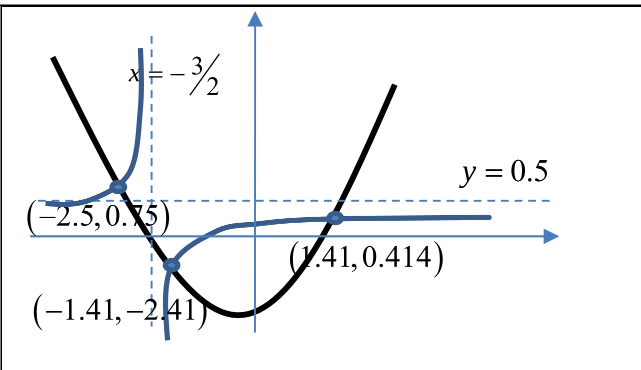
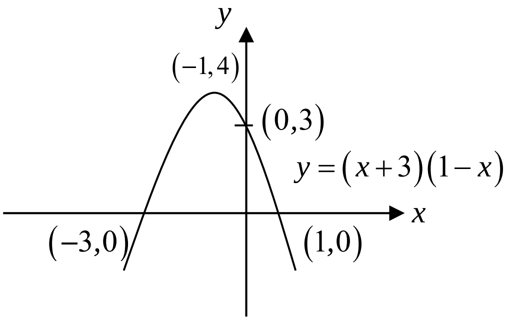
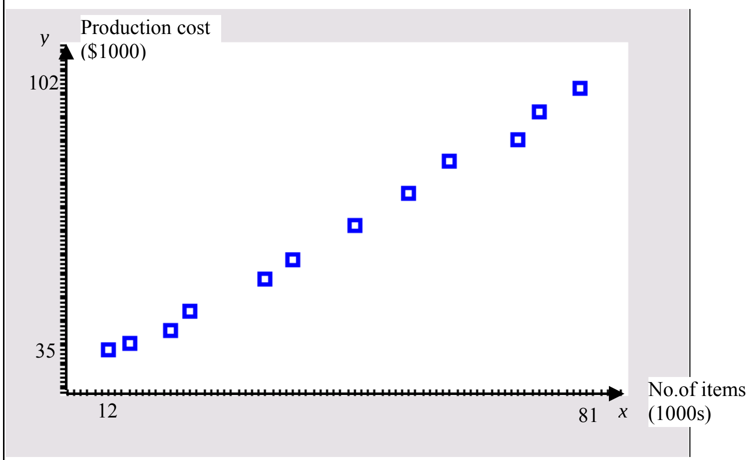
2017 C2 H1 Prelim

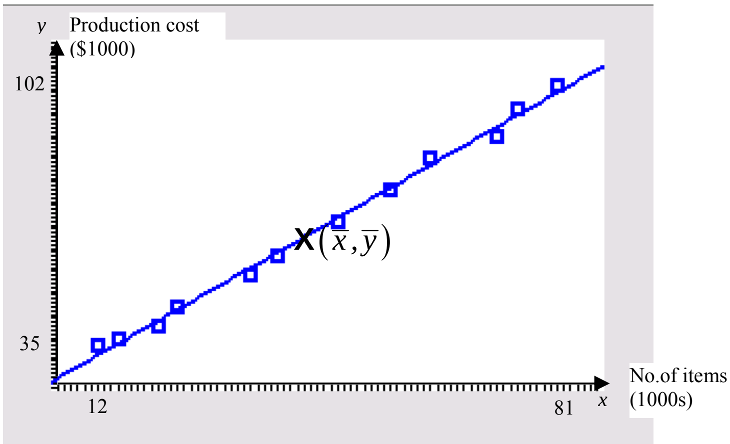
	Solutions
1	Let x, y and z be the original selling price per pack of organic quinoa, organic feed eggs and chia seeds in dollars. $3x+0.85y+2z=72.28$ --- (1) $2x+2(0.85)y+5z=93.85$ --- (2) $6x+3(0.85)y+3z=145.43$ --- (3) $x=\$14.90, y=\$11.49, z=\$8.90$
2(a)	$\frac{\mathrm{d}}{\mathrm{d}x}(x+\ln x)^2=2(x+\ln x)\left(1+\frac{1}{x}\right)$ $=\frac{2}{x}(x+\ln x)(x+1)$
2(b)	$\frac{\mathrm{d}}{\mathrm{d}x}\mathrm{e}^{\left(\frac{1}{\sqrt{2-x}}\right)}=\frac{1}{2(2-x)^{\frac{3}{2}}}\mathrm{e}^{\left(\frac{1}{\sqrt{2-x}}\right)}$
3	 $-2.5\le x<-1.41$ or $-1.41\le x\le 1.41$
4(i)	Using long division $\frac{2x^2+1}{x-4}=2x+8+\frac{33}{x-4}$ OR $\frac{2x^2+1}{x-4}=\frac{(Ax+B)(x-4)+C}{x-4}$ $2x^2+1=(Ax+B)(x-4)+C$ Compare coefficient: $2=A, B=8, C=33$
4(ii)	$\int_5^6 \frac{2x^2+1}{x-4}\,\mathrm{d}x=\int_5^6 2x+8+\frac{33}{x-4}\,\mathrm{d}x$ $=\left[x^2+8x+33\ln\lvert x-4\rvert\right]_5^6$ $=84+33\ln 2-65$ $=19+33\ln 2$

