can vary, find the value of r for which A is stationary. [3]

The surface of the figurine is to be painted.

(d) Justify why the value of r found in **part (c)** will make the figurine the most cost-efficient to paint. [3]

11 A circle C passes through the points $A(1, 1)$ and $B(9, 5)$.
Its centre P lies on the line $y = x + 1$.

(a) Showing all your working, find the equation of the circle C. [6]

(b) Show that the x-axis is a tangent to the circle. [2]

(c) The tangent to the circle at the point $D(8, 8)$ cuts the x-axis at the point R. Find the area of the triangle ODR. [4]

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End of Paper 2