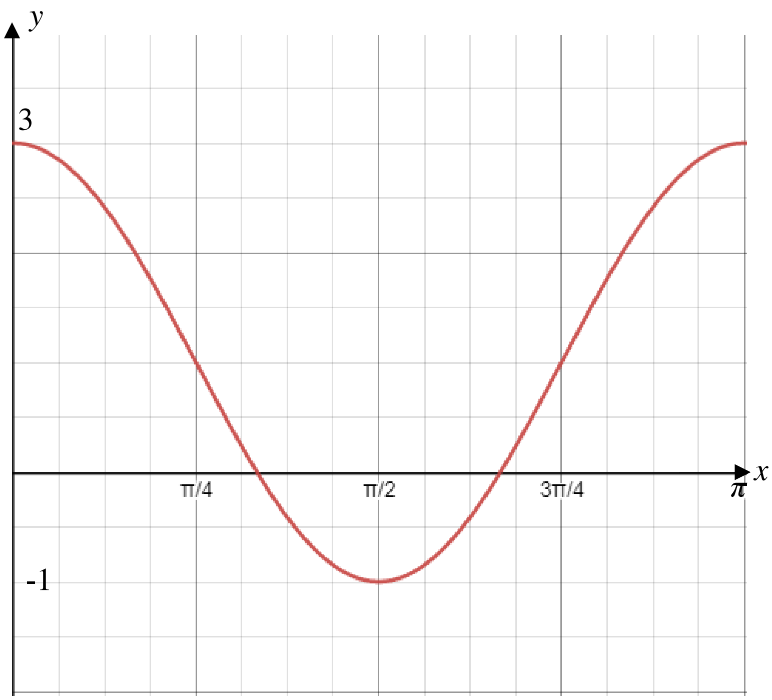
2021 AM 4E Prelim P1 Marking Scheme

Solutions:		
1i	$\sqrt{1-p^2}$ obtained $-\sqrt{1-p^2}$	
1ii	$\dfrac{-\sqrt{1-p^2}}{p}$	
2	$x^2+12x+9>4x+k$ $x^2+12x-4x+9-k>0$ $x^2+8x+9-k>0$ For the equation to be always positive, no real roots $b^2-4ac<0$ $(8)^2-4(1)(9-k)<0$ $64-36+4k<0$ $28+4k<0$ $4k<-28$ $k<-7$	
3	$\dfrac{dy}{dx}=\dfrac{(2x+1)6e^{2x}-6e^{2x}}{(2x+1)^2}$ $=\dfrac{12xe^{2x}+6e^{2x}-6e^{2x}}{(2x+1)^2}$ $=\dfrac{12xe^{2x}}{(2x+1)^2}$ $=\dfrac{4x3e^{2x}}{(2x+1)^2}$ $=\dfrac{4xy}{2x+1}$ $\therefore k=4$	
4i	$A=\pi r^2$ $\dfrac{dA}{dr}=2\pi r$ $\dfrac{dA}{dt}=2\pi r\times\dfrac{dr}{dt}$ $=2\pi\times5\times3$ $=30\pi\text{ cm}^2\text{ s}^{-1}$	
4ii	$\pi r^2=4\pi$ $r=2$	

	$\frac{dA}{dt} = 2\pi r \times \frac{dr}{dt}$ $= 2\pi \times 2 \times 3$ $= 12\pi \text{ cm}^2 \text{ s}^{-1}$	
5i	Amplitude = 2 Period = π	
5ii	Minimum value = −1 Maximum value = 3	
5iii		
6i	When $P = 3000$, $t = 0$, $3000 = 5000 + Ae^{k(0)}$ $3000 - 5000 = A$ $A = -2000$ (shown)	
6ii	$P = 5000 - 2000e^{kt}$ When $P = 3800$, $t = 10$, $3800 = 5000 - 2000e^{k(10)}$ $e^{10k} = \frac{1200}{2000}$ $\ln e^{10k} = \ln\left(\frac{3}{5}\right)$ $k = \ln\left(\frac{3}{5}\right) \div 10$ $k = -0.0511$ (3 sig. fig.)	