5(i)	
5(ii)	$y=(x+3)(1-x)$ $\quad =x+3-x^2-3x$ $\quad = -x^2-2x+3$ $x+k=-x^2-2x+3$ $x^2+3x+k-3=0$ $b^2-4ac>0$ $3^2-4(k-3)>0$ $9-4k+12>0$ $4k<21$ $k<5.25$ $\{k\in\mathbb{R} : k<5.25\}$
5(iii)	$y=(x+3)(1-x)=x+5$ $-x^2-2x+3=x+5$ $x^2+3x+2=0$ $(x+2)(x+1)=0$ $x=-2, x=-1$ $\text{area} = \int_{-2}^{-1}\left(-x^2-2x+3-x-5\right)\mathrm{d}x$ $=\int_{-2}^{-1}\left(-x^2-3x-2\right)\mathrm{d}x$ $=\left[-\frac{x^3}{3}-\frac{3x^2}{2}-2x\right]_{-2}^{-1}$ $=\left(\frac{1}{3}-\frac{3}{2}+2\right)-\left(\frac{8}{3}-\frac{12}{2}+4\right)$ $=\frac{1}{6}\text{units}^2$ Or

	$\text{area} = \int_{-2}^{-1} (-x^2 - 2x + 3)\text{d}x - \frac{1}{2} \times (3+4) \times 1$ $= \left[-\frac{x^3}{3} - \frac{2x^2}{2} + 3x \right]_{-2}^{-1} - \frac{7}{2}$ $= \frac{1}{6}\text{units}^2$
6(i)	$\text{Area} = 3\left(\frac{1}{2}\right)x^2 \sin\left(\frac{\pi}{3}\right) + 2xy = \frac{3\sqrt{3}}{4}x^2 + 2xy$ $\frac{3\sqrt{3}}{4}x^2 + 2xy = 15$ $2xy = 15 - \frac{3\sqrt{3}}{4}x^2$ $2y = \frac{15}{x} - \frac{3\sqrt{3}}{4}x$ Let P be the perimeter. $P = 5x + 2y$ $= 5x + \frac{15}{x} - \frac{3\sqrt{3}}{4}x$ $\frac{\text{d}P}{\text{d}x} = 5 - \frac{3\sqrt{3}}{4} - \frac{15}{x^2}$ For cost to be minimum, perimeter has to be minimum. $\frac{\text{d}P}{\text{d}x} = 0 \Rightarrow 5 - \frac{3\sqrt{3}}{4} - \frac{15}{x^2} = 0$ $\frac{15}{x^2} = 3.700962 \Rightarrow x^2 = 4.053$ $x = 2.0132$ $\therefore x = 2.01, \quad y = 2.42$ $\frac{\text{d}^2P}{\text{d}x^2} = \frac{30}{x^3} > 0$ Therefore P is a minimum.

6(ii)	Let N be the perimeter of the fence in integral value Cost from Company $A = 90N$ Cost from Company $B = 10(95) + 84(N - 10)$ $= 110 + 84N$ $110 + 84N < 90N$ $6N > 110$ $N > 18.3 \Rightarrow N > 18$
6(iii)	When $x = 2.0132$, $y = 2.4177$ $P = 14.902$ Since $14.902 < 18$, therefore it is cheaper to choose Company A.
7(a)	No. of words that can be formed $= 7! - 1$ $= 5039$
(b)	No. of words if 3 vowels are altogether $= 3! \times 5!$ $= 720$ No. of words $= 5040 - 720$ $= 4320$
(c)	No. of words $= {}^4C_2 \times 2! \times 5!$ $= 1440$
8(i)	Let B be the event that the lunch box is produced by production line B. Let F be the event that the lunch box is faulty. $\mathrm{P}(F \cap B) = \mathrm{P}(B) \times \mathrm{P}(F\|B)$ $= \mathrm{P}(F) \times \mathrm{P}(B\|F)$ (*) $\mathrm{P}(B)(0.03) = 0.05(0.4)$ $\mathrm{P}(B) = \frac{2}{3}$
8(ii)	Let A be the event that the lunch box is produced by production line A. $\mathrm{P}(A \cap F) = 0.05 \times 0.6 = 0.03$ $\mathrm{P}(F\| A) = \frac{0.03}{\frac{1}{3}} = 0.09$
8(iii)	$\mathrm{P}(B \cap F') = \frac{2}{3} - 0.02 = 0.64667$ $\mathrm{P}(B\|\text{ only 1 faulty}) = \frac{0.64667 \times 0.02 \times 2}{0.95 \times 0.05 \times 2}$ $= 0.272$
9(i)	Let X denote the number of diners, out of 20, who choose a burger. $X \sim \mathrm{B}(20, 0.05)$ $\mathrm{P}(X > 3) = 1 - \mathrm{P}(X \le 3) = 0.0159$

9(ii)	$P(X < n) > 0.9$ --- (1) $P(X \le n-1) > 0.9$ --- (2) Using GC, $P(X \le 1) > 0.736$ $P(X \le 2) > 0.925$ $\therefore$ smallest value of n is 3.
9(iii)	Let Y denote the number of diners, out of 20, buying a drink in the cafe. $Y \sim B(20, p)$ $20p(1-p) = 4.55$ --- (1) $p^2 - p + 0.2275 = 0$ $p = 0.35$ or 0.65 Since $p > 0.5$, $p = 0.65$
10(i)	
10(ii)	Product moment correlation coefficient $r = 0.998$ which indicates a strong positive linear correlation between the number of items produced per month by the company together with the total cost of production
10(iii)	$\bar{x} = 45, \quad \bar{y} = 65$

10(iv)	$y = 0.98x + 20.99$ - 0.98 gives the rate at which the production costs are increasing i.e. for every additional item produced, the production cost increases by \$0.98 - OR For every increase in 1000 items produced, there is an increase in the total production cost by 980 dollars. - \$20,990 is the fixed cost of production. 
10(v)	$y = 0.9781x + 20.985$ When $x = 70$, $y = 0.9781 \times 70 + 20.985 = 89.452$, an estimate for the production cost of 70 thousand items is \$89452. Since $x = 70$ lies within the data range $12 \le x \le 81$ and $\lvert r \rvert$ is close to 1, therefore this estimate is reliable.
10(vi)	If x items(1000s) are produced, Total income = $\$2.20 \times x$ The total cost for producing x items is $y = 0.9781x + 20.985$ If there is no loss, $2.20x \ge 0.9781x + 20.985$ $1.2219x \ge 20.985$ $x \ge 17.17407$ Therefore the min number of items to be produced per month is 17175 items.
11(i)	Let X be the weight of a packet of Calhwa potato chips and μ denotes the population mean weight of a packet of potato chips in grams $H_0: \mu = 84$ $H_1: \mu < 84$