6iii	$P = 5000 - 2000e^{-0.0511t}$ Since $e^{-0.0511t} > 0$, $-2000e^{-0.0511t} < 0$. $5000 - 2000e^{-0.0511t} < 5000$ Thus the number of fishes can never reach 5000.	
7i	Let $\angle BAE = x$ $\angle ACB = x$ (tangent-chord theorem) $\angle CAD = x$ (alt. $\angle$s, $CE \,//\, DA$) Thus $\angle BAE = \angle CAD$	
7ii	In triangles *BAE* and *DAC*, $\angle BAE = \angle CAD$ [from part (i)] $\angle CDA = 180° - \angle ABC$ (opp. $\angle$s of cyclic quad) $\angle CDA = 180° - (180° - \angle ABE)$ ($\angle s$ on a str. line) $= \angle ABE$ Hence, triangles *BAE* and *DAC* are similar. (AA similarity test): Encouraged to write. Not necessary to penalize)	
7iii	$\angle ACB = x$ [from part (i)] $\angle BAE = \angle BEA$ (base $\angle s$ of isos. triangle) $= \angle DCA$ (ΔBAE similar to ΔDAC] $= x$ $\because \angle ACB = \angle DCA = x$, the line *AC* bisects the angle *BCD*.	
8	$(1+ax)^6 = 1 + 6ax + 15a^2x^2 + \ldots$ $(1+bx)(1+ax)^6 = (1+bx)(1+6ax+15a^2x^2+\ldots)$ $= 1 + (6a+b)x + (15a^2 + 6ab)x^2 + \ldots$ $6a + b = 0$	

	$15a^2+6ab=-\frac{21}{4}$ $5a^2+2ab=-\frac{7}{4}$ Sub $b=-6a$ into $5a^2+2ab=-\frac{7}{4}$ $5a^2-12a^2=-\frac{7}{4}$ $-7a^2=-\frac{7}{4}$ $a=\pm\frac{1}{2}$ When $a=\frac{1}{2}, b=-3$ (reject) When $a=-\frac{1}{2}, b=3$	
9a	$(4^x)(8^y)=1$ $2^{2x+3y}=2^0$ $2x+3y=0$ $3y=-2x \quad -(1)$ $125^y\div\left(\sqrt[3]{5}\right)^x=\frac{1}{\sqrt{5}}$ $5^{3y-\frac{x}{3}}=5^{-\frac{1}{2}}$ $3y-\frac{1}{3}x=-\frac{1}{2} \quad -(2)$ Sub. (1) into (2), $2x+\frac{1}{3}x-\frac{1}{2}=0$ $2\frac{1}{3}x=\frac{1}{2}$ $x=\frac{3}{14}$ $\therefore y=-\frac{1}{7}$	
9b	$V=\pi r^2h$ $\left(12+4\sqrt{2}\right)\pi=\pi\left(\sqrt{2}-1\right)^2h$	

$$h=\frac{12+4\sqrt{2}}{\left(\sqrt{2}-1\right)^2}$$

$$h=\frac{12+4\sqrt{2}}{3-2\sqrt{2}}$$

$$h=\frac{\left(12+4\sqrt{2}\right)\left(3+2\sqrt{2}\right)}{\left(3-2\sqrt{2}\right)\left(3+2\sqrt{2}\right)}$$

$$h=\frac{36+24\sqrt{2}+12\sqrt{2}+16}{9-8}$$

$$h=52+36\sqrt{2}$$

10i

$$y=x(2x-4)^3$$

$$\frac{\mathrm{d}y}{\mathrm{d}x}=(x)3(2x-4)^2(2)+(2x-4)^3(1)$$

$$6x(2x-4)^2+(2x-4)^3=0$$

$$(2x-4)^2(8x-4)=0$$

$$(2x-4)^2=0 \quad \text{or} \quad (8x-4)=0$$

$$2x=4 \qquad 8x=4$$

$$x=2 \qquad x=\frac{1}{2}$$

$$y=0 \ \text{ or } \ y=13\frac{1}{2}$$

Coordinates: $(2,0)$ and $\left(\frac{1}{2},-13\frac{1}{2}\right)$

10ii

Using First Derivative Test,

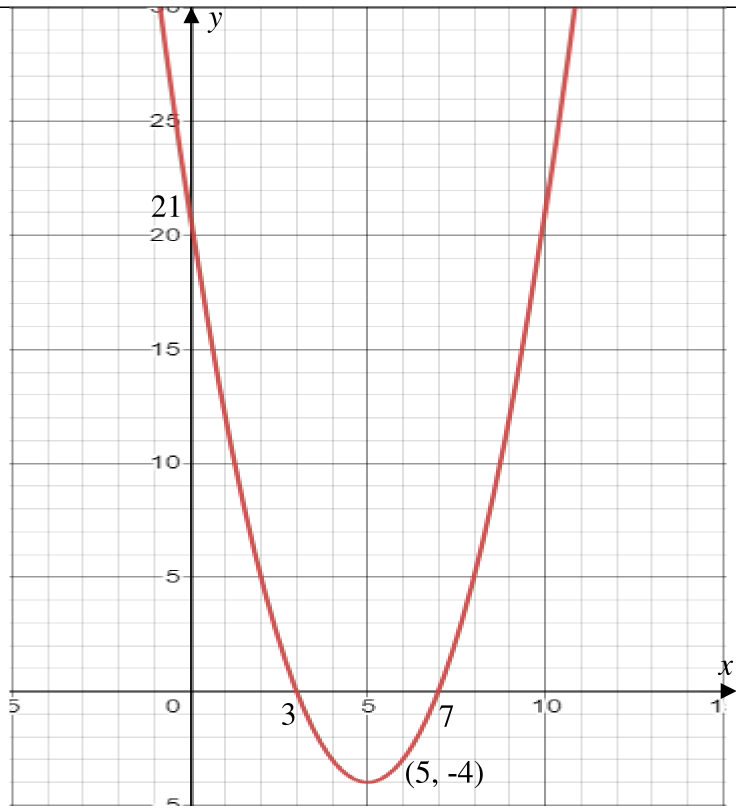
x	2^-	2	2^+
$\frac{\mathrm{d}y}{\mathrm{d}x}$	+	0	+