11(ii)	At 1% level of significance, under H_0, since $n = 100$ is large, by Central limit theorem, $\overline{X} \sim N\left(84, \frac{5^2}{100}\right)$ approximately Test statistic $Z = \frac{\overline{X} - \mu}{\sqrt{\frac{\sigma^2}{n}}} \sim N(0,1)$ $p = 0.0139 > 0.01$, we do not reject H_0 and conclude that at the 1% level of significance, there is insufficient evidence to say that the average weight of a packet of potato chips is less than 84 grams
11(iii)	When the level of significance is set at 1%, there is 1% chance that we wrongly conclude the mean weight of a packet of potato chips is less than 84 grams when in fact the mean weight of a packet of potato chips is at least 84 grams.
11(iv)	Since the sample size $n = 100$ is sufficiently large, the **sample mean** weight of the packets of potato chips will be normally distributed by the Central Limit Theorem. Therefore it is not necessary to assume the weight of packets of potato chips follow a normal distribution.
11(v)	$H_0: \mu = 84$ $H_1: \mu \neq 84$ Level of significance: 5% Under H_0, since n =100 is large, by Central limit theorem, $\overline{X} \sim N\left(84, \frac{5^2}{100}\right)$ approximately. Test statistic $\lvert Z \rvert = \frac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \sim N(0,1)$ Rejection region: Reject H_0 if $z \le -1.95996$ or $z \ge 1.95996$ Since there is sufficient evidence, at 5% level significance to conclude that the average weight of the potato chip has changed, $\frac{t-84}{\frac{5}{\sqrt{100}}} \le -1.95996$ or $\frac{t-84}{\frac{5}{\sqrt{100}}} \ge 1.95996$ $\Rightarrow 2t - 84 \le -1.95996$ or $2t - 84 \ge 1.95996$ $t \le 83.020$ or $t \ge 84.979$ Range of t : $t \le 83.0$ or $t \ge 85.0$

12(a)	$\mathrm{P}\left(X<15\right)=0.841\Rightarrow \mathrm{P}\left(Z<\dfrac{15-\mu}{\sigma}\right)=0.841$ --(1) $\Rightarrow \dfrac{15-\mu}{\sigma}=0.99858$ $\mathrm{P}\left(9<X<15\right)=0.682\Rightarrow \mathrm{P}\left(\dfrac{9-\mu}{\sigma}<Z<\dfrac{15-\mu}{\sigma}\right)=0.682$ $\Rightarrow \mathrm{P}\left(Z<\dfrac{9-\mu}{\sigma}\right)=0.841-0.682=0.159$ ---(2) $\Rightarrow \dfrac{9-\mu}{\sigma}=-0.99858$ --- (2) Solving (1) & (2), $\mu=12$ and $\sigma=3.00$ **Alternatively** By observation , $\mu=12$. $\mathrm{P}\left(X<15\right)=0.841$ Using GC, $\sigma=3.00$
12(bi)	$S \sim \mathrm{N}\left(\mu,3^2\right)$ $\mathrm{P}\left(\lvert S-\mu\rvert>2.5\right)$ --- (1) $=\mathrm{P}\left(\left\lvert\dfrac{S-\mu}{3}\right\rvert>\dfrac{2.5}{3}\right)=\mathrm{P}\left(\lvert Z\rvert>\dfrac{2.5}{3}\right)=2\times\mathrm{P}\left(Z>\dfrac{2.5}{3}\right)$ --- (2) $=0.405$
b(ii)	$\mathrm{P}\left(S>11\right)=0.75$ --- (1) $\mathrm{P}\left(Z\le\dfrac{11-\mu}{3}\right)=0.25$ $\dfrac{11-\mu}{3}=-0.67449$, $\mu=13.0$ °C
b(iii)	$\mathrm{P}\left(17.5<T<23\right)=0.786$
b(iv)	Find $\mathrm{P}\left(0\le T-\dfrac{S_1+S_2}{2}<10\right)$ Let $W=T-\dfrac{S_1+S_2}{2}$ $\mathrm{E}\left(W\right)=8$ $\mathrm{Var}\left(W\right)=9.34$ $W \sim \mathrm{N}\left(8,9.34\right)$ $\mathrm{P}\left(0\le T-\dfrac{S_1+S_2}{2}<10\right)=0.73915=0.739\left(3\text{s.f}\right)$
(v)	Assume that the minimum and maximum temperatures are independent of each other. It is unrealistic because the weather, e.g. wind direction, rainy weather, etc, will affect both the minimum and maximum temperature of the city.