(2, 0) is a point of inflexion.

x	$\frac{1}{2}^-$	$\frac{1}{2}$	$\frac{1}{2}^+$

	<table><tr><td>$\frac{dy}{dx}$</td><td>$-$</td><td>0</td><td>+</td></tr></table> $\left(\frac{1}{2}, -13\frac{1}{2}\right)$ is a minimum point	
11i	$v = \frac{ds}{dt}$ $= 3t^2 + 10t - 2$	
11ii	$a = 6t + 4$ $v = \int a \, dt$ $= 3t^2 + 4t + c$ When $t = 1$, $v = 4$. $c = -3$ $\therefore v = 3t^2 + 4t - 3$ $s = \int v \, dt$ $= t^3 + 2t^2 - 3t + c$ When $t = 0$, $s = 6$ $c = 6$ $\therefore s = t^3 + 2t^2 - 3t + 6$	
11iii	$s_Q = s_P$ $t^3 + 2t^2 - 3t + 6 = t^3 + 5t^2 - 2t + 4$ $3t^2 + t - 2 = 0$ $(3t - 2)(t + 1) = 0$ $t = \frac{2}{3}$ or $t = -1$ (reject) At $t = \frac{2}{3}$, v_P = positive v_Q = positive Therefore, *P* and *Q* are travelling in ***<u>same</u>*** direction	
12ai	$x^2 - 10x + 21$	

	$= x^2 - 10x + (-5)^2 - (-5)^2 + 21$ $= (x-5)^2 - 4$ Since $(x-5)^2 \geq 0$, $(x-5)^2 - 4 \geq -4$ Thus $x^2 - 10x + 21$ can never be smaller than -4	
12aii	$(5, -4)$ Minimum	
12aiii	$x^2 - 10x + 21 = 0$ $(x-7)(x-3) = 0$ $x = 7$ or $x = 3$	

12aiv		
12b	$6x^2 + \frac{x}{a} - \frac{1}{b^2} = 0$ Discriminant $= \left(\frac{1}{a}\right)^2 - 4(6)\left(-\frac{1}{b^2}\right)$ $= \frac{1}{a^2} + \frac{24}{b^2}$ Since $\frac{1}{a^2} > 0$ and $\frac{24}{b^2} > 0$, Thus, $\frac{1}{a^2} + \frac{24}{b^2} \neq 0$ and there are no values of a and b for which the equation has equal roots.	
13i	Gradient of $PQ = \frac{4-(-2)}{1-(-1)}$ $= 3$	

	Gradient of perpendicular bisector of $PQ = -\frac{1}{3}$ Midpoint $=\left(\frac{-1+1}{2},\frac{-2+4}{2}\right)$ $=(0,1)$ $y=-\frac{1}{3}(x)+c$ $1=-\frac{1}{3}(0)+c$ $c=1$ Can infer from previous working that y-intercept is 1 Equation of perpendicular bisector of PQ is $y=-\frac{1}{3}x+1$.	
13ii	At x-axis, $y = 0$. $0=-\frac{1}{3}x+1$ $\frac{1}{3}x=1$ $x=3$ Coordinates of R are (3, 0)	
13iii	Since PS is parallel to x-axis and P is the point $(-1,-2)$, y-coordinate of S is −2. $-2=3x-9$ $3x=7$ $x=\frac{7}{3}$ Coordinates of S are $\left(\frac{7}{3},-2\right)$.	
13iv	Area of $PQRS = \frac{1}{2}\begin{vmatrix} 3 & 1 & -1 & \frac{7}{3} & 3 \\ 0 & 4 & -2 & -2 & 0 \end{vmatrix}$ $=\frac{40}{3}$ units^2